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PREFACE

This volume of Quarterly TRANSACTIONS is Part III of Volume 50, issued in September 1931. It contains the papers and discussions presented at the Middle Eastern District Meeting of the A. I. E. E. Pittsburgh, Pa., March 11-13, 1931 and at the North Eastern District Meeting, Rochester, N. Y. April 29-May 2, 1931, and in addition, two contributions, "An Electric Analog of Friction" by H. H. Skilling and "Effects of Electric Shock—II" by W. B. Kouwenhoven and O. R. Langworthy.

An index of authors appears in the back of the volume. The complete subject and authors' annual indexes of Volume 50 will appear in the December Quarterly.

Burn-Off Characteristics of A-C. Low-Voltage Network Cables

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Synopsis.—This paper describes tests made to determine the characteristics of the clearance of faults in several types of a-c. low-voltage network copper-conductor cables installed in accordance with present standards of underground construction, and includes a discussion of test results together with other relative data studied. The investigation included the burning clear of various types of faults in buried non-magnetic sheath cables, and non-metallic sheath and lead-covered cables for duct installation. A comparison is made of the burn-off characteristics of the several cable constructions and their installations.

The amount of short-circuit current and length of time required to clear the different types of faults in No. 4/0, 350,000- and 500,000-cir. mil cables have been determined by test and by calculations in some instances. Conclusive data were not obtained to clear up all present conflicting opinions related to the burn-off characteristics of a-c. low-voltage network cables. However, a study of the subject matter would seem to establish at least a partial basis in regard to burn-off characteristics for determining the conductor size of low-voltage network mains and the spacing of network transformers. Desirable developments for a-c. network cables indicated by the results of this investigation are emphasized.

* * * * *

I. Introduction

OPERATION of underground, low-voltage network systems, designed to supply continuous electric service, has proved the value of data derived from a thorough investigation into the inherent characteristics of a given design to clear faults. The maximum reduction of service outages, prevention of manhole fires, explosions and other system disturbances due to the occurrence of faults are therefore receiving increased consideration of operating and design engineers.

The clearing of faults in the cables of a network depends upon; (1) the relative rates of generation and dissipation of heat in the fault and in the cable conductors joining the fault; (2) physical conditions at the fault location; and (3) the effect of gases generated if an arcing condition is established.

The heat generated in the fault is a function of the contact resistance and the square of the short-circuit current flowing into the fault. The heat is dissipated principally by conduction and radiation, previous to the existence of an arc. The clearance of the fault at its initial location will depend usually upon its inability to dissipate the heat generated, provided that the relative amounts of heat generated and dissipated in the conductors joining the fault are such that the latter conductors will not fail first. The thermal capacity of the fault, relative to the heat generated, is in this case less than that of the cable conductors. In this class, a high fault resistance is associated with a low rate of heat dissipation. As will be seen from the discussion in this paper, particular consideration has been given to this class of faults, because most faults occurring in practise first exist as a fault of this class.

If the relative rates of generation and dissipation of heat in the cable conductors joining the fault are such that the fusing temperature is attained in the conductors before the fault itself burns clear, clearing will take place at a location other than the initial location of the fault. Relative to the amounts of heat generated in the fault and in the conductors, the thermal capacity of the fault is greater than that of the cable conductors.

The cables used generally in the past for low-voltage underground network systems have been rubber or paper insulated and lead covered. Operating experience with low-voltage (115/199, 120/208 and 115/230 volts) underground systems of standard construction, and previous tests, prove that faults between the lead sheath and a conductor have cleared instantly or within a few seconds with little injury to the faulty cable and generally no serious injury, if any, to adjacent cables. Previous tests have shown also that a copper-to-copper fault, in network systems of phase-to-phase voltages of the order of 208 volts, formed by loosely wrapping the conductors with binding wire, or by a spike as large as ⅜ of an inch in diameter, being driven into stranded 500,000-cir. mil conductors will usually clear in a short interval of time (less than 30 seconds) with a total minimum value of current of approximately 2,500 amperes or greater (within certain current limits depending on cable construction). The quick clearance of the faults depends to a great extent on the relative positions of the cables in the duct at the fault location. It seems questionable at present as to whether the greater number of explosions and manhole fires occurring in network systems have been due to ignition of sewer and escaped illuminating gases or gases generated from cable materials subjected to short-circuit conditions. One preliminary study[2] has indicated that most explosions occurring during a five-month period of careful observation were probably due to the volatilization of cable or joint insulation and rapid combustion due to the intense heat of the electric arc.

The use of cables suitable for burying in the ground

1. Both of the New York and Queens Electric Light and Power Company.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

2. "Study on Failures in A-C. Low-Voltage Network Cables," by J. M. Comly, Brooklyn Edison Company, Inc., and J. A. McHugh, The New York Edison Company.

without conduit for low-voltage network distribution is a new field of distribution engineering in this country. The burn-off characteristics of this cable, made up in several types of construction and installed under several proposed conditions, were unknown previous to the making of tests described herein. Before the installation of any buried cables in a network system, even as an experimental investigation, it was desirable that tests be made to determine the clearing of faults which might occur at rare intervals in such a system.

II. Scope

An attempt has been made to determine the burn-off characteristics of cables installed under actual service conditions. A total of 86 tests was made to check some of the results of previous investigations, and in addition, to determine the advantages or disadvantages of new cable constructions installed under several conditions.

Copper conductor cables with various insulations and coverings were used for test. It is recognized that other pure metals or metal alloys may have better burn-off characteristics than copper. Cables with conductors of metal other than copper constitute a field for a separate investigation, and are not further discussed in this paper.

It has been the purpose of this paper to study the burn-off characteristics of the several types of copper-conductor cable under consideration with special emphasis on the following:

1. The amount of current and length of time required to burn clear copper-to-copper and ground faults representative of those occurring in practise in the types

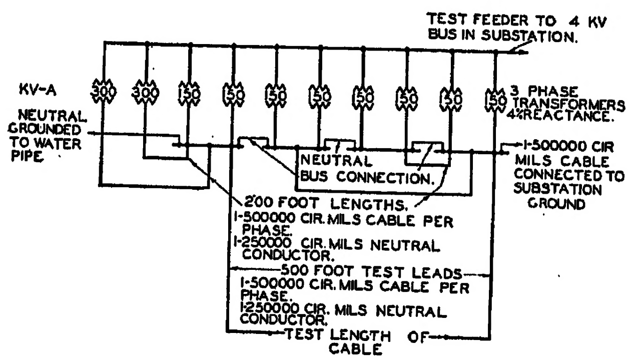

Fig. 1—Circuit Diagram—Tests Made May 19, 1929

and sizes of cables proposed and commonly used for a-c. underground low-voltage network distribution.

2. A comparison of the self-clearing of three-phase, phase-to-phase, phase-to-neutral, and ground faults in the several types of cables tested.

3. The relative burn-off characteristics of multiple-conductor and single-conductor buried cables.

4. The effect on burn-off characteristics of the spacing of buried cables.

5. The amount of flame, smoke and gas due to the cable insulation and covering burned under short-circuit conditions

6. The advantages, if any, of paper insulation compared with rubber insulation.

7. The burn-off characteristics of non-metallic sheath cables for duct installation compared with lead-covered cables.

8. The burn-off characteristics of lead-covered cables in fibre duct, tile duct, and iron pipe.

9. A comparison of the explosibility of gases generated in the duct line under the conditions of Item 8.

III. Test Equipment

Provision was made for isolating a 13,200-volt feeder connected to an isolated substation bus section to which

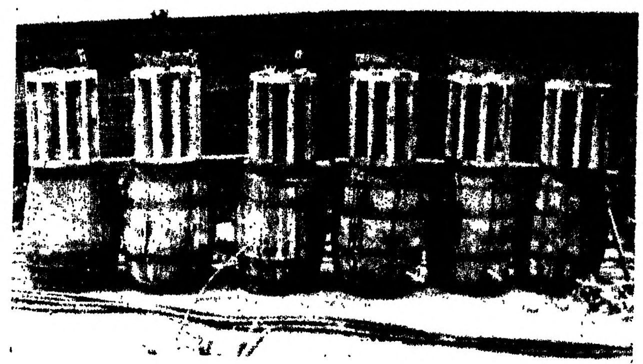

Fig. 2—Water-Cooled Resistors Removed from Barrels

a-c. 60-cycle power was supplied. The voltage was reduced to 4,000 volts, phase-to-phase, through a 4,800-kva. oil-insulated, self-cooled transformer. Between the 4,000-volt bus and the 4,000 230-volt network transformers, the secondary terminals of which were connected to the low-tension test circuit, were connected three single-phase ten per cent voltage regulators. The distance from the 4,000-volt substation bus to the network transformers was approximately 300 circuit-feet. The network transformers, consisting of two 300-kva. and eight 150-kva. transformers, were connected for some of the tests, as shown in Fig. 1, so that 900 kva. capacity supplied power to each side of the fault in the test cable. This test set-up is nearly equivalent to a network layout for city blocks, 200 by 1,000 ft., having a 300-kva. transformer installed at each intersection and a fault at the center of the long side of the block.

Cable reel reactors, water-cooled wire resistors, (Fig. 2) and spaced lengths of cable were used in several combinations to limit the short-circuit current to the desired values. High-speed graphic ammeters, mounted on a temporary wooden panel, were used for recording current values. The secondary bus voltage of the transformers was determined by means of indicating voltmeters.

Each test length of buried cable was located approximately 18 in. beneath the surface of the ground. When the cables were spaced, the neutral weatherproof, braided conductor was placed at one side of the trench and the three cables for buried installation were laid in parallel positions. The length of buried cables subject

to burn-off was approximately 17 ft. Various joints and service taps, simulating field conditions were made in the lengths of cable under test.

The cables for duct installation were pulled into four-inch ducts between distribution boxes, made for test purposes and located approximately 75 ft. apart. These boxes were approximately 24 in. wide and 28 in. in depth and length. They were equipped with standard reinforced concrete covers. Standard three-way joints were made in the boxes at one end of the test length. At the other end the cables were not spliced, but were pulled directly through the boxes (Fig. 10).

The various types of faults and cable constructions tested will be described under the heading, Discussion and Analysis of Test Results.

IV. Test Procedure

All signals for control operations were given by the blowing of a whistle, a code having been agreed upon for each operation.

The graphic meter charts were placed in operation before the oil circuit breakers in the test feeder were closed so that a complete record of current values was obtained. The secondary bus voltage of the network transformers was read from the indicating meters in order to detect any trouble that may have developed in the test equipment, and to provide comparative data for an analysis of results. The bus voltage in the substation was also noted throughout each test.

Careful observation was made of the flames, smoke and explosion, if such occurred.

Each test length of cable was carefully inspected after the tests were completed to determine the condition of the insulation and coverings, and the location where clearing took place.

Samples of gas, generated in representative tests on cables in ducts, were collected by means of high percentage rubber tubing and inverted bottles of water during the arcing conditions. Analyses of these samples were made for comparative purposes.

The short circuit was energized in all tests, except two, until the fault was cleared.

V. Discussion and Analysis of Test Results

After making several tests in which cable reel reactors were used to limit the short-circuit current, it was suggested that the highly inductive circuit might possibly aid the sustenance of the arc. Accordingly, tests were made using long cable leads to limit the current. Results indicated that the high inductance of the test circuit first used did not affect the arc at the low test voltage, 133/230 volts, and with those values of current used. The ratio of calculated resistance to the calculated reactance of the test circuit varied between the approximate limits of 0.20 and 0.70. This question was not given any further consideration in the remaining tests conducted.

Tests on Multiple-Conductor Buried Non-Magnetic Sheath Cables. Eleven tests were made on five-conductor cables constructed as indicated in Fig. 3A. Three tests were made on five-conductor cables constructed as indicated in Fig. 3B. As shown in both figures, the insulating material was a rubber compound. In both constructions, the maximum separation of phase conductors was 0.3 in., the minimum separation, 0.2 in.

A heavy iron clamp, squeezing the conductors (No. 4/0 phase conductors, No. 2/0 neutral) together, the insulation and covering having been removed throughout a length of approximately six inches, was used in making the faults in four tests. This produced a fault having a thermal capacity, relative to the amount of heat generated, greater than that of the cable conductors. Consequently, at values of total initial short-circuit current of approximately 4,000 amperes these faults did not clear; but the insulation was charred throughout the test length between the joints, and failure tended to take place at the joints where two or more conductors joined the one at fault.

At values of total current, approximately 11,000 amperes flowing into the fault, there was such concentration of heat at the edge of the clamp that fusing of the conductors took place, causing failure before the insulation between conductors in the adjacent lengths of cable was destroyed enough to allow a progression of the fault. Clearing of the fault took place at this value of current in approximately 35 seconds.

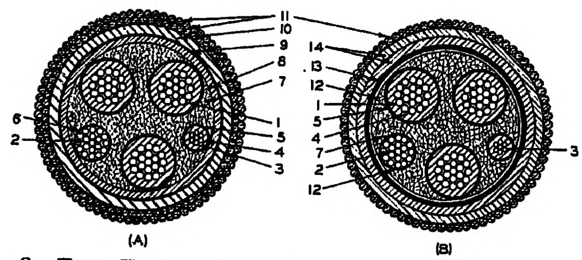

Fig. 3—Two Types of Construction of Five-Conductor Buried Non-Magnetic Sheath Cables

Note:
1—No. 4/0 copper conductor, 19 strands
2—No. 2/0 copper conductor, 19 strands
3—No. 6 copper conductor, 7 strands
4—Special rubber compound
5—Rubber-filled cotton tape
6—Weatherproof cotton braid
7—Impregnated jute filler
8—Saturated asbestos braid
9—Paper separator
10—Sisal braid
11—Helically applied jute wrap
12—Impregnated fabric tape
13—Lapped bronze tape
14—Impregnated fibre tape

In one test, a pneumatic cable stabber was used to produce a fault in an energized test length of five-conductor cable, Fig. 3A. The blade of the stabber severed the cable except for a small section of the covering on one side. The total values of initial short-circuit current in the three-phase conductors and neutral were 2,000, 1,280, 1,600 and 800 amperes, respectively. The fault cleared instantly, approximately an inch of the blade being destroyed by the arc.

A fault was also produced in the test length of cable mentioned in the foregoing paragraph by driving a bullpoint into it. Clearance took place instantly, the total initial values of short-circuit current in the phase

conductors being 1,900, 2,200 and 2,500 amperes. It is reasonable to assume that a fault produced by driving a pick axe into the cable would clear in a similar manner.

Another fault was made in a test length of cable of the construction indicated in Fig. 3A by baring the conductors and arranging a 2 by 4-in. plank so as to force the conductors together when the plank was struck with a sledge hammer. The insulation between conductors to be short-circuited was removed throughout a length of approximately two inches. This method produced a contact between conductors and a phase-to-phase fault of this type cleared in 20 seconds with a total value of current varying from 0 to 2,480 amperes. A second test on the same cable with a two-phase-to-neutral fault, made in the same manner, failed to clear at the point of fault with a total value of current of about 3,800 amperes. It seems that a point contact was formed in the first case, which was cleared easily with the low value of current. But, in the second case, a fault of greater contact surface than that of a point contact must have been formed; and the current obtained was not sufficient for clearing. Also, it was possible that the collection of molten copper established a metallic path between conductors, which resulted in burning the insulation throughout the length of the test cable, making subsequent dielectric breakdown easy. In the latter test, an arcing condition was produced which existed approximately 35 minutes until nearly the entire length of the test cable was destroyed.

Two tests on multiple-conductor cables were made with the conductors grounded to a water pipe which was connected to the city mains. A fault made by grounding one of the phase conductors and the neutral on the water pipe cleared in approximately two minutes at the location of the fault with a total value of current, remaining nearly constant, of 3,750 amperes. There was considerable arcing and emission of steam due to water, from the hole burned in the pipe, coming in contact with the arc. The fault was entirely submerged in water before clearing took place. The other phase conductors were not involved in the fault. In the second test in which one phase conductor was grounded to the water pipe, the maximum current obtained during the test had a total value of 2,900 amperes. This fault did not clear due to the low value of current (approximately 400 amperes) that resulted during the first 9.5 minutes of the eleven-minute period that the circuit breaker of the test feeder was left closed.

In the majority of the tests, the fault was made up by baring the conductors for a length of about six inches, forcing them together, and taping tightly overall with friction tape to hold the conductors in contact and completely covering the bared portion of the conductors. This method gave a contact length between conductors of approximately three inches. This is probably the worst type of fault that would occur initially in the low-voltage mains of a network system, except when a fault is produced by mistake in making connections to the network switch (or similar operations), or when the insulation and covering may be charred throughout a considerable length of cable by low values of short-circuit current, allowing the conductors to come in contact. Unless otherwise stated, faults mentioned in the discussion which follows will be assumed to be of this type (three-inch contact fault).

A three-phase-to-neutral fault in five-conductor cable of the construction indicated in Fig. 3A burned for 42.5 minutes with a total initial value of current of 3,200 amperes. Another burned for 35 minutes with a total initial value of current of approximately 3,900 amperes.

Five burn-off tests were made on four-conductor buried cable containing three No. 4/0 phase conductors and one No. 2/0 neutral; and two burn-off tests were made on three-conductor buried cable containing three No. 4/0 phase conductors, with a single-conductor, No. 2/0 weatherproof braided cable as a neutral. Test results obtained with these two latter types indicate that the burn-off characteristics are the same as for the five-conductor buried cables tested under the same conditions. The latter statement is supported by the following data. A test length of four-conductor, paper-insulated cable with a three-phase-to-neutral fault burned for 36 minutes with an initial value of total short-circuit current of 6,200 amperes; and cleared in the splice box at one end of the test length, and at the point where the test leads were connected at the other end. Similarly, a test length of the three-conductor cable burned for 23 minutes with an initial value of total short-circuit current of 4,800 amperes.

The four-conductor cable was insulated with sulfinated oiled paper. The three-conductor cable was insulated with a rubber compound of the same composition as that of the insulation of the five-conductor cable, Fig. 3A. The separation of conductors in both constructions was the same as that of the five-conductor cable.

From the tests made on the multiple-conductor cables, the data obtained indicate that, with values of total initial short-circuit current of 6,200 amperes and less flowing into the fault, self-clearing of the fault is very unsatisfactory; and when an arcing condition has been established, the fault tends to progress from its initial location, completely destroying the cable and leaving molten metal in the earth where the cable was buried. The destruction of the cable produced by such an arcing condition is indicated in Fig. 4.

For values of total initial short-circuit current between 4,000 and 6,500 amperes clearance under such arcing conditions usually takes place at joints where two or more conductors join the one at fault. Without actual test data available, it is difficult to predict what minimum value of current is required to clear an arcing fault in the multiple-conductor cable; but, it is evident

from the data obtained that such value of current is higher than the corresponding value for single-conductor buried cable, as shown from information which follows.

With the cables buried approximately 18 in. beneath the ground surface and with no protective covering, arcing faults produced flames of a maximum height of nearly 5 ft. above the ground. Smoke and volatilized

FIG. 4—REMAINS OF FOUR-CONDUCTOR BURIED CABLE AFTER TEST

Total value of current varied from 0 to 6,200 amperes during period of 33 minutes, clearing fault

materials were blown from the holes in the soil formed by escaping gases.

Single-Conductor Buried Non-Magnetic Sheath Cables. Following are the constructions of single-conductor buried non-magnetic sheath cables tested for burn-off characteristics and the number of tests made on each.

Construction	No. of Tests
1. No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound One wrap of double-faced rubber-filled cotton tape Two wraps of tarred jute applied in reverse directions served with asphalt compound One treated burlap wrap, in reverse direction to last jute wrap Two wraps of heavy asphalted jute Soapstone finish........................	3
2. No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound One wrap of single-faced rubber-filled cotton tape Serve of heavy asphaltum compound One wrap of asphalted jute Two rubber-filled fiber armoring tapes Serve of heavy asphaltum compound One wrap of jute saturated with asphalt compound Soapstone finish........................	3
3. No. 4/0 copper conductor, 19 strands 110 mils of special rubber compound One wrap of single-faced rubber-filled cotton tape 5 mils of bronze tape Serve of tar pitch Two wraps of fiber tape Serve of tar pitch One wrap of jute Special flame-proof compound Soapstone finish........................	5
4. No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound 125 mils of 60 per cent rubber compound	
5. No. 4/0 copper conductor, 19 strands 80 mils of special intermediate rubber compound One wrap of single-faced rubber-filled cotton tape Serve of asbestos base caulk Impregnated asbestos braid Serve of asbestos base caulk Two wraps of non-hydroscopic kraft armor Serve of asbestos base caulk One wrap of asphalted jute Serve of special finishing compound Soapstone finish........................	5
6. No. 4/0 copper conductor, 19 strands 20 mils of vulcanized sulfinated oiled paper One wrap of single-faced rubber-filled cotton tape Remainder of covering, same as No. 5.......	12
7. No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound One wrap of single-faced rubber-filled cotton tape 95 mils of 40 per cent rubber compound One wrap of impregnated burlap tape Serve of special asphalt compound One wrap of impregnated burlap tape Serve of special asphalt compound One wrap of impregnated duck tape Serve of special finishing compound Soapstone finish........................	3
8. No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound One wrap of single-faced rubber-filled cotton tape Serve of special asphalt compound One wrap of tar paper tape Serve of special asphalt compound One impregnated burlap tape Serve of special asphalt compound One impregnated burlap tape Serve of special asphalt compound One impregnated duck tape Serve of special finishing compound Soapstone finish........................	3

The greater number of these tests were made with the three No. 4/0 phase cables and a No. 2/0 weatherproof braided neutral cable placed at random in the trench and buried. These tests were made in both wet and relatively dry soils. The fault was not submerged in water in any of these tests.

The results indicate that, with the No. 4/0 single-conductor buried cables spaced approximately 1.5 to 2 in., cover to cover, a total minimum initial value of current of approximately 6,000 amperes, in the test circuit used, is required to clear satisfactorily the three-inch contact fault. (Fig. 5). The clearing time required with this value of current may generally be expected to be in the order of one minute.

With total values of initial short-circuit current

FIG. 5—SINGLE-CONDUCTOR BURIED CABLES SHOWING FAULT CLEARED AT INITIAL LOCATION

Total value of current of 5,800 amperes cleared fault in 40 seconds

somewhat less than 6,000 amperes and with the cables buried together at random, it was found that long time arcing conditions were generally produced. The clearing action under such conditions was usually found to be the same as that experienced with the multiple-conductor buried cables. When no protective covering was used with the lower values of current, flames broke through the ground to various heights, the maximum of which was ten feet. These flames were burning gases from the cable coverings and insulation and continued to burn as long as the arc produced such gases.

In order to obtain some indication as to the flames and smoke due to burning of the insulation and coverings and the effect of the gases, so produced, on the clearing action of the fault, two tests were made on No. 4/0 bare conductors buried in the ground. In one test, the conductors were buried in a relatively dry mixture of clay and sand with a spacing between conductors of 1 to 1.5 in. A three-phase-to-neutral fault cleared in 6 min. and 20 sec., approximately one foot on each side of the location of the initial fault, with a total initial value of current of 5,000 amperes. A second test was made with the conductors spaced and buried in relatively dry loam soil containing some cinder and waste material. A three-phase-to-neutral fault cleared in six and one-half minutes, at points approximately 18 in. on each side of the location of the initial fault, with a total initial value of current of 5,000 amperes. No smoke or flame broke through the ground during either of these tests.

It seems that the conductors at the fault location, in these two tests, must have been heated to a fused state and an arc was formed which must have aided in clearing away the metallic path between conductors. In the case of insulated conductors under the same conditions, the insulation would probably be carbonized at this value of current which would aid the existence of the fault. Therefore, because of the carbonization of the insulation and covering of a completed cable under the same test conditions, gases are not generated in any appreciable quantity in the presence of the arc. Hence, it seems that the gases are not present in sufficient quantity to aid in extinguishing the arc by their deionizing[3] effect.

When an arcing condition exists, causing a progression of the fault, clearing will usually take place at the point where two or more cables join the one at fault. If the joints of all conductors are enclosed in one junction box filled with an asphaltic compound, the compound is ignited when exposed to the arc and aids in sustaining it. When each joint is enclosed in a separate junction box as indicated in Fig. 6, clearing may be expected to take place at such a location.

Spacing the cables in the trench will reduce the period of burning considerably, since this condition makes it more difficult for the arc to subsist; and also, it aids in preventing the spread of single-phase faults to the other

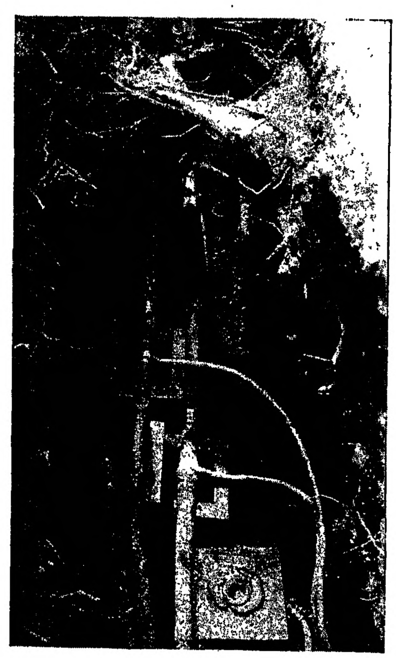

FIG. 6—METHOD OF MAKING SERVICE CONNECTIONS TO BURIED CABLES

phases. It was found that the separation between cable coverings must be at least 1.5 to 2 in. to improve appreciably burn-off characteristics.

A 2 by 10-inch creosoted yellow pine plank located and buried about two inches above the cables prevented flames from bursting through the surface of the

3. *Arcs in Low-Voltage A-C. Networks*, by J. Slepian and A. P. Strom, see page 847.

ground. Such a covering in a network system has the additional advantage of protecting the buried cables against injury from the tools of workmen.

Some tests were made with the cables buried in relatively dry soil, and other tests were made under the same conditions except that the soil was water-soaked. No difference in burn-off characteristics under these two conditions was observed.

As accurately as could be determined from the test results, the amount of current and time required for clearing was the same for the several constructions tested.

Tests on samples of constructions, 3, 5, and 6 show that these three buried non-magnetic sheath cables contained less inflammable material than that in the other makes tested, and therefore produced less smoke and flame under short-circuit conditions.

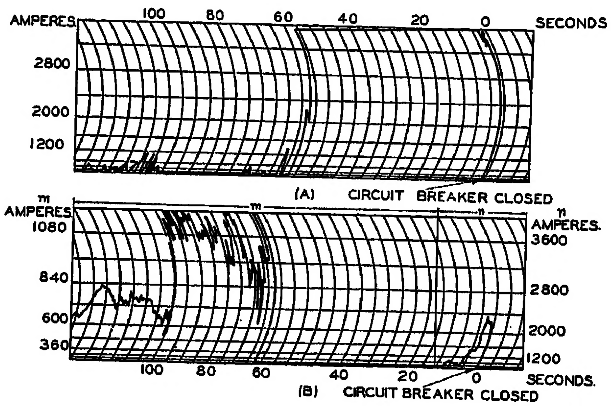

Fig. 7—Typical Charts Showing Variation of Current During Part of Short-Circuit Period

As accurately as could be observed, tests made on constructions 5 and 6 indicated that the vulcanized sulfinated oiled paper insulation of the latter cable possessed no advantage in burn-off characteristics over the rubber insulation of the former, the coverings being the same in the two cables.

There are not sufficient test data on construction 3 to determine whether or not the bronze tape has any effect upon the burn-off characteristics. It seems probable that the bronze tape in the covering would help to prevent the spreading of single-phase faults into other phases, if the heat of the burning conditions is not too intense. The melting point of bronze is approximately 900 deg. cent.

Representative test data supporting the foregoing discussion on burn-off characteristics of single-conductor buried cables are included in Table I.

In Fig. 7 are shown two sections of meter charts, indicating typical current variations during the short-circuit period.

Single-Conductor Cables for Duct Installation. Following are the cable constructions for duct installation which were tested for burn-off characteristics and the number of tests made on each.

Construction	No. of Tests
A. Non-Metallic Sheath Cables	
1. No. 4/0 copper conductor 19 strands	
80 mils, 30 per cent A. S. T. M. rubber compound	
One single-faced rubber-filled cotton tape	
Weatherproof cotton braid............	4
2. No. 4/0 copper conductor, 259 strands	
15 mils of felted asbestos	
Three varnished cambric tapes	
30 mils of felted asbestos	
45-mil asbestos braid..................	2
3. No 4/0 copper conductor, 19 strands	
80 mils of intermediate rubber compound	
One double-faced rubber-filled cotton tape	
Serve of asbestos base caulk compound	
60-mil, saturated asbestos braid	
Serve of asbestos base caulk compound	
70-mil, saturated sisal braid	
Flameproof moisture resistance compound...........................	2
4. No. 4/0 copper conductor, 19 strands	
5 mils of vulcanized sulfinated oiled paper	
One double-faced rubber-filled cotton tape	
(Remaining construction same as construction 3)......................	1
B. Lead-Covered Cables	
1. No. 4/0 copper conductor, 19 strands	
80 mils, 30 per cent A. S. T. M. rubber compound	
One single-faced rubber-filled cotton tape	
80 mils of lead......................	6
2. 350,000 cir. mils, copper conductor, 37 strands	
95 mils of oiled paper	
95 mils of lead......................	4
3. 500,000 cir. mils, copper conductor, 37 strands	
95 mils of oiled paper	
95 mils of lead......................	7

In each of these tests, three single-conductor phase cables of the construction under test and the bare neutral conductor were pulled into the 75-ft. duct length installed approximately two and one-half ft. beneath the surface of the ground. No. 4/0 neutral conductors were used with the 350,000- and 500,000-cir. mil phase conductors, and a No. 2/0 neutral with the No. 4/0 phase conductors. In none of these tests was the fault submerged in water.

The results obtained from tests on the non-metallic sheath cables do not disclose any particular advantage of any one of these types, as compared to the others. The amount of smoke, flame and disturbance produced in each case indicated the same relation to the clearing value of current required. The tests were insufficient in number to supply conclusive data on which to base a thorough comparison with lead-covered cables

TABLE I—REPRESENTATIVE DATA FROM BURN-OFF

Duration of short-circuit current	Current flowing into fault-amperes											
	One side of fault from bus No. 1				Other side of fault from bus No. 2				Total			
Min.—sec.	A	B	C	Neutral	A	B	C	Neutral	A	B	C	Neutral
5—24	2,120	2,290	2,130	0	2,050	2,240	2,060	0	4,170	4,530	4,190	0
27—36	0–1,300	0–1,850	0–1,980	0–670	0–1,270	0–1,170	0–1,200	0–600	0–2,570	0–3,020	0–2,680	0–1,270
3— 0	0	0	0	0	0	0	0	0	0	0	0	0
36— 0												
Momentary	2,580	2,440	2,500	0	2,560	2,440	2,640	0	5,140	4,880	5,140	0
10— 0	0–1,000	0–1,050	0–800	0	0–800	0–800	0–800	0	0–1,800	0–1,850	0–1,600	0
10—30	0–1,000	0–1,050	0–800	0	0	0	0	0	0–1,000	0–1,050	0–800	0
3— 0	0	0	0	0	0	0	0	0	0	0	0	0
23—30												
0—40	2,520	3,060	2,930	400	2,980	3,120	2,760	0	5,500	6,180	5,690	400
5—20	0	0	0	0	0	0	0	0	0	0	0	0
6— 0												

1—See Fig. 1

However, in addition to valuable data obtained, the proper lines of development in such cable constructions are indicated.

A four-inch fiber duct was used in all of the tests involving No. 4/0 phase conductors.

The results obtained from the tests made on No. 4/0

FIG. 8—SINGLE-CONDUCTOR CABLES IN DUCT SHOWING FAULT CLEARED AT INITIAL LOCATION

Total value of current of 6,000 amperes cleared fault instantly

rubber insulated, lead covered cables were essentially the same as those obtained on the No. 4/0, non-metallic sheath types. A total minimum initial value of current of approximately 6,000 amperes is required to clear the three-inch contact fault at its initial location in any of those types of No. 4/0 cables when tested under the conditions described, Fig. 8. Results indicate that the clearing time would rarely exceed one minute for this value of current. When the fault failed to clear at the initial location, the cables between the distribution boxes were for the most part destroyed, Fig. 9. The greater separation of cables in the distribution boxes, and the tension at the duct edge due to the short length of suspended cable must have aided clearing which generally took place at these points with values of total initial current of approximately 4,000 amperes.

The four tests made on construction B, 2, 350,000-cir. mil paper insulated, lead-covered cable, indicate that the three-inch contact fault in cables of this conductor size and type requires a total minimum initial value of current of approximately 8,000 amperes at 133/230 volts for clearing at the initial location; and

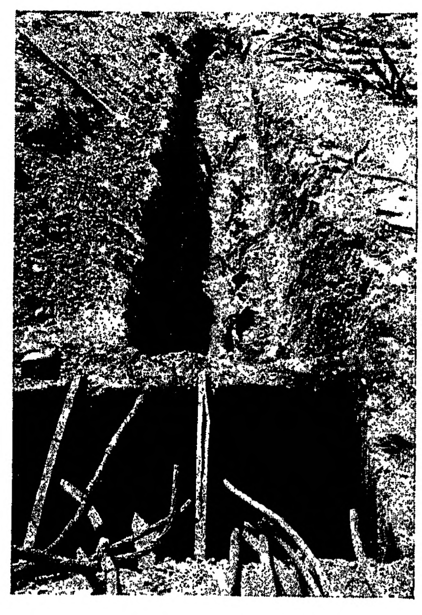

FIG. 9—REMAINS OF SINGLE-CONDUCTOR CABLES AND DUCTS AFTER TEST

Left—Total value of current varied between 0 and 5,000 amperes for a period of 11.5 minutes

Right—Total value of current varied between 0 and 3,600 amperes for a period of 21.5 minutes

TESTS MADE ON SINGLE-CONDUCTOR BURIED CABLES

Duration of short-circuit current	Bus voltage									
	Bus No. 1			Bus No. 2						
Min.—sec.	A-B	B-C	C-A	A-B	B-C	C-A		Test remarks	Description of cables and fault (See page 5 for cable construction)	Results
5—24	226	228	226	229	226	228	Max.	Current nearly constant	Construction No. 6	Fault was cleared at point one foot from joints at one end and just beyond joints at other end of test length. Cables destroyed and flames broke through ground to height of 3 feet between points of clearance.
27—36	199	201	200	215	212	214	Min.	Period of arcing	Phase conductors and neutral short-circuited. Taped fault.	
3— 0								Time allowed for fault to re-establish.		
36— 0										
Momentary	235	235	238	224	233	234	Max.	Initial current decreased rapidly	Construction No. 5	Clearance took place at point one foot from the initial fault on one side and at a point three feet from the initial fault on the other side. Flames broke through ground to a height of 3 feet.
10— 0	220	218	220		222	222	Min.	Period of arcing	Phase conductors and neutral short-circuited. Taped fault.	
10—30								Fault was cleared on side fed from Bus 2		
3— 0								Time allowed for fault to re-establish.		
23—30										
0—40	214	214	214	214	212	212	Max.	Current nearly constant	Construction No. 5	Fault was cleared at initial location.
5—20	172	172	172	194	192	192	Min.	Time allowed for fault to re-establish.	Phase conductors and neutral short-circuited. Taped fault.	
6— 0										

that the clearing time is in the order of one minute or less. These tests were also made with the cables installed in four-inch fiber duct.

Test results show no difference in burn-off characteristics of 500,000-cir. mil rubber-insulated, lead-covered cables in fiber and tile ducts; and the data obtained indicate no difference in burn-off characteristics of 500,000-cir. mil cable in fiber duct and iron pipe. However, there are not enough data to state conclusively the difference in burn-off characteristics between the two latter constructions. These test results indicate that a total minimum initial value of current of approximately 10,000 amperes at 133/230 volts will satisfactorily clear the three-inch contact fault in 500,000-cir. mil cables in approximately 1.25 min. or less. Figs. 10 and 11 show the condition of cables subjected to short-circuit currents less than the minimum clearing value of the three-inch contact fault, except Test 83, Fig. 11, in which test the total short-circuit current was approximately 10,000 amperes.

Fig. 10—Distribution Boxes at One End of Duct Section

Asbestos board partitions removed. Cables were subjected to values of current below the clearing values of 3-inch contact fault

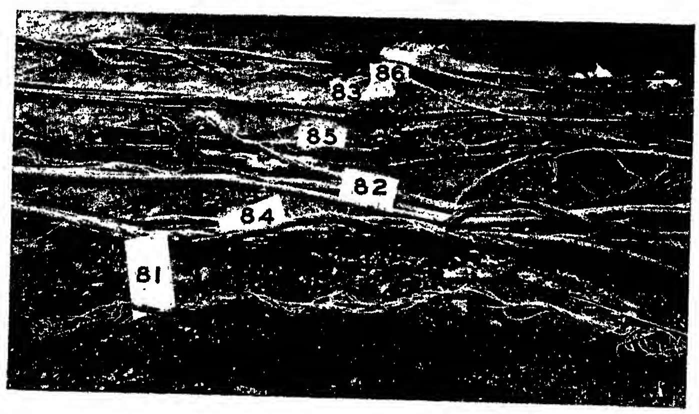

Fig. 11—500,000-Cir. Mil Paper-Insulated Lead-Covered Cables Removed from Ducts After Test

Note cables of Test No. 83—total value of current of 10,400 amperes cleared three-inch contact fault in 1.25 minutes. Remaining cables shown were subjected to total initial values of current between 7,300 and 9,000 amperes

No attempt was made in these tests to determine the maximum values of current at which arcing faults in the several cable constructions might be extinguished.

The clearing of only one ground fault in lead-covered cable, conductor-to-sheath, was tested. This fault was made by driving a screw into the cable and was cleared instantly. A study of available operating and test data shows that ground faults between the conduc-

tor and sheath of lead-covered cables have cleared in all instances. The extensive tests made by the Siemens-Schuckert Company[4] show that such faults will clear in a 243/420-volt network system.

The moving of cables, installed in ducts, due to magnetic forces under short-circuit conditions depends upon the relative sizes of the cables and duct. In a smaller duct the cables have less chance to separate and therefore the arc may take longer to be started and the fault may progress throughout a greater distance. As the duct size for a given group of cables is decreased, a condition is reached finally where the construction simulates multiple-conductor cables.

Rust, grease, and foreign material on the inside surface of an iron pipe, being an insulator, might in many cases prevent grounding of a phase conductor, the insulation and coverings being burned away, if the arc has not previously burned this coating on the duct wall. However, no data have been obtained which prove the practical aspect of this statement.

It has been suggested that the resistance of the cupric oxide formed on the bare conductors, after the insulation and coverings have been burned away, may aid in keeping the initial fault localized. Tests have been made on a copper conductor, producing a coating of the oxide varying in thickness throughout as wide a range as can be obtained by heating the conductor in air. The breakdown voltage of the oxide film varied from a minimum of 15 volts to a maximum of 38 volts. It is probable that the bare conductors under short-circuit conditions do not establish continuous electrical contact throughout the duct, but that the conductors may be forced together either by gravity, magnetic forces or both so as to establish a new fault.

The effect of molten metal in the bottom of the duct upon time of burn-off depends upon the relative positions of the cables. In a fiber duct this may establish a conducting path between two or more of the circuit conductors. In an iron pipe, the same conditions might result, or, in addition, a ground might be established to one or more of the phase conductors, irrespective of the neutral cable construction. At values of current somewhat less than the minimum clearing values, it was noted that the lead sheaths of the lead-covered cables in many cases were mostly melted off the phase conductors. This was due to heat, produced partly by induced currents in the sheath and also by the wattage loss in the conductor.

Samples of gases were collected from the duct line during two of the tests in fiber duct, and one in tile duct. An analysis was made of these gases, the results of which are shown in the following table:

TABLE II—ANALYSIS OF GASES COLLECTED FROM DUCTS AND DISTRIBUTION BOXES DURING TESTS

Constituent gases evolved	Relative Percentage of Each Gas				
	Test No. 81 east end D. B.[1]	Test No. 84 east end D. B.[1]	Test No. 84 center of duct line	Test No. 85 east end D. B.[1]	Test No. 85 west end D. B.[1]
Kind of duct	Fiber	Fiber	Fiber	Tile	Tile
Carbon dioxide	1.0	2.4	7.7	7.7	7.1
Unsaturated hydrocarbons	0.4	0.8	1.8	1.9	2.1
Oxygen	20.3	16.6	11.8	13.0	13.2
Carbon monoxide	0.7	2.5	3.3	3.3	3.5
Nitrogen	77.6	77.7	75.4	74.1	74.1

1. D. B. = distribution box

It is evident that the principal source of the oxygen and nitrogen shown in the above results was the air. The results therefore show that an amount of the order of 50 per cent of the gases generated from the cable materials was combustible. There is very little difference in the content of the gaseous mixture given off under short circuit conditions in tile and fiber ducts. The fiber duct itself contains considerable volatile matter, but this does not alter to any marked extent the gaseous products under short-circuit conditions. The results also show that the gas at the center of the duct line was nearly the same as that in the distribution boxes.

Since the samples of gases collected during these tests were from paper-insulated cables, it was desirable to obtain an analysis of the gases given off from rubber-insulated cables. This was done in the laboratory by wrapping a resistance wire about a 500,000-cir. mil, rubber-insulated cable, from which the lead sheath had been removed, and collecting the gases given off when the insulation was heated in this manner in open air. An analysis of the gases given off from paper-insulated cable under the same conditions was also made. The results of these analyses are shown in the following table.

TABLE III—ANALYSIS OF GASES COLLECTED IN LABORATORY

Constituent gases evolved	Relative percentage of each gas	
	Rubber insulation	Oiled-paper insulation
Carbon dioxide	10.9	10.3
Unsaturated hydrocarbons	6.4	6.9
Carbon Monoxide	9.2	9.5
Oxygen	10.9	10.6
Nitrogen	62.6	62.7

Also, in consideration of these results, it should be borne in mind that the principal source of the oxygen

4. "New Developments in the Operation of Polyphase Low-Voltage Network Systems," by Hans Besold and Otto Mueller, *Elektrotech. Zeitsch.*, July 3, 1930. p. 953.

and nitrogen was the air. Approximately 60 per cent of the remaining gases was combustible. It seems probable that some of the unsaturated hydrocarbons burned, forming carbon dioxide and water. The results indicate that there is no appreciable difference in the inflammability or explosibility of the gases given off from rubber and oiled paper-insulated cables under simulated short-circuit conditions.

The unsaturated hydrocarbons and the carbon monoxide are the constituents in the gaseous mixture which produce burning and explosion when they unite with oxygen. The force of explosion depends upon the amount of these gases which collects in the duct line before ignition. Should there be air currents in the duct line so that the gases may escape as they are generated, the violence of the explosions may be considerably reduced with probable elimination of explosions altogether.

The occurrence of explosions during the tests did not prove conclusively the relative characteristics of the different sizes and types of cables as regards explosions occurring during short-circuit conditions, due to varying test conditions. However, these results do support the indications of one preliminary study on underground cable failures, namely, that explosions were mostly due to the volatilization of cable or joint insulating materials. The covers were in place during all the tests on the different types of No. 4/0 cables; but the wind condition varied producing stronger air currents through the ducts in some tests than in others. The distribution box covers were partly broken in some tests, leaving holes through which gas may have escaped. Also, the covers were left ajar during some of the tests because of hazards introduced. In those tests in which it is known that gases had not collected in the duct line during previous tests, explosions occurred from 5.5 to 15 minutes after the initial short circuit.

From observation of the tests on cables installed in fiber ducts, there was more smoke produced during the tests on the non-metallic sheath types than during the tests on the lead-covered cable. Under the same conditions, there seemed to be little difference in the amount of smoke produced from the four non-metallic sheath constructions. Inspection of the fiber ducts after some of the tests on the non-metallic sheath types, however, showed that a considerable amount of the duct material had been burned.

In several of the tests on 500,000-cir. mil cables, the lead sheaths were bonded together in the distribution boxes at one end of the test cables, 75 ft. in length. Omission of the bonds at the other distribution box did not produce pitting of the sheath under short-circuit conditions. No voltage due to sheath currents was indicated by voltmeters connected between the lead sheaths, and between the lead sheaths and neutral. However, the distance from the point of bonding to a possible fault location in the usual system layout would be considerably greater, should bonding be done only in manholes at street intersections. It was not possible to obtain conclusive data regarding this question.

Representative data obtained from tests on cables for duct installation are included in Table IV.

Calculated Clearing Time of Faults in Cables. As stated in the foregoing, a fault in cables may be one of the two general classes:

1. A fault having a thermal capacity, relative to the heat generated, less than that of the cable conductors.
2. A fault having a thermal capacity, relative to the heat generated, greater than that of the cable conductors.

Because of the extremely variable character of faults of the first class, it is impracticable to attempt to calculate the clearing time unless the available short-circuit current and the fault resistance are known. The three-inch contact fault, previously described, belongs to this class. Calculations based upon approximate voltage and current readings indicate that its resistance varied in those No. 4/0 conductor cables tested between the limits of 0.004 and 0.02 ohm. The wide variation was due in part to the change of the fault between the time of making it and the time it was tested, and in part to the difficulty of producing the same contact surface between conductors in each test.

In connection with faults of the second class, it is possible to make calculations,[5] within certain current limits, based upon the physical, thermal, and electrical characteristics of the cable, to determine the time required for fusing the conductors joining the fault. In these calculations, it is assumed that the cable insulation and covering remain practically intact so that their heat dissipating characteristics are not altered during the fusing period, and that the fault does not progress from its initial location.

As seen from the representative test data included herein, the short-circuit current remained nearly constant as long as an arcing condition was not established. Taking a constant value of current and assuming the temperature coefficient of resistance to remain constant between the cold temperature of the conductor (in the order of 20 deg. cent.) to the fusing temperature of the conductor, fusing curves can be determined as shown in Fig. 12. Fusing curves may be obtained by similar calculations, assuming a constant voltage applied to a given length of cable. As neither a constant-voltage nor a constant-current condition results when a fault occurs in a network, average values of watts per foot per thousand circular mils have been plotted against fusing time. For a given circuit condition, approximate relations between initial watts, average watts and fusing current may be established.

5. "Current Capacity of Wires and Cables," by George E. Luke, *Electric Journal*, Vol. 20, p. 127. *Underground Alternating Current Network Distribution for Central Station Systems*, by A. H. Kehoe, A. I. E. E. Trans., Vol. 43, p. 853, Fig. 27.

TABLE IV—REPRESENTATIVE DATA FROM BURN-OFF TESTS MADE

Duration of short-circuit current	Current flowing into fault-amperes											
	One side of fault from bus No. 1				Other side of fault from bus No. 2				Total			
Min.—sec.	A	B	C	Neutral	A	B	C	Neutral	A	B	C	Neutral
0— 5	2,200	2,560	2,440	0	2,320	2,440	2,540	0	4,520	5,000	4,980	0
8— 5	0–2,080	0–2,080	0–2,080	0–1,190	0–1,940	0–1,980	0–2,080	0–1,060	0–4,020	0–4,060	0–4,160	0–2,250
3—30	0	0	0	0	0–1,940	0–1,980	0–2,080	0–1,060	0–1,940	0–1,980	0–2,080	0–1,060
3—20	0	0	0	0	0	0	0	0	0	0	0	0
15— 0												
Momentary	2,520	2,760	3,260	2,400	2,880	3,220	3,120	2,440	5,400	5,980	6,380	4,840
5— 0	0	0	0	0	0	0	0	0	0	0	0	0
5— 0												
2— 0	2,120	2,260	2,200	0	2,040	2,160	2,010	0	4,160	4,420	4,210	0
13— 0	0–2,080	0–2,080	0–2,080	0–1,200	0–1,680	0–2,080	0–1,960	0–1,180	0–3,760	0–4,160	0–4,040	0–2,380
3— 0	0	0	0	0	0	0	0	0	0	0	0	0
18— 0												
2—30	2,110	2,280	2,190	0	2,040	2,150	1,980	0	4,150	4,430	4,170	0
15— 0	0–1,720	0–2,080	0–2,080	0–1,120	0–1,940	0–2,280	0–1,880	0–1,140	0–3,660	0–4,360	0–3,960	0–2,260
3— 0	0	0	0	0	0	0	0	0	0	0	0	0
20—30												
0—32	4,080	4,100	3,800	0	4,150	4,080	3,540	0	8,230	8,100	7,340	0
5—28	0	0	0	0	0	0	0	0	0	0	0	0
6— 0												
4—16	4,320	4,320	4,350	0	4,500	4,500	4,200	0	8,820	8,820	8,550	0
2—58	4,320	3,000	4,350	0–1,700	0–4,200	1,800	3,500	0–3,200				
2— 8	4,320	4,300	0	0	0–4,200	4,200	0	0–3,200				
9—18	0	0	0	0	0–1,200	0–1,200	0	0–1,000	0–1,200	0–1,200	0	0
9—20	0	0	0	0	0	0	0	0	0	0	0	0
28— 0												
1—16	5,100	5,000	5,100	0	5,300	5,470	5,100	0	10,400	10,470	10,200	0
0—11	0	0	0	0	0	0	5,100	0	0	0	5,100	0
10—33	0	0	0	0	0	0	0	0	0	0	0	0
12— 0												

Note: Tests were made with dry ducts only.

Calculations show the fusing times of paper and rubber-insulated cables of the same conductor size, covering and thickness of insulation to be so nearly the same that the difference may be neglected.

In the few tests in which an arcing fault was not produced and in which the fault was of the second class, the calculated fusing time checked with time determined by test within a maximum error of ten per cent. Sufficient test data were not obtained to examine thoroughly the accuracy of the three curves and their applicable portions. It seems probable that the portions of the curves shown in solid line might be applicable for checks against test results. The curves at least show an interesting theoretical comparison.

VI. Conclusions

A study of test results and other data leads to the following conclusions:

1. Faults included in the class of point contacts and occurring in single-conductor a-c. low-voltage network cables, operating at voltage values of the order of 120/208 volts, will usually clear within a period of several seconds if a minimum value of current of approximately 3,000 amperes is available, irrespective of

ON SINGLE-CONDUCTOR CABLES FOR DUCT INSTALLATION

Duration of short-circuit current	Bus voltage									
	Bus No. 1			Bus No. 2						
Min.—sec.	A–B	B–C	C–A	A–B	B–C	C–A		Test remarks	Description of cables and fault (see page 7 for cable construction)	Results
0— 5	232	232	234	232	234	232	Max.	Current nearly constant	No. 4/0 non-metallic sheath cables in 4″ fiber duct. Cable construction, A, 2	Fault was cleared in boxes at each end. Cables between boxes destroyed. Fiber duct burned.
8— 5	230	232	230	224	226	222	Min.	Period of arcing		
3—30								Cleared on side fed from bus No. 1	Phase conductors and neutral short-circuited. Taped fault.	Explosion blew covers of boxes at one end of duct 20′ above ground. Heavy smoke and flames from boxes at each end.
3—20								Time allowed for fault to re-establish.		
15— 0										
Momentary	214	214	214	214	212	212	Max.		No. 4/0 non-metallic sheath cables in 4″ fiber duct. Cable construction, A, 1.	Fault was cleared instantly at initial location.
5— 0	170	176	176	190	200	190	Min.	Time allowed for fault to re-establish.		
5— 0									Phase conductors and neutral short-circuited. Taped fault.	
2— 0	228	229	228	228	228	228	Max.	Current nearly constant	No. 4/0 non-metallic sheath cables in 4″ fiber duct. Cable construction, A, 1.	Cables between boxes destroyed. Fiber duct burned. Fault was cleared in boxes at each end. Heavy black smoke from boxes. Flames from one box.
13— 0	201	204	201	216	216	216	Min.	Period of arcing		
3— 0								Time allowed for fault to re-establish.	Phase conductors and neutral short-circuited. Taped fault.	
18— 0										
2—30	228	228	226	228	228	228	Max.	Current nearly constant	No. 4/0 lead covered cables in 4″ fiber duct. Cable construction, B, 1	Cables between boxes destroyed. Fiber duct burned. Fault was cleared in boxes at each end. Heavy black smoke from boxes. No flames.
15— 0	200	203	200	216	214	214	Min.	Period of arcing		
3— 0								Time allowed for fault to re-establish.	Phase conductors and neutral short-circuited. Taped fault.	
20—30										
0—32	218	219	218	218	217	230	Max.	Current nearly constant	350,000-cir. mil. lead-covered cables in 4″ fiber duct. Cable construction, B, 2.	Fault was cleared at initial location.
5—28	158	166	163	204	208	208	Min.	Time allowed for fault to re-establish.		
6— 0									Phase conductors and neutral short-circuited. Taped fault.	
4—16				234	236	236	Max.	Current nearly constant	500,000-cir. mil. lead-covered cables in 4″ tile duct. Cable construction, B, 3.	Fault was cleared in box at one end of duct length and at point between initial fault and box at other end of duct. Conductors welded together at initial fault. Heavy black smoke from boxes. Explosions at both ends, covers of boxes at one end blown 20 feet above ground.
2—58				198	200	200	Min.			
2— 8								C phase conductor was cleared.		
9—18								Side of fault fed from bus No. 1 was cleared.	Phase conductors and neutral short-circuited. Taped fault.	
9—20								Time allowed for fault to re-establish.		
28— 0										
1—16				234	238	238	Max.	Current nearly constant	500,000-cir. mil. lead-covered cables in 4″ iron pipe. Cable construction, B, 3.	Fault was cleared at initial location.
0—11				198	200	200	Min.			
10—33								Time allowed for fault to re-establish.		
12—.0									Phase conductors and neutral short-circuited. Taped fault.	

the conductor size. Tests and operating data show that ground faults between the copper conductor and the sheath of lead-covered cables have been cleared in all cases with voltage values of 115/199 to 243/420 volts, inclusive.

2. Copper-to-copper faults, in single-conductor a-c. low-voltage (120/208 volts) network cables, made by the taping the bared conductors overall and thereby obtaining a contact length of approximately three inches between conductors will clear at the location of the fault within a period of the order of one minute (usually less) provided the following *minimum* values of short-circuit current flowing into the fault are maintained during the period of short circuit:

TABLE V

Conductor size	Constant values of short-circuit current, amps. Per conductor in test circuit	Total
No. 4/0	3,000	6,000
350,000 cir. mils	4,000	8,000
500,000 cir. mils	5,000	10,000

3. The amount of current and length of time required to burn clear a given type of fault in the buried

non-magnetic sheath cables, dropped into the trench at random, are the same as those for that type of fault in the same sizes and general types of cables installed in four-inch non-metallic ducts. A spacing of approximately two inches between adjacent coverings of single-conductor non-magnetic sheath cables buried in relatively compact soil improves the burn-off characteristics.

4. The time required to fuse a-c. low-voltage network cables with nearly constant values of current can be calculated accurately enough for practical purposes provided that the total time does not exceed eight minutes.

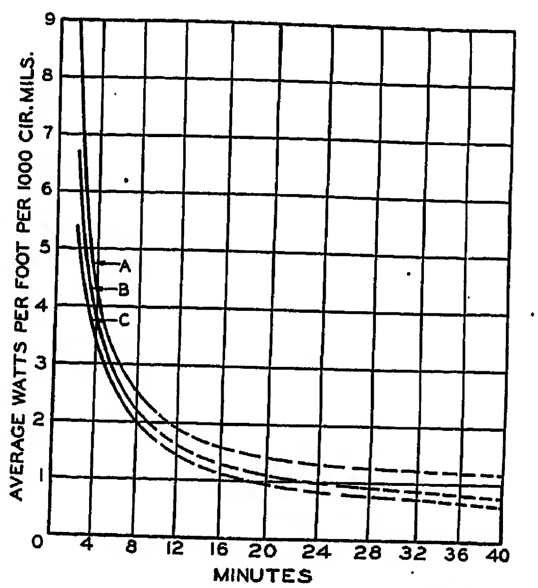

Fig. 12—Fusing Curves for Low-Voltage Cables

A—No. 4/0 conductor, 80 mils of paper insulation, 80 mils of lead
B—350,000-cir. mil conductor, 95 mils of paper insulation, 95 mils of lead
C—500,000-cir. mil conductor, 95 mils of paper insulation, 95 mils of lead

5. The burn-off characteristics of multiple-conductor buried non-magnetic sheath cables make this construction undesirable for low-voltage a-c. network distribution. The better burn-off characteristics of the single-conductor buried cables are attributed to the wider separation of conductors and to the coverings impregnated with flameproof compound.

6. As accurately as could be observed in tests on buried cables, vulcanized sulfinated oiled paper compared to rubber insulation improves burn-off characteristics very little. Analyses show no appreciable difference between oiled paper and rubber insulations the explosibility and inflammability of gases produced from lead-covered cables. With either insulation, explosions due to gases evolved from the cable materials by short circuits in present standard cables installed in standard four-inch duct systems may occur if the fault exists longer than five minutes, provided explosive gases do not already exist in the duct line.

7. Sufficient data are not available from tests thus far made to determine whether or not single-conductor non-metallic sheath cables, insulated with materials now used and installed in one duct, have better burn-off characteristics than lead-covered cables under the same conditions.

8. Sufficient data have not been obtained to determine conclusively the difference in the effect on burn-off characteristics of single-conductor cables in iron pipe and the same cables in fiber or tile duct.

9. Analyses of gases produced under short-circuit conditions show that the materials of fiber duct in standard use do not increase the inflammability or combustibility of the gases.

The foregoing conclusions are based upon operating data and results of tests made on cables buried in relatively dry and wet soil and installed in relatively dry ducts. The results of these tests indicate the desirable burn-off characteristics of low-voltage cables for underground network systems. The development of new insulating materials for cables having these characteristics will require further research.

Acknowledgment

The writers wish to acknowledge their indebtedness to those persons who made several helpful suggestions in connection with this study, and also, to the several manufacturing companies for their advice and cooperation in the supply of cable for some of the tests made.

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Discussion

W. R. Bullard: I believe the most important conclusion which has been drawn in this paper, is that single-conductor buried cable of the non-metallic armored type has appreciably better burn-off characteristics than the other types of construction tested. However, a very important feature of such construction is that with such cable the necessity for good burn-off characteristics might be expected to be very largely eliminated. That is, with single-conductor, non-metallic sheathed cable buried with a separation of one or more inches, the possibility of the occurrence of short circuits would be very remote. An insulation failure would, of course, result in the flow of some leakage current to ground, but it would be very unusual for such leakage current to reach sufficient magnitude to constitute a short circuit.

This type of construction, therefore, offers excellent possibilities for increased reliability of service for a-c. network systems. Unfortunately, one important factor connected with such construction is unknown. This is the useful life of the non-metallic sheathed cable. It is to be hoped that this feature will be diligently investigated and that the development will progress to the point where it will be known with certainty just what may be expected from the different types now on the market under various local installation conditions.

There are now in service or being installed several low-voltage distribution systems employing this type of construction, notably Mr. Sutherland's system and one in Santiago, Chile. The latter has a nominal service voltage of 220/380 volts, three-phase, four-wire. The experience gained from the operation of such systems will, of course, be very helpful in obtaining the necessary information for the further development of a-c. networks in general.

One further point which should be noted in connection with this paper is that the conditions of test were, in general, more severe than conditions ordinarily encountered in practise. That is, the tying together of the conductors produces considerably lower fault resistance than is ordinarily encountered under actual operating conditions. Numerous tests made under actual service conditions have shown considerably better burn-off characteristics than those listed in the paper, even in the case of systems utilizing voltages of the 220/380 class.

Wm. A. Del Mar: The non-metallic type of trench cable has undoubtedly come to stay in connection with the secondary network system. Every time that a new type of cable has appeared, the question of its permanence has been raised, and has usually stayed with us, in one form or another. It is, therefore, natural that it should be raised in connection with this type; —all the more so, since the essential peculiarity of the type is the omission of the very covering which has been relied upon to protect the insulation from disintegrating agencies, namely, the lead sheath. There are about as many types as there are manufacturers, so that experience with one type is no criterion of the performance of any other. Some of these types undoubtedly possess the characteristics that make for permanence, while others may not.

Some of this cable is purchased on specifications which merely describe one type or another to the exclusion of all others, while some is purchased on the basis of performance tests without obvious reference to the type. Unfortunately there are nearly as many opinions about such tests as there are performance type specifications. It is about time that some coordination be effected in this field.

Looking over these performance tests, we find tests to assure adequate resistance to the following agencies of deterioration:

(a) Water, with special reference to prevention of water-logging of rubber insulation.

(b) Acids, with special reference to installations in marshy land.

(c) Alkali, with special reference to installations in alkaline soil.

(d) Abrasion, with special reference to handling.

(e) Impact, with special reference to damage by excavators.

(f) Shear, with special reference to handling.

(g) Dielectric stress, with special reference to corona-formation, a matter of no interest in connection with secondary networks, but important for series lighting cables.

(h) Bending, with special reference to terminal and splicing conditions.

(i) Burning, with special reference to entering houses, and arc extinction.

Some of these agencies of deterioration obviously are of little significance, but too much stress cannot be laid on protection from water as rubber will not last in contact with soil water.

The protective materials used in the various types of cable are cellulose (in the form of paper, jute, cotton, sisal, asbestos reinforcement, etc.) asbestos, semi-solid hydrocarbons (such as asphalt, oxidized and vulcanized oils, pitch, etc.), rubber, waxes, and oils.

The protective structures used are tapes, braids, wrappings, and extruded sheaths.

The problem of the operating engineer is to give proper weight to the various requirements, the devising of tests should be a joint effort of the operating and manufacturing engineers, and the designs of cables to meet these tests should be left to the cable maker.

B. M. Jones: A large part of the test data in this paper bears out conclusions that the Duquesne Light Company arrived at as a result of tests made early in 1928. It was interesting to us that these tests by Messrs. Sutherland and MacCorkle showed no material difference in the time or currents required to burn clear of fault in fiber, clay, or iron ducts.

I would like to call attention to one particular feature which should not be overlooked in planning and designing a low-voltage a-c. network, and that is the providing of capacity to burn clear a fault in a weak spot on the network under three extreme emergency conditions, which are—(1) the short circuit being of the most severe type of copper-to-copper contact; (2) the fault occurring when one particular high-tension supply feeder is out of service thus reducing the short-circuit capacity available at the weak spot in the network; (3) the fault occurring at this weakest spot.

Assuming 6,000 amperes required to burn clear a fault of various sizes of cables, and with a transformer bank out of service thus leaving a weak spot in the network, to which spot 6,000 amperes flows from one direction and 3,500 amperes from another, direction, the fault would burn clear from the end supplied with 6,000 amperes and would not burn clear from the end supplied with 3,500 amperes.

Obviously, it would be necessary to increase the short-circuit current flowing from the weak side by increasing the transformer capacity in that direction, which would cost a considerable amount of money in some cases and this additional capacity would be required solely and only for the purpose of increasing the short-circuit amperes to burn clear a fault upon the simultaneous happenings of *three extremely remote contingencies*.

While the demands are for better and better service particularly in the business areas that are suitable for low-voltage a-c. networks; costs must be recognized and the day has arrived when the engineer must justify his expenditures as well as his design.

Most of the engineers and operators realize that electrical equipment in general is supplied with a "back-up unit," and there is a generator ready for service in case the largest one fails, a spare transformer bank or unit, an automatic circuit breaker to back up another one and relay schemes to back each other up, but there is not another back-up for the first back-up; so the providing for three extremely remote happenings, if it costs any money, is very questionable.

The engineers and operators considering a low-voltage a-c. network must make recommendations as to how much of a gamble they shall take on having these three extremely remote contingencies happen at the same time, or how much money they are willing to spend to be ready to handle these rare simultaneous occurrences, and of course the management's sanction should be obtained on either of these two schemes.

J. M. Comly and J. A. McHugh: During a six months' period 29 cases of fires and explosions caused by faults on a-c. low-voltage network mains cables were reported and of these none were found to give evidence of the presence of sewer or illuminating gases. Five cases involved explosions, nineteen cases involved sustained fires, and five cases involved only brief fires.

Four of the five brief fire cases showed negligible burning of insulation. One case resulted in considerable damage to insulation of No. 6 B&S cable used as main.

Seventeen cases of sustained fire showed from 3 to 3,400 ft. of cable insulation damaged for an average of approximately 420 ft. Most of these faults developed under dry conditions. In the two cases where 2 ft. or less of insulation was damaged the faults were submerged. The five failures involving explosions showed from 180 ft. to 560 ft. of cable insulation damaged for an average also of approximately 420 ft. In four of the five cases the underground system was dry in the vicinity of the explosion.

A comparison of the reported clearing conditions and the calculated available short-circuit currents indicates a considerable variation in fault resistance and this appears to be supported by the observed physical effects. Fourteen of the twenty-nine cases involved failure of network switches or fuses to clear.

Arcs in Low-Voltage A-C. Networks

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and

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Synopsis.—The extinction of a-c. arcs at current zero is reviewed, and arc reignition characteristic and circuit reignition characteristic are defined. From a study of arc reignition characteristics of short arcs remote from insulation it is concluded that such arcs are incapable of interrupting practical low-voltage a-c. network circuits. The extinction of arcs in practical network cables is then ascribed to the deionizing action of gas blasts coming from decomposing adjacent insulation. Experiments with arcs in cables, and arcs between parallel plates remote from insulation, and closely bounded by insulation, confirm this view.

Inorganic insulating materials may also assist in arc extinction by generating gas blasts by their decomposition. Of the various inorganic materials tried, boric acid was the most effective.

Charring of organic insulation may be expected to cause it to lose its arc extinction aiding characteristic.

* * * * *

I. Introduction

THE great success of the low-voltage a-c. cable network described by Kehoe,[1] which is finding increasing application as a system of distribution in metropolitan areas is dependent fundamentally upon the capacity for self clearing of faults on the network which this system enjoys. The tying together of the secondaries of many transformers into a network ensures that at any point of fault there will be a sufficiently large current to melt apart any contacting metallic conductors and to start an arc. The low a-c. voltage of the network ensures that the arc will be extinguished quickly at a normal zero of current. Thus the success of the system depends upon the properties of the arc which occurs at a cable fault. A close study of these arcs and how they are extinguished would seem to be worth while, particularly as the extension of this system to higher voltages which seems necessary for economy and proper voltage regulation where loads are very heavy, leads to conditions in which the extinction of the arc is far less certain than in lower voltage systems.

II. The Extinction of A-C. Arcs

Arc Reignition and Circuit Reignition Characteristics.

Very considerable advance has been made in recent years in the theory of extinction of a-c. arcs. The continuance of the a-c. arc is believed to depend on the repeated reignition of the arc for each new half cycle, after a current zero. Just after a current zero the arc space is losing ionization and recovering the ability to withstand voltage. On the other hand voltage tending to break down the arc space again is building up across the arc space terminals, following some natural transient of the external circuit. If the dielectric strength of the deionizing arc space, as measured in volts which the space can withstand, grows faster than the voltage which the external circuit impresses upon the terminals, then the arc will be extinguished. If the voltage supplied by the external circuit builds up faster than the dielectric strength of the arc space, then the arc will be reignited.

From the standpoint of arc extinction, therefore, the arc is best described in terms of a curve between the recovered dielectric strength of the arc space measured in volts, and the time after current zero measured in microseconds. Such a curve may be called the "arc reignition characteristic." Likewise, from the standpoint of arc extinction, the external circuit is best described in terms of a curve between the voltage which would appear across the arc terminals if the current remained zero after the particular current zero under consideration, and the time after that current zero measured in microseconds. Such a curve may be called the "circuit reignition characteristic." The arc is extinguished at a current zero if the arc reignition characteristic lies completely above the circuit reignition characteristic. The arc is reignited if the circuit reignition characteristic is anywhere above the arc reignition characteristic.

The circuit reignition characteristic may be calculated according to well-known electrical engineering principles. It depends upon circuit constants, *i. e.*, magnitudes and interconnection of reactances, resistance, capacitances, etc., and upon circuit conditions existing at the current zero, *i. e.*, excitation and phase position of synchronous machines, momentary currents in windings, momentary charges upon conductors, etc. It is independent of the arc itself except in so far as the arc voltage previous to the current zero may have influenced circuit conditions. Circuit reignition characteristics, for various simple cases have been published.[2–6]

The arc reignition characteristic cannot be calculated even approximately except in a few special simple cases in the present state of the science. It depends on conditions internal to the arc space, such as nature and geometrical arrangement of electrodes, the nature of the gas, presence of magnetic fields, and gas blasts. It does not depend upon the external circuit except in so far as the current previous to the current zero may be influenced and thus cause the conditions internal to the arc space to be altered.

The arc reignition characteristic is determined fundamentally by the manner of disappearance of the ions in the arc space immediately after current zero.

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Ions disappear from the space, (1) by direct recombination in the arc space itself, (2) by discharge to the electrodes, (3) by recombination on surrounding walls or in surrounding cooler gases, (4) by recombination in cooler gases mixed turbulently into the arc space by gas blasts. (2) appears to be most important in short arcs, and (3) in longer arcs. The importance of (4) in practical devices such as fuses and oil circuit breakers where intense gas blasts are generated by the decomposition of material under the heat of the arc has only recently begun to be appreciated.[2]

III. The Short Arc Without Gas Blast

The arc reignition characteristic for a short arc in the absence of gases coming from decomposing neighboring materials, and with what were believed to be relatively cool, non-vaporizing copper electrodes has been given by Slepian,[2] and is reproduced as curve *A* in Fig. 1. Noteworthy in this curve is the extremely rapid, almost instantaneous recovery by the arc space of a dielectric strength of 240 volts, and the slower further recovery of dielectric strength. The theory

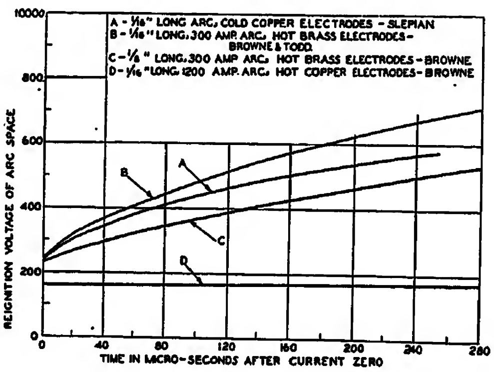

Fig. 1—Arc Reignition Characteristics

offered by Slepian also predicted that the immediately recovered dielectric strength would be independent of the arc length and arc current, but would depend on the nature of the (incoming) cathode, and the gas at the cathode. The slope of the arc reignition characteristic would depend on arc length and current if ions disappeared from the arc space by means other than direct recombination in the arc path, and if these means were affected by current and arc length.

Browne and Todd[7] have found similar arc reignition characteristics for short arcs with metal electrodes but with the metal at the arc terminals hot, and probably vigorously boiling. Their results for brass electrodes are given in curves *B* and *C*, in Fig. 1. The immediately recovered dielectric strength was 230 volts, and the slope of the characteristic for the 1/16-in. 300-ampere arc was nearly the same as that of the cold electrode copper arc given by Slepian. The slope was found to decrease with increase of arc length, however, as shown by curve *C*, Fig. 1, and also to decrease with increase of arc current, although complete arc reignition characteristics were not given for larger currents.

Curve *D*, Fig. 1 gives unpublished results of a limited number of tests made by Browne for a heavier current stationary arc with copper electrodes. Here is noted the extremely low value of the immediately recovered dielectric strength, about 160 volts, and the extremely small slope of the characteristic for the first few hundred microseconds.

IV. Extinction of Arcs in A-C. Networks

According to the tests of Kehoe[1] the arc following a fault in an a-c. network cable will always be extinguished at 220 volts, r. m. s. At 1,000 volts r. m. s. currents up to 500 amperes and at 2,000 volts r. m. s. up to 100 amperes were also successfully interrupted by the arc. In the tests at higher voltages, however, the currents were limited by resistances so that it may not be concluded that extinction of the arc would have taken place in a reactive circuit of like current. At 220 volts, however, we may conclude that the arc would be extinguished under all circuit conditions.

Similar results have been obtained by Besold and Mueller[8] although here again, the currents were apparently limited by resistance so that it is not certain that the results will apply also for reactive circuits.

These results are entirely compatible with the earlier data on arc reignition characteristics such as curves *A*, *B*, and *C* of Fig. 1. For example, on curve *C*, the immediately recovered dielectric strength of the arc space is 230 volts r. m. s. However, on curve *C*, dielectric strength of 311 volts is recovered in 60 microseconds, and since, in practical a-c. network circuits it will take longer than this for maximum open-circuit voltage to build up from arc voltage after current zero, the arc will not reignite after current zero.

The more recent results of Browne, however, indicated in curve *D*, and relating to heavier current arcs between copper electrodes, and which should be more directly applicable to cable fault arcs, are not compatible with these results. According to curve *D*, arcs with currents exceeding 1,200 amperes should not be extinguished with more than 125 or 130 volts r. m. s. in a low power-factor circuit. We are then driven to the conclusion that in a-c. cable fault arcs additional deionizing agents must exist which are not active in ordinary short arcs between metal electrodes.

These additional deionizing agents must come from the insulation used in cables, and the authors believe that it comes from the intense blasts of gas directed through the arc from the decomposing insulation adjacent to the arc. That gas blasts through an arc are strongly deionizing has been long known, and gas blasts coming from decomposing insulation are believed to be responsible for the effectiveness in circuit interruption of the expulsion fuses and the oil circuit breaker. The cable fault arc is *in* the insulation in the same way and sense as the oil circuit breaker arc is *in* the oil. In both cases the arc is in a gas pocket or bubble, but this gas and the arc in it are not in a quiescent state

but are being most violently stirred by the turbulent mixing into it of large volumes of fresh, relatively cool, unionized gas coming from the decomposing insulation or oil. This turbulent inpouring of fresh gas causes the extinction of the arc in circuits of higher voltage than would otherwise be possible.

The gases coming from the thermal decomposition of the insulation in cables would generally be regarded as a nuisance. Since they are inflammable, under certain circumstances, they may even constitute an explosion hazard. Nevertheless they are probably necessary in the functioning of present day low-voltage a-c. networks.

V. Tests on Arcs in Cables

While the ideas expressed in the preceeding sections were developing, it seemed worthwhile to get some first hand knowledge of the characteristics of arcs in cables. Accordingly copper-to-copper short circuits were made in short lengths of oil impregnated paper insulated cable. The cable used was 600-volt, three-conductor, 500,000-cm. lead-covered cable, manufactured by the Habirshaw Cable Co. The arcs were started in several different ways, namely, (1) by twisting together two strands of the conductors to be short-circuited; (2) by driving a nail through the stranded conductors; (3) by tying the conductors together with No. 20 bare copper wire. The tests were made in a 550-volt r. m. s., 60-cycle circuit in which the current was limited only by the reactive impedance of the transformers and cables. Currents varied from 3,000 to 10,000 amperes. In some of these tests the short circuit was made near one end of the cable, power being supplied from the opposite end. In others, the short circuit was made near the center of the cable section, power being supplied from both ends of the cable, in order to eliminate the strong magnetic blowout effect that is present with the first method of short-circuiting. In most of these tests the cable was drawn into a 3¾-in. fibre duct, the ends of which were closed by packing with rags.

It was found that in general the arcs would be extinguished in one to fifteen half cycles. Occasionally, however, the arc would not be extinguished, and in such a case provision had been made to throw in various values of resistance in shunt with the arc to see what influence would be exerted on the extinction of the arc. When the arc persisted it was found that resistance in shunt with even as low a value as one ohm had no influence on the continuance of the arc.

The arc voltage was found to be usually of the order of 100 volts when the short circuit was made 5 or 6 in. away from the insulation, and about 350 volts when the arc occurred near the insulation. The first of these figures is approximately what would be expected for arcs of corresponding length in air between copper electrodes, but the second is very much too high for an arc drawn in air and unexposed to any gas blasts. This arc voltage was high enough to raise the power factor of the circuit considerably, which favored the arc extinction. A study of the oscillograms shows that in some cases power factors of from 0.3 to 0.6 were reached.

In general it was found that when the arc was closely bounded by insulation, the arc would be promptly extinguished, but in those tests where the insulation had been carefully cleaned away about six or more inches from the arc, then the arc would persist. As previously mentioned, it was also found that the arc voltage was about three to four times as large when the arc was closely bounded by insulation, as when remote from insulation.

These results speak eloquently of the large part the insulation plays in the extinction of the arcs, and show how persistent the arcs are when not subjected to the influence of the insulation.

VI. Tests on Short Arcs Free from Insulation

It seemed worth while to obtain some data on the extinction of short arcs between copper electrodes free from insulation, for currents larger than those used

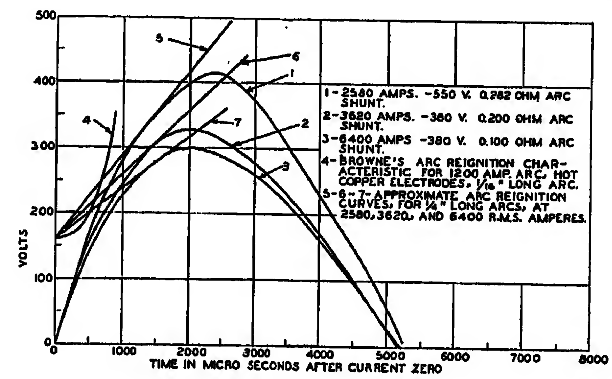

Fig. 2—Circuit Reignition Characteristics on Tests with Hot Copper Electrodes

by Browne. Accordingly, a pair of copper plates ¼ in. thick and 4 in. by 27 in. were used, spaced apart ¼ in. To keep the gas pressure between the plates low, ½-in. diameter holes, spaced 1.41 in. between centers were punched in the plates.

The arc was started by a No. 20 copper fuse wire drawn through one pair of holes. The current was obtained from transformers and was limited only by air-cored reactors where it was not the short-circuit current of the transformers.

Tests were made at various voltages and currents, and values of resistance which when in shunt to the arc would just cause it to go out were determined. With these values of resistance the circuit reignition characteristic should just be tangent to the arc reignition characteristic. Numerous circuit reignition characteristics were calculated, and several are shown in Fig. 2. In this figure, the curve of Browne from Fig. 1 is redrawn. Also, probable arc reignition characteristics for the larger currents and spacing are indicated by drawing tangents from the 160-volt point on the

voltage axis to the various circuit reignition characteristics.

Arc voltages in these tests were found to vary from 40 to 100 volts.

These tests show decisively that the simple short arc between copper electrodes free from insulation is inadequate to meet the fault clearing requirements of low-voltage a-c. networks.

VII. Tests with Short Arcs Bounded by Organic Insulation

Further tests were made with the electrode arrangement of the preceding section, but with oil soaked paper clamped between the copper plates. A ⅛-in. hole was cut in the paper through which the fuse was passed for starting the arc.

At 550 volts the arc was extinguished for all values of current tried, up to 11,000 amperes. Similar results were obtained as the voltage was increased up to 750 volts. At 1,150 volts, the arc persisted. The arc voltage was found to vary from 300 volts at low currents to 600 volts for high currents.

As the size of the hole in the paper through which the arc initiating fuse passed was increased, the influence of the oiled paper became less intense, and when the hole had a diameter of ⅜ in., the arc persisted at 550 volts, and the arc voltage was only about 200 volts.

Tests were also made in which the paper was impregnated with vaseline, and with glycerine. The results were practically the same as for the oiled paper.

The above tests again affirm the important part played by the insulation in the extinction of cable arcs.

VIII. Tests with Short Arcs Bounded by Inorganic Insulations

As has been mentioned before, the gases resulting from the decomposition of organic insulation, and which are effective in extinguishing the arc, are in themselves combustible, and therefore under certain circumstance may constitute an explosion hazard. It seemed worth while, therefore, to make some preliminary tests using materials which would give off incombustible gases such as water or carbon dioxide.

The following materials were tried:

Material	Decomposition products	
	Gases	Solid
Boric Acid, $H_3 BO_3$	H_2O	B_2O_3
Gypsum, $CaSO_4\ 2H_2O$	H_2O, SO_2, O_2	CaO
Borax $Ha_2B_4O_7\ 10H_2O$	H_2O	$Na_2B_4O_7$
Sodium Bicarbonate, $NaHCO_3$	H_2O, CO_2	$NaOH$
Magnesium Carbonate $M_3CO_3\,3H_2O$	H_2O, CO_2	MgO
Basic Magnesium Carbonate $3MgCO_3Mg(OH)_23H_2O$	H_2O, CO_2	MgO
Magnesium Oxide, MgO		MgO
Ammonium Carbonate $(NH_4)_1\ CO_3H_2O$	H_2O, CO_2, NH_3	
Aluminum Ammonia Sulfat. $Al_2(SO_4)_3(NH_4)_2\ SO_424H_2O$	H_2O, SO_2, NH_3	Al_2O_3

The various materials in the form of powder were packed between the copper plates described in sections VI and VII, and a fuse wire pulled through a pair of holes in the two plates. Tests were then made, as in preceding sections. The arc was permitted to make as large a hole as it would in the closely packed powdered material.

Before comparing the results obtained with the various materials, it will be well to consider the characteristics which are found for arcs subjected to deionization from decomposing bounding walls. Fig. 3, which gives results for basic magnesium carbonate, is typical for the relation between the volts which can be interrupted by a given length of arc, and the current. As the current is increased from small values, the volts which can be interrupted decrease, reach a minimum and then increase again. This relation is rather to be expected if it is reflected that for very large currents the gas blast will be very intense, and that for very small currents the arc will be particularly sensitive to deionizing influences.

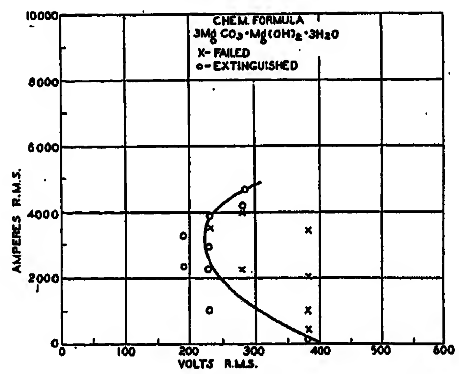

Fig. 3—Basic Magnesium Carbonate as Arc Extinguisher

Used between parallel copper plates spaced ¼ in. apart, without arc shunting resistance

Hence starting with low voltages, all currents can be interrupted by the arc. Above a certain voltage, however, (the minimum voltage on the curve of Fig. 3) there will be two limits of current, such that between these limits the arc will not be extinguished. For values of current less than the lower limit and for values of current greater than the larger limit, the arc will interrupt the circuit, but for intermediate values of current, the arc will persist. Such relations as these are found generally in expulsion fuses, and oil circuit breakers.

If now, the voltage of the circuit is kept constant and only the circuit reignition characteristic is altered, similar results are obtained. Below a certain "speed" of circuit all currents are interrupted, whereas for higher "speeds" there will be an intermediate range of current for which the arc will persist.

This is illustrated in Fig. 4, which shows the results of tests using gypsum powder. All these tests were

at 550 volts, with varying values of resistance shunting the arc giving different circuit reignition characteristics. For shunting resistances less than 16 ohms, the arc interrupted all values of current, but for resistance greater than 16 ohms there would be an intermediate range of current for which the arc would persist. When the arc was unshunted the arc would persist for currents from 2,800 amperes down to 400 amperes, the smallest tried, although undoubtedly for a sufficiently small

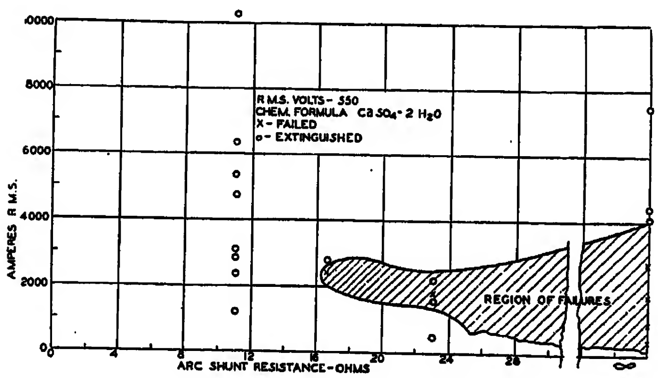

Fig. 4—Gypsum as an Arc Extinguisher

Used between parallel copper plates spaced ¼ in. apart

current the arc would have been extinguished. For currents larger than 4,100 amperes the arc always interrupted the circuit.

As it so happens that most of the data for the different powders were obtained at 550 volts with varying resistance shunts, we may compare their efficacy by plotting in one figure curves similar to that of Fig. 4. This has been done in Fig. 5. No curve is given for boric acid, as the arc interrupted all values of current even when unshunted, except for one test which did not repeat itself. The curve for gypsum is taken from Fig. 4, and as is seen consists of the boundary of the region of arc persistency. Because of the smallness of the number of test points in Fig. 4, the location of this boundary is rather vague, so that although a definite curve is shown in Fig. 5, no accurate quantitative value should be attached to it. Similar remarks apply to the curves for the other powdered materials, which were obtained from test data similar to that shown in Fig. 4. For the other materials only the lower branch of the curve is shown, as the other branch if it existed, occurred at values of current greater than those available for these tests. For comparison, the curve for air is also included in Fig. 5.

From all these data we conclude that boric acid made the best arc extinguishing powder being as good as oiled paper over the range of these tests. Next, although considerably inferior, is gypsum. Then far inferior, though better than plain air, are the rest of the materials tried.

IX. Requirements of Gas-Generating Arc-Extinguishing Substances

In considering the various substances which may be placed adjacent to an a-c. arc to cause its extinction by the generated gas blast, one would expect that that substance would be best which generated the largest amount of gas in a given volume, or which generated the largest amount of gas for a given energy input from the arc. However, the results of the preceding sections show that some other factor must also be taken into account. For example, boric acid and oiled paper, which proved to be the best among the substances tried, are certainly inferior to ammonium carbonate, $(NH_4)\ (CO_3H_2O)$, one of the worst of the substances tried from the standpoint of gas generation per unit volume or per unit energy.

This other factor probably relates to the state of the surface of the bounding material just after the moment of arc extinction. The gases generated from the substance, may ensure that the arc will not reignite in the arc space itself, but that with the arc not reigniting, the substance itself must be able to stand the voltage without breaking down. Organic insulation, in general, seems to meet this requirement. Some of the insulation close to the arc is decomposed into gases, but the surface of the material which is left remains clean, and has good insulating quality.

In the case of the ammonium carbonate, however, because this substance melts below the temperature at which rapid decomposition takes place, the surface

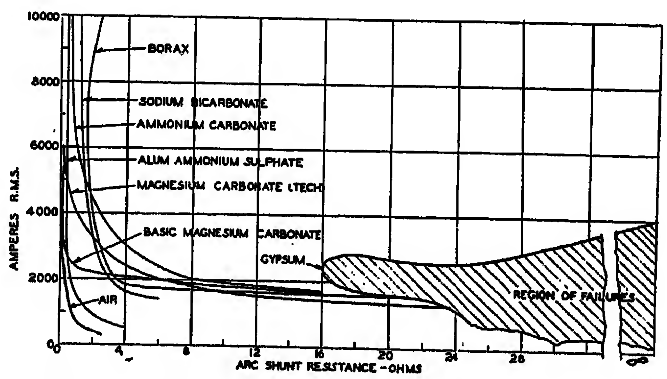

Fig. 5—Powder Fillers as Arc Extinguishers

Used between parallel copper plates spaced ¼ in. apart r. m. s. volts 550

of the material which is left after a half cycle of arcing undoubtedly consists of a molten film of the ammonium carbonate. Furthermore, molten ammonium carbonate is highly conducting. Just after current zero, the arc is undoubtedly extinguished momentarily, and open-circuit voltage momentarily appears across the electrodes. This voltage causes a considerable current to flow in the molten surface film, which is brought immediately to a violent boil and gas evolution, and the resulting tearing of this film while carrying current will restart the arc.

Boric acid also has its melting point below that at which rapid decomposition takes place, but the electrical resistivity of the molten boric acid is very high. Hence, just after the arc extinction at current zero, when open-circuit voltage is restored, the current sent through the molten surface film will be small, even for voltage gradients as high as those used in the tests, namely 550 volts r. m. s. across 1/4 in. The current will be so small, that the power will be insufficient to keep the boric acid film hot, and it will recongeal, thus permanently opening the circuit.

While organic insulation in general has the property of being left with a surface of good dielectric quality after exposure to an arc, there is a circumstance important in practical a-c. network operation under which the insulation may be expected not to have this property. This is when the insulation has been subjected to a moderately high temperature so that it is more or less completely charred before the arc begins to act upon it.

Such a circumstance may arise for example when a very low-resistance fault forms at a point in a network where there is available only a small short-circuit current, which may take a very long time to melt apart the low-resistance fault. Because of this long time, the whole cable leading to the fault may be heated and the insulation charred. By the time the arc forms, the organic insulation is no longer in condition to cause its extinction and the arc burns along the cable until it reaches some point where the insulation, due to lower temperatures, is still in condition to be effective in causing extinction of the arc.

The charred insulation is ineffective in causing the extinction of the arc first because it has lost its more volatile constituents so that it produces much less gas per unit volume than before, and second, because now after exposure to the arc, the surface which is left is almost entirely free carbon and a good conductor while hot. With normal organic insulation, the rapid evolution of gas seems to scour away the surface and keep it clean of carbonaceous residues, but the charred insulation has lost this property.

X. Summary and Conclusions

1. Short arcs between copper electrodes and not adjacent to insulation are inadequate to meet the fault clearing requirements of low-voltage a-c. networks of more than 130 volts.

2. Actual low-voltage networks of higher voltage than 130 volts depend for the extinction of arcs formed at faults upon the strongly deionizing action of gas blasts coming from decomposing adjacent insulation.

3. The extinction of arcs in cables is strongly affected by the degree of remoteness of the insulation from the arc. In a 1/8-in. hole in oiled paper insulation, a 1/4-in. long arc will interrupt current at 750 volts, whereas in a 3/8-in. hole it will fail to interrupt current at 550 volts.

4. Inorganic insulating materials giving off noncombustible gases are also effective in aiding arc extinction. Of the various materials tried, boric acid was the best.

5. Charring of organic insulation may be expected to cause it to lose its arc extinction aiding properties.

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Discussion

W. R. Bullard: The statement at the end of the paper is interesting in that the arc extinguishing features of network cables would be destroyed by the pre-carbonization of the cable insulation. This fact was discovered about four years ago during some tests made by the Cia Cubana de Electricidad in Havana, Cuba under the direction of the writer and was subsequently published in the *Electrical World*. I feel that most of the few sustained arcs which have occurred in actual operation of a-c. networks have probably been due to this particular phenomenon.

The portion of this paper dealing with the arc extinguishing properties of non-inflammable gases is very interesting and it is appropriate to inquire whether practical application may not be found for the knowledge thus gained. Intensive study in this direction would seem to be in order since the elimination of inflammable gas production during short-circuit conditions would be highly desirable, whereas our present knowledge based on the work outlined in this paper would indicate that the production of gases of some kind is necessary for proper arc extinguishing characteristics.

W. G. Dow: Three of the arc reignition characteristics shown in Fig. 1 of the paper converge to a reignition voltage of 230 at the extreme left. In contrast with this, the tests described in our paper see page 854 indicate convergence to a value between 340 and 380 volts. The regularity with which the short-time reignition voltage repeated itself in our work was in fact very striking. There are several possible explanations for the difference between the 230-volt figure in one paper and the 340- to 380-volt values in the other.

1. Slepian and Strom's arc reignition characteristics should correspond, from the method used in their determinations, to the lower envelope curves of Figs. 15 and 16 in our paper. Thus while their work makes no distinction between various individual restriking voltages, it does give rise to curves which mark the boundary between the region on the graph in which reignition cannot occur (below our bottom envelope) and that in which it may occur (above our bottom envelope).

Our work gives rise to a lower envelope curving *upward* toward its left end; their arc reignition characteristics show no such behavior. It seems to us that their method of test would not permit discovery of such an upward curvature. Their arc reignition characteristic is drawn tangent to all of a family of circuit reignition characteristics having voltage and resistance values such as to just fall short of producing reignition. The resistance shunt type of circuit which they used produces a circuit reignition curve which is always rising. No such curve could ever be tangent to that part of an arc reignition characteristic which might lie to the left of a minimum such as our lower envelope has at about 25 microseconds.

Tests of the type used by Slepian, Strom, and Browne[1] in locating the extreme left end of their arc reignition characteristics would be made with relatively low-circuit voltages (below 300 volts peak) and high-shunt resistances. Now if the true curve actually has such a minimum point as our lower envelope exhibits, the peak voltage of the test circuit might be considerably lower than the short-time reignition voltage, and yet be sufficient to cause reignition at from 20 to 30 microseconds after the current zero. If the shunt resistance is adjusted until reignition just does occur, it will still be occurring at or near the minimum point on the curve. The results of the test would in that case have little or no relation to the true value of reignition voltage required at one or two microseconds after the current zero.

In brief, if the true arc reignition characteristic does have a minimum value at a time later than zero:

(a) The method of test used by Slepian and Strom may not discover such a minimum.

(b) The value of short-time reignition voltage derived from their tests (230 volts) will be considerably too low.

2. In determining the family of circuit reignition curves to which their arc reignition curve is tangent, no account was taken of the distributed capacity, nor of the possibility that the current might stop prior to reaching its zero value. In order to operate on moderate voltage circuits with the large currents used in their tests, the series inductance must have been rather small. This would tend to minimize the effect of the arc-failure current being greater than zero. A very small value of distributed capacity, however, increased the probability of a dip to a negative glow before voltage recovery.

In general, both of the effects just mentioned, if considered, would tend to shift all of the arc reignition characteristics determined by Messrs. Slepian, Strom, and Browne[2] toward the right.

3. All of the arc reignition characteristics appearing in the paper by Messrs. Slepian and Strom result from tests using currents of 300 amperes or more. In many cases the arcs were in rapid motion. In our work the currents were between 25 and 30 amperes, and the arc relatively stationary. These rather striking differences may of course provide the true explanation for the differences between the two sets of arc reignition characteristics. In fact, it seems rather surprising that the two sets of tests do not give more divergent results than they do.

Perhaps the most interesting part of their work is that relating to the effects of the presence of impregnated paper and of boric acid in accelerating deionization. I would like to point out in this connection something that is very commonly lost sight of: namely, that the presence of any surface whatever, liquid or solid, conducting or non-conducting, is a great aid to recombination of electrons and positive ions wherever the ion concentration is at all great. Recombination in the volume of a gas not adjacent to a surface is always much less important than that at any surface. Ions disappear at the walls of an ionized region much as water flows over the edge of a dam; that is, practically every ion or electron that reaches the walls in the course of its random motions at once ceases to play any further role as a current carrier.

The existence of turbulent motion due to evolved gases will of course freshly expose various parts of the ionized region to surfaces, as well as exerting a cooling action which makes more difficult the production of new ions to replace those that recombine.

1. See discussion by T. E. Browne, Jr. page 868.

2. *Loc. cit.*

Reignition of Metallic A-C. Arcs in Air

BY S. S. ATTWOOD,* Member, A. I. E. E.
W. G. DOW* Associate, A. I. E. E.
and
W. KRAUSNICK† Member, A. I. E. E.

Synopsis.—Developments made in circuit breakers in the last two years have emphasized the necessity for obtaining experimental evidence of the current-voltage-time relationships that exist during the period when the a-c. arc between metallic electrodes passes through its cyclic current zero. Twenty-nine cathode-ray oscillograms of these relationships are presented.

During the current zero period the arc electrode voltage is determined by the circuit constants and rises until the electrode voltage reaches the breakdown or reignition value, which is determined by the deionizing influences at work while the arc is extinguished. Alteration of the circuit constants permits a variation in the rate of voltage rise with a consequent change in the reignition voltage.

Permanent extinguishment of the arc occurs when the gap breakdown voltage has risen, due to deionizing influences, to a value that cannot be reached by the electrode voltage controlled by circuit constants.

The action of a circuit breaker in extinguishing an arc is greatly influenced by the presence of adjacent load circuits and by the presence of distributed inductance and capacity in the connecting lines.

Introduction

RECENT work on the general subject of arc extinction as related to switch and circuit breaker action has emphasized the need for more accurate information concerning the conditions existing in an a-c. arc during the period of current zero. In particular, experimental evidence has been needed to show the rate at which the arc voltage rises toward a reignition value in the new circuit direction. This paper presents a study of the voltage and current relationships near the cyclic current zero in arcs between metallic electrodes, at moderate values of current and voltage, based on some two hundred cathode-ray oscillograms. The cathode-ray oscillograph is necessary inasmuch as the voltage may change at the rate of several hundred millions per second.

Records were taken with various circuit conditions, and with different electrode materials, shapes, and spacings. The electron beam in the oscillograph was initiated by a surge circuit controlled by a rotating contact synchronously driven from the same voltage source as that which supplied the arc. The most consistent results were obtained with an arc between copper plates 8 in. sq. and ¾ of an in. apart. This arrangement gives an arc of constant length and one which reaches its current zero at nearly the same part of each half cycle. For most of the work the power source was a transformer supplying 440 volts at 60 cycles. However, for certain circuit arrangements requiring high reignition voltages a 660-volt supply was used. The currents used were 25 amperes or less.

This investigation shows that when a small value of arc current is reached the current breaks sharply to substantially zero, where it remains until the discharge starts again with the opposite polarity. During this period the voltage across the arc electrodes rises according to the usual circuit equations until breakdown occurs, yielding first a glow discharge in the arc gap and later an arc with the new polarity. Variation of the circuit constants offers the opportunity to alter the rate of the voltage rise. It is found that a voltage rising slowly must rise to a relatively large value before breakdown occurs. The curve of breakdown voltage against time has been discussed by Dr. Slepian from a theoretical basis.[1] This study presents the experimental evidence that permits this curve to be found and shows the modifications that must be made in it.

Moreover these oscillograms show the influence that nearby loads exert upon arc extinction in circuit breakers and emphasize the necessity for further study

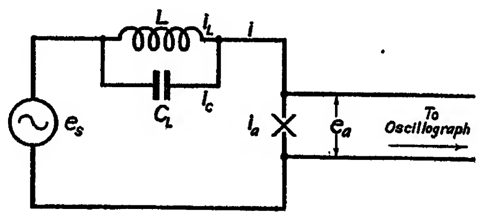

FIG. 1—CIRCUIT FOR UNSHUNTED ARC

of deionizing agents in the arc gap and of the influence of long lines with distributed constants upon arc extinction.

Arc Current—Voltage Relations

I. Unshunted Arc

The arc current in the tests made was limited by an air core series reactor of sufficient size to make the current lag nearly 90 deg. behind the supply voltage. Fig. 1 shows the circuit composed of the transformer of voltage e_s connected through the reactor L, with its small (but not negligible) distributed capacitance C_L, to the arc. The arc voltage e_a was measured by the oscillograph. The time relationships existing between the supply voltage e_s, the arc voltage e_a and the arc current i_a are shown qualitatively in Fig. 2. Except

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Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

1. *Extinction of a Long A-C. Arc*, J. Slepian, A. I. E. E. TRANS., Vol. 49, 1930, p. 421.

at instants near the current zero the current is practically a sine wave nearly 90 deg. behind the supply voltage. Throughout this part of the cycle the arc current i_a is practically the same as the current in the inductance, i_L.

Eventually the arc current reduces sinusoidally to a small value (order of 0.1 ampere) I_1 at which it suddenly fails, falling very sharply to practically zero, where it remains until the arc again is made with current in the opposite sense. In some cases this transition period to the arc in the new direction has lasted for 600 microseconds, which is 7 per cent of one-half cycle at 60 cycles. Throughout this period the supply voltage is practically constant ($e_s = E$).

The current i_L in the inductance cannot drop instantaneously and therefore surges into its own distributed capacitance. From $t = 0$ to $t = t_1$ the voltage e_a may be computed by the usual circuit equations, inasmuch as the arc current is practically zero. See Appendix A.

$$e_a = E - E_m \cos(\omega t + \theta) \qquad (1)$$

where

$$\omega = \frac{1}{\sqrt{L C_L}} \qquad E_m = \sqrt{(E - E_1)^2 + \left(\frac{I_1}{\omega C_L}\right)^2}$$

E = the maximum of the supply voltage.
E_1 = the arc voltage when the arc fails.
I_1 = the arc current when the arc fails.
e_a = the arc voltage during the period of $t = 0$ to $t = t_1$

$\theta = \tan^{-1} \dfrac{I_1/\omega C_L}{E - E_1}$ = angle from arc failure to negative maximum.

Particular note should be made of the fact that both the magnitude and the phase position of the negative voltage maximum are determined largely by the magnitude of the arc current when the arc fails. The frequency of the oscillation represented by this equation was of the order of 30,000 cycles per sec. in the circuits used. During this period from $t = 0$ to $t = t_1$ the current has changed from an extremely small negative value (much less than the 0.1 ampere arc-failure current) through zero at the zero of voltage to a small positive value. The current appears to change at approximately a uniform rate throughout this interval.

At time t_1 the voltage across the electrodes no longer rises. In fact it drops quickly to a voltage e_g, where it remains practically constant for a period of from zero to as long as 500 microseconds. The steady voltage e_g is indicative of the existence of a glow state (high-voltage small current arc). The glow voltage is of the order of 350 volts. During the glow the current though still very small rises at a considerably greater rate than during the preceding period. Eventually the glow current rises to a value sufficient to maintain an arc proper (low-voltage, relatively high-current phenomenon). This change occurs at $t = t_2$. At this moment the electrode voltage drops with great rapidity from the glow value of about 350 to the arc value of about 75, the latter depending on the length of the arc. Throughout the remainder of the half cycle the arc voltage remains constant, but the arc current exhibits its sine character.

Typical oscillograms illustrating this circuit condition for arcs between iron, aluminum, and copper electrodes respectively are shown in Figs. 3A, B, and C. In these three figures the starting glow voltages are 360, 310, and 340. The breakdown voltages preceding the glows are 380, 320, and 380 occurring in 24, 27, and 20 microseconds respectively. There appears to be far less difference between these kinds of electrodes, at least, than between various circuit conditions for only one electrode material.

It has been shown that the negative electrode voltage dip is due to the sharp break in the negative arc current and depends for its magnitude upon the value of the current at the moment it fails. It is possible for the current to fail earlier in the cycle at a larger negative value. In this case the negative electrode voltage may drop very sharply, in practically linear manner with time, to a sufficient value to produce a negative glow which lasts for a few microseconds and dies out. After this the electrode voltage follows the usual cosine curve given by equation (1). Fig. 3D illustrates the two types of negative voltage dip that may occur. This particular oscillogram is one of an 0.0096 μf. capacity shunted across the arc, but similar negative dips with the unshunted arc have been observed frequently.

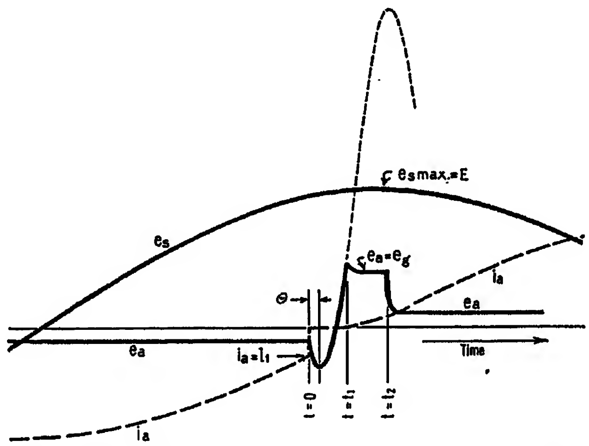


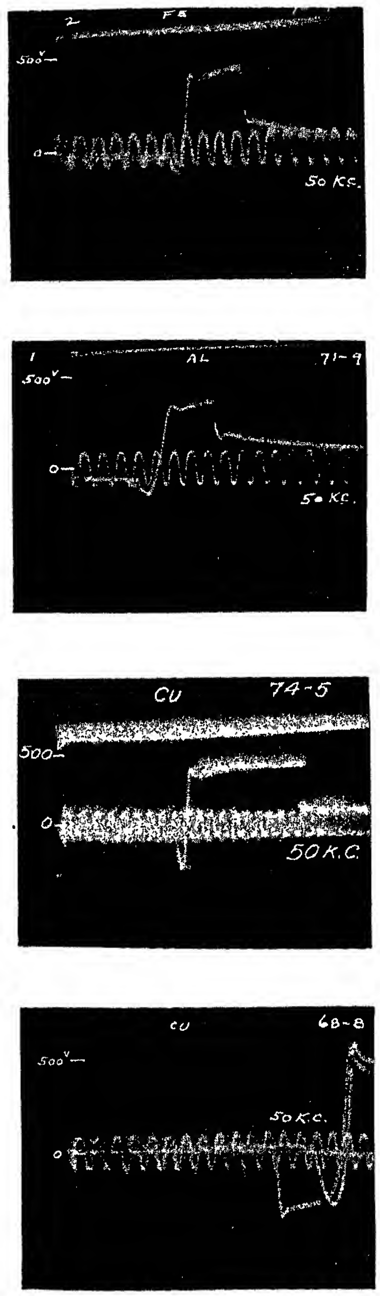
FIG. 2—UNSHUNTED ARC CURRENT AND VOLTAGE

The sudden changes of the volt-ampere relationships in the electrical discharge that is passing at low-current values are best understood in terms of the volt-ampere characteristics of the arc and glow discharge in air at atmospheric pressure. Compton[2] gives the type of characteristic shown in Fig. 4A, based on the use of direct current over long time intervals. The oscillo-

2. *The Physical Nature of the Electric Arc*, Compton, A. I. E. E. TRANS., 1927, p. 868.

grams presented in this paper clearly show the existence of both the arc and glow states at low arc current values, but they indicate that the shape of the volt-ampere characteristic is appreciably different from that shown in Fig. 4A.

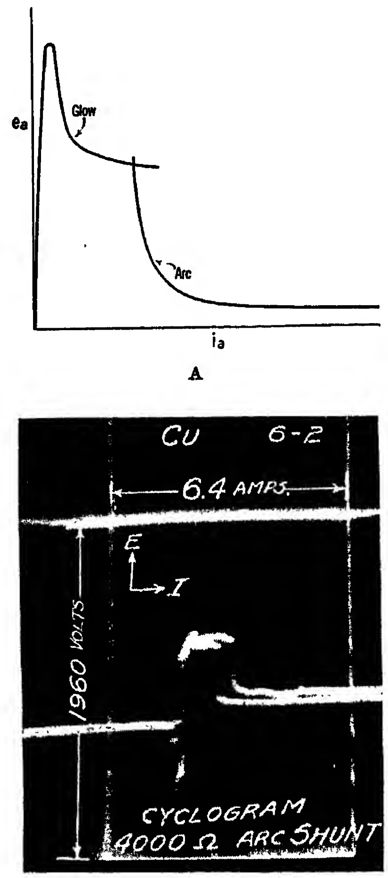
In the a-c. arc, with conditions changing with great



A B C D

FIG. 3—VOLTAGE OSCILLOGRAMS FOR *Fe*, *Al* AND *Cu*, UNSHUNTED ARCS. 3D ILLUSTRATES NEGATIVE DIP (ROUND CURVE), AND NEGATIVE GLOW (PEAKED CURVE)

rapidity, the arc current may fail at larger or smaller values, the larger values producing a negative glow, the smaller values failing to do so. A positive arc voltage produces a very small current growing with the voltage until the glow voltage is reached, when the current rises at practically constant voltage. This condition continues until the arc current has reached a value sufficient to maintain an arc proper, whereupon the voltage drops sharply (about 400 to 75) and the current returns to its sinusoidal variation at constant arc voltage. Fig. 4B is a cyclogram of arc voltage against arc current illustrating this point. This cyclogram was taken with a 4,000 ohms shunt across the arc, but it is representative of the unshunted arc case. It should be noted in the cyclogram that the curve has a different shape for the two polarities in the alternating-current case, while an extension of Fig. 4A to include negative



B

FIG. 4—VOLTAGE CURRENT CYCLOGRAMS

A. Direct current and long time (Compton)
B. A-c. short-time oscillogram

currents for the direct-current situation would be a perfect reflection of the curve as shown but with reversed polarity.

II. RESISTANCE SHUNTED ARC

The circuit used for the resistance shunted arc is shown in Fig. 7A. The electrode voltage during the arc current zero for a resistance shunted arc must be written in two forms, depending upon whether the resistance R is greater or less than the critical resistance R_c, which is equal to $\frac{1}{2}\sqrt{\frac{L}{C_L}}$. See Appendix B.

For resistances larger than the critical resistance the electrode voltage becomes

$$e_a = E - e^{-\frac{1}{2RC_L}t}\left[(E - E_1)\cos\omega t + \frac{1}{\omega C_L}\left(-I_1 + \frac{E - E_1}{2R}\sin\omega t\right)\right] \tag{2}$$

$$\omega = \sqrt{\frac{1}{L C_L} - \frac{1}{4R^2 C_L^2}}$$

$$R > \frac{1}{2}\sqrt{\frac{L}{C_L}}$$

See Appendix B-I.

Figs. 5A, B and C illustrate this condition. For the first two of these cases with an infinite resistance equation (2) reduces to the simpler form of equation (1). Equation (2) holds for Fig. 5C inasmuch as the 10,000-ohm arc shunt is greater than the critical resistance of 6,125 ohms.

The circuit conditions back of all of the oscillograms of Fig. 5 were identical except for the shunt resistance, and the maximum value of the line voltage varied only slightly from 975. The series reactor possessed an inductance L of 0.066 henrys and a distributed capacity C_L of 440 $\mu\mu$f. This distributed capacity may easily be calculated by applying equation (1) to one of the curves of Fig. 5B. From the magnitude and phase position of the negative dip maximum ω may be computed and hence C_L found. The arc was struck between the centers of two 8 in. by 8 in. parallel copper plates spaced ¾ of an in. apart. The arc was semi-enclosed to prevent effects from stray air currents.

For resistances smaller than the critical resistance the electrode voltage takes the form

$$e_a = E - e^{-\frac{1}{2RC_L}t}\left[(E - E_1)\cosh\beta t + \frac{-I_1 + \frac{E - E_1}{2R}}{\beta C_L}\sinh\beta t\right] \tag{3}$$

$$\beta = \sqrt{\frac{1}{4R^2 C_L^2} - \frac{1}{L C_L}}$$

$$R < \frac{1}{2}\sqrt{\frac{L}{C_L}}$$

See Appendix B-II.

Figs. 5D, E, F, G and H for shunts of 4,000, 2,000, 1,200, 800 and 500 ohms respectively may be represented by equation (3).

For purposes of computation equation (4) may be used in place of equation (3) from which it has been reformed.

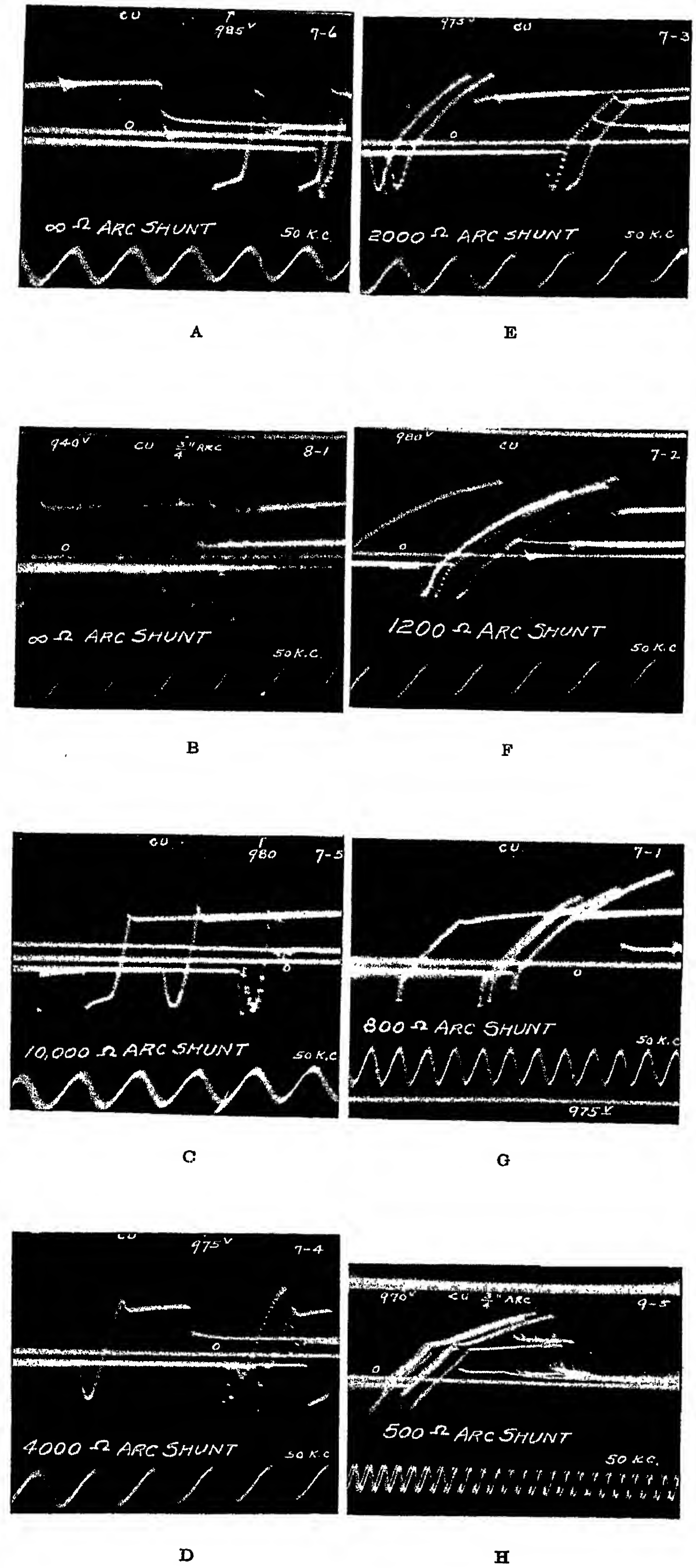


FIG. 5—VOLTAGE OSCILLOGRAMS FOR RESISTANCE SHUNTED ARC. 25-AMPERE, 660-VOLT, ¾-IN. ARC BETWEEN FLAT COPPER ELECTRODES

$$e_a = E - \frac{1}{1-\Delta_1}\left\{ e^{-\frac{Rt}{L}(1+\Delta_3)}\left[(E-E_1)\left(1-\frac{\Delta_1}{2}\right) - I_1 R\right] - e^{-\frac{t}{RC_L}\left(1-\frac{\Delta_1}{2}\right)}\left[(E-E_1)\frac{\Delta_1}{2} - I_1 R\right]\right\} \quad (4)$$

where

$$\alpha = \frac{4 R^2 C_L}{L}$$

$$\Delta_1 = 1 - 2 R C_L \beta = 1 - \sqrt{1-\alpha} = \frac{\alpha}{2} + \frac{\alpha^2}{8} + \frac{\alpha^3}{16} + + +$$

$$\Delta_3 = \frac{2\,\Delta_1}{\alpha} - 1 = \frac{\alpha}{4} + \frac{\alpha^2}{8} + + +$$

As t increases the second term in the large bracket becomes very small, and when $t > \frac{\log_e 10}{\beta}$ the error introduced by its omission is less than one per cent.

The 500-ohm shunt of Fig. 5H is the lowest resistance at which it was possible to operate the arc. At and near this border line resistance the electrode voltage shows a peculiar hump at approximately the glow voltage. Sometimes the electrode space breaks into the glow at

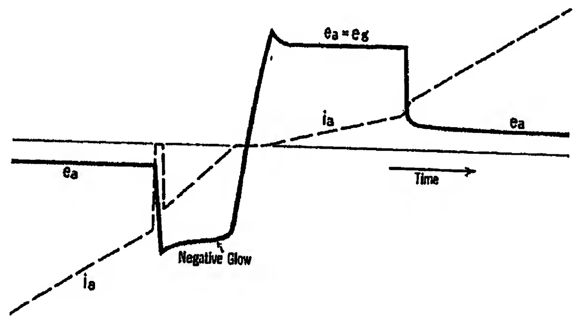


Fig. 6—Voltage and Current for Resistance Shunted *Cu* Arc with Negative Glow

this stage and at other times the voltage rises to a higher breakdown point, then drops to glow or arc value.

The arc current curve takes the general form shown in Fig. 2 if the electrode voltage goes through its round negative dip. However, if the sharp voltage drop leading to a negative glow occurs (Fig. 6), the current drops to zero simultaneously but at once rises to a value sufficient to sustain a negative glow, after which it drops gradually throughout the existence of the negative glow, reaching zero at the end of the glow period. From this time on until the positive breakdown voltage is reached the current is extremely small and the voltage equations given hold.

The reactor current can be obtained by mechanical integration from the electrode voltage curve, provided the reactor distributed capacity is relatively small.

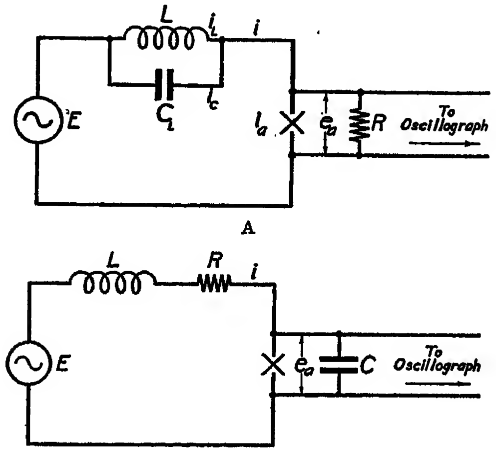
$$E - e_a = L\frac{d\,i}{d\,t}, \text{ so that}$$

$$i = \int (E - e_a)\, d\,t + K$$

Subtracting the current through the shunt resistance gives the arc current.

$$i_a = i - i_R = \int (E - e_a)\, d\,t + K - \frac{e_a}{R} \quad (5)$$

The effect of the arbitrary constant may be eliminated



B

Fig. 7—Circuits for Resistance and Capacity Shunted Arcs

A Arc shunted by resistance
B Arc shunted by capacity

by choosing the arc current as being zero when the arc voltage is zero.

III. Capacity Shunted Arc

The circuit used for the capacity shunted arc is shown in Fig. 7B. The small distributed capacity of the series reactor C_L may be neglected in comparison with the capacity across the arc C, inasmuch as the latter is far the greater.

The electrode voltage is given by equation (6), for the case of $\frac{R^2}{4\,L^2} < \frac{1}{L\,C}$. See Appendix C.

$$e_a = E + e^{-\frac{R}{2L}t}\left[-(E-E_1)\cos\omega t + \frac{I_1 - \frac{R\,C}{2\,L}(E-E_1)}{\omega\,C}\sin\omega t\right] \quad (6)$$

$$\omega = \sqrt{\frac{1}{L\,C} - \frac{R^2}{4\,L^2}} \qquad \frac{R^2}{4\,L^2} < \frac{1}{L\,C}$$

The studies presented in this paper involved a series resistance so low that it might be neglected. Making $R = 0$ equation (6) reduces to the form shown in equation (7), which is identical with equation (1)

A

B

C

D

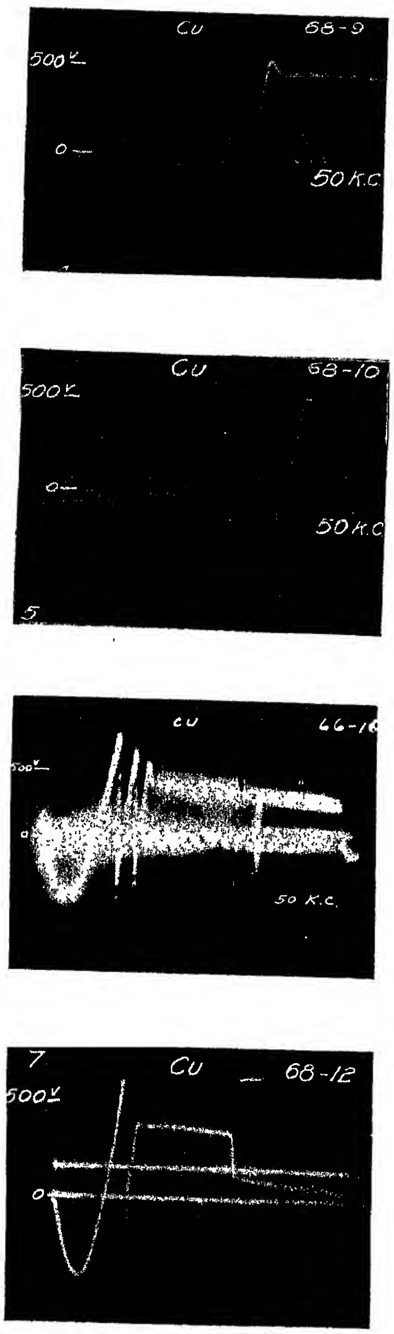


Fig. 8—Voltage Oscillograms for Capacity Shunted *Cu* arc

providing the capacity of the coil is now replaced with that placed across the arc.

$$e_a = E - E_m \cos(\omega t + \theta)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\theta = \tan^{-1} \frac{I_1}{\omega C (E - E_1)} \qquad (7)$$

$$E_m = \sqrt{(E - E_1)^2 + \left(\frac{I_1}{\omega C}\right)^2}$$

The electrode voltage, with capacity shunt, may exhibit the rounding negative dip or the sharp drop that leads to the negative glow, as shown in Figs. 8A and B. In these figures is shown also the possibility of having either a glow or an arc state immediately following the gap breakdown. The general character of these curves is similar to those for resistance shunted arcs. In the latter however the positive glow state almost universally appears.

All four electrode voltages shown in Fig. 8 were obtained by using a capacity of 0.0125 μf. Figs. 8C and D illustrate a state that exists only with a relatively large capacity. In these figures it is seen that the voltage following the initial breakdown may break to a negative value or may develop into a sawtooth wave. Since the electrode and the capacity are shunted, the extremely rapid drop in voltage indicates a short time discharge of the condenser, or spark through the arc space. The negative voltage overshoot may be due to the small inductance of the condenser arc path or to some internal arc ionic condition. In the case of Fig. 12D, where the timing is stretched out sufficiently to permit measurement, the average arc current was found to be 13 amperes during the first spark and approximately 8 amperes during each of the succeeding sparks.

Following the spark the electrode voltage, starting from approximately zero, rises again according to equation (7) but at a steeper slope because the initial current I_1 in the series reactor path has grown considerably since the first breakdown. Spark after spark may follow until the series reactor current has grown to a point where a glow or an arc may be maintained in the gap. A sharp crackling and hissing noise accompanies the operation of this type of arc.

The oscillograms shown in Figs. 9A and B give the arc current and voltage for identical conditions, except that it was necessary to introduce a small coil into the arc path in order to measure the current. The capacity was 0.0125 μf. The oscillograms were taken only a few seconds apart. The current picture shows clearly the initial break in the current, the prolonged current zero period and the spark that occurs whenever the electrode voltage drops suddenly. The current curve has been offset from the zero line. The arc current curve has approximately the same slope as the series reactor current for the condition of the arc short-circuited, as it should.

Figs. 10A and B are oscillograms of a cyclogram and an arc voltage taken a few seconds apart under the same conditions as those of the preceding figure. The

new point to be observed is the loop on the cyclogram which represents the current-voltage relation during the transition from the glow to the arc. Probably the loop should be pointed in the current direction; the rounding may be partially due to the inductance of the small coil inserted in the arc path for measurement purposes. The records on Fig. 11A, taken at intervals of a few seconds, show the current and voltage with the same capacity (0.0125 μf.) when a sawtooth occurs.

A

B

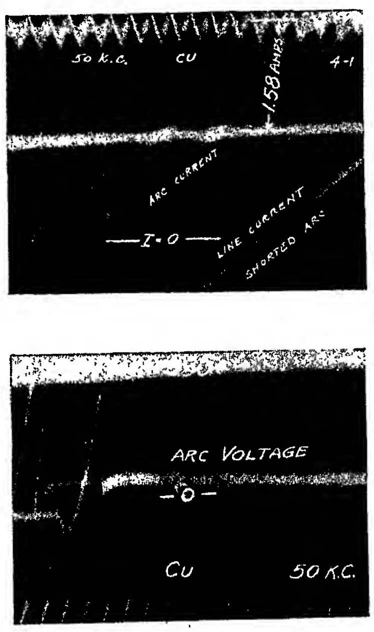


FIG. 9—CURRENT AND VOLTAGE OSCILLOGRAMS FOR 0.0125 μf SHUNT

Fig. 11B is the complex type of cyclogram that may be expected in this case.

The effects of varying the capacity shunted across the arc are shown in Fig. 12. Below 0.01 μf. the gap goes through the glow to reach the arc stage, but for larger capacities the sawtooth replaces the glow interval and the voltage changes require longer and longer periods. For the larger capacities the negative voltage dip is missing, not because the current continues until I_1 is zero but because the term $\frac{I_1}{\omega\, C}$ in equation (7) becomes negligibly small compared to $E - E_1$.

ARC-FAILURE CURRENT VALUES

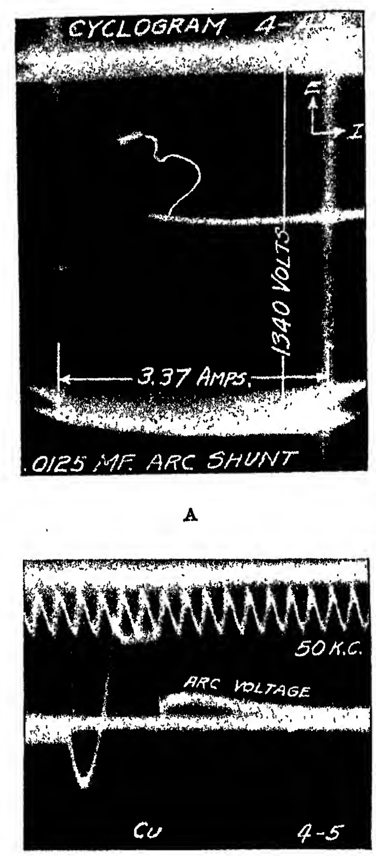
A knowledge of the numerical values of arc-failure currents is desirable. Now as soon as the arc stops momentarily at the end of a half-cycle the voltage recovery curve starts sharply downwards (increasing negative values) with a slope given by

$$\left(\frac{d\,e_a}{d\,t}\right)_{t=0} = \frac{I_1}{C} \tag{8}$$

as shown in the appendixes.

Theoretically this provides a measure of I_1, which is the arc-failure current for any particular record. Practically the slope is changing with considerable rapidity at this moment, so that it is impossible to obtain an accurate measure of the current by this means.

The trace ordinarily continues downward to a negative maximum which provides a good measure of the arc-failure current. Equation (9), obtained by equating the derivative of equation (4) to zero and rearranging, shows the relation between I_1 and the extreme of the negative dip, for the logarithmic case with a resistance shunt.



A

B

FIG. 10—CYCLOGRAM AND VOLTAGE FOR 0.0125 μf SHUNT

$$I_1 = \frac{1 - \frac{\Delta_1}{2}}{R}\left[(E - E_1) - (E - e_m)\, T^{\frac{\Delta_2}{2}}\right] \tag{9}$$

where

$$T = e^{2\omega t_m} = \frac{\dfrac{2\,I_1\,R}{\Delta_1} - (E - E_1)}{\left(\dfrac{I_1\,R}{1 - \dfrac{\Delta_1}{2}}\right) - (E - E_1)} \tag{10}$$

e_m = negative maximum voltage.

t_m = time at which it occurs.

$$\Delta_2 = \frac{1}{2\,R\,C_L\,\beta} - 1 = \frac{1}{\sqrt{1-\alpha}} - 1$$

$$= \frac{\alpha}{2} + \frac{3\,\alpha^2}{8} + \frac{5\,\alpha^3}{16} + +$$

Δ_1, β and α have the meanings previously assigned.

$$R < \frac{1}{2}\sqrt{L/C_L}$$

Equation (10) can readily be put into a form giving a direct solution for I_1 in terms of t_m, but in the records obtained t_m could not be accurately measured. It is easy, however, to measure e_m closely. I_1 is then obtainable by a trial and error solution of equation (9), in which $T^{\frac{\Delta_2}{2}}$ is only slightly sensitive to changes in I_1. The greatest value of $T^{\frac{\Delta_2}{2}}$ for all of the records studied was about 1.2, obtained in one of the 4,000-ohm cases; its minimum was of course unity.

For very small values of α, such as occur in the present study when R is 500 ohms, Δ_1 and Δ_2 can be neglected entirely, giving the still simpler form of equation (11).

$$I_1 = \frac{1}{R}\,(e_m - E_1) \tag{11}$$

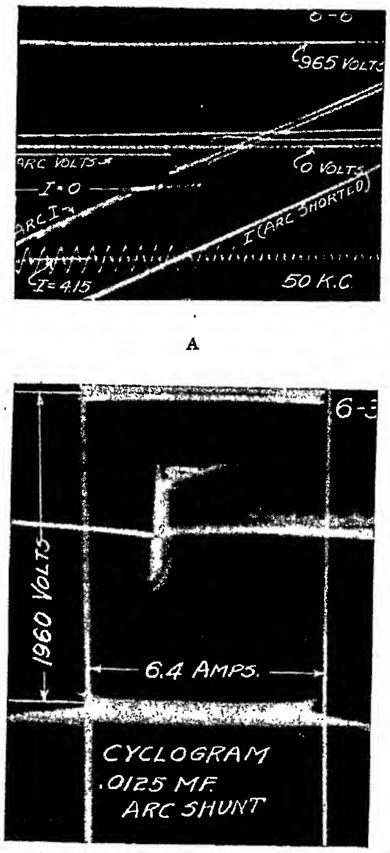
For the harmonic case with a resistance shunt equation (12), obtained in a manner similar to equation (9), provides the best approach to I_1.

$$I_1 = -\sqrt{\frac{C_L}{L}\left[(E-e_m)^2\,e^{\frac{t_m}{RC_L}} - (E-E_1)^2\left(1-\frac{1}{\alpha}\right)\right]} + \frac{E-E_1}{2\,R} \tag{12}$$

where

$$\frac{t_m}{2\,R\,C_L} = \frac{I_1}{I_1 - \dfrac{(E-E_1)\,\alpha}{2\,R}}$$

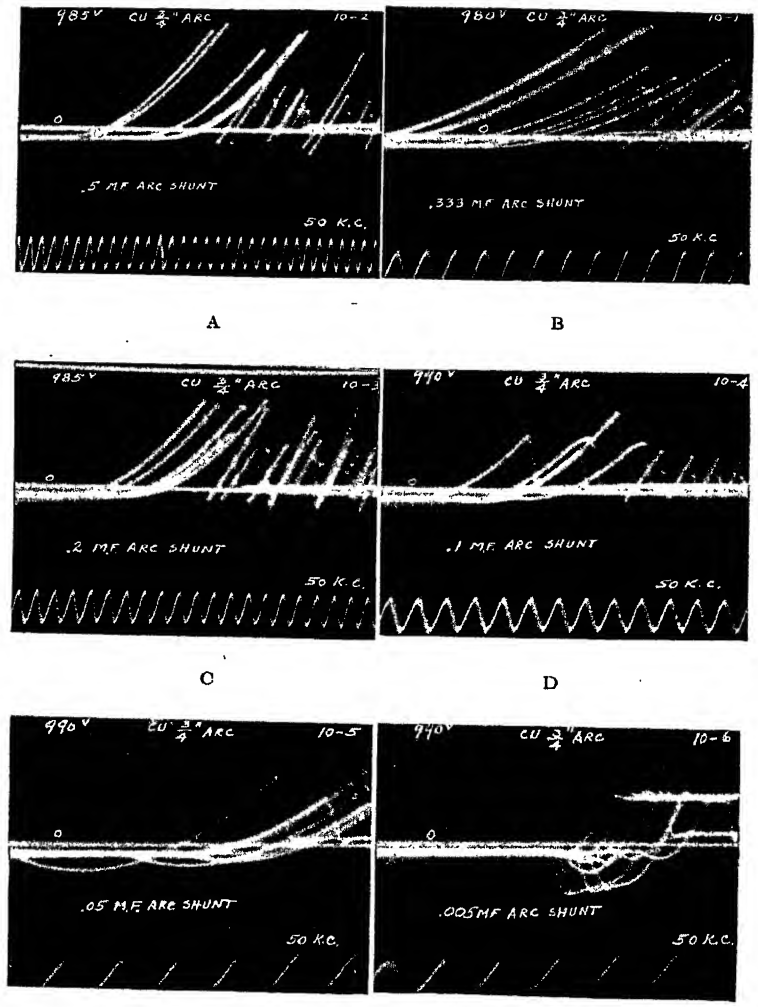
$$R > \frac{1}{2}\sqrt{\frac{L}{C_L}}$$



A

B

FIG. 11—CYCLOGRAM, CURRENT AND VOLTAGE FOR 0.0125 μf SHUNT



A B

C D

E F

FIG. 12—VOLTAGE OSCILLOGRAMS FOR CAPACITY SHUNTED ARC. 25-AMPERE, 660-VOLT, ¾-IN. ARC BETWEEN FLAT COPPER ELECTRODES

As in the preceding case, a trial solution is required if t_m cannot be accurately measured.

For the capacity shunted arc, $E - e_m$ becomes E_m in equation (1), which is very readily solved for I_1.

Of course if the arc-failure current is large enough so that the negative maximum that it would naturally reach exceeds the reignition voltage, a negative glow forms, and the normal extreme indicated by the circuit equations is never reached. This makes impossible an accurate determination of the arc-failure currents

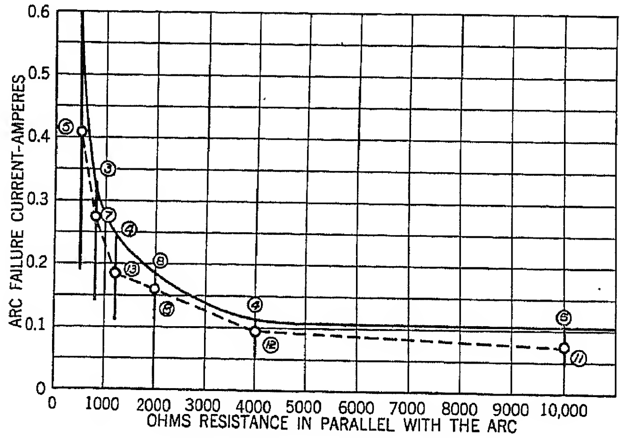


FIG. 13—ARC-FAILURE CURRENTS FOR RESISTANCE SHUNTS

for such cases; all that can be said is that they exceed a certain value.

Complete calculations have been made of the arc-failure currents from those traces that did not indicate a negative glow, the determinations being based on the extreme negative dip of the voltage recovery curve. The values obtained vary widely, but nevertheless exhibit a consistent trend in relation to shunt resistance values used. This is illustrated in Fig. 13, in which a heavy vertical line extends from the extreme lower to the upper value of arc-failure current observed with each resistance condition studied. The solid curve marking the upper ends of these lines merely indicates the boundary between currents that will and those that will not produce a negative glow.

Dotted lines join points that have been plotted to indicate the average of those arc-failure currents that did not produce a negative glow. Of course the omission of the glow cases prevents the averages from being of great significance, since every omission indicates an arc failure at a current near to or above the maximum of those contributing to the average. There is nevertheless a sufficiently definite trend to make the plot of interest. The figure adjacent to each point is the number of records contributing to the average, and at the top of each heavy vertical line is another figure which stands for the number of records made indicating a negative glow.

Fig. 13 indicates that the resistance acts in some way to affect the most likely value of arc-failure current, and to change the likelihood of very small or very large values. A low resistance is favorable toward arc failure at a relatively high value of arc current. A large capacity seems to have a similar tendency.

The maximum arc-failure current evaluated, considering all the observations with resistance and with capacitance in parallel with the arc, was less than 0.6 ampere. Some that resulted in a negative glow may have been slightly greater. The minimum determined was about 0.03 ampere. Values less than 0.05 ampere were very unusual.

ARC LEAKAGE WITH RESISTANCE SHUNTS

A peculiar behavior of considerable interest is exhibited most clearly in Fig. 5G. There is a sudden change in the slope of the voltage rise curve just before it reaches zero on the return from the negative peak. This is quite pronounced in the 1,200-ohm and 800-ohm records, and actually exists to a greater or less extent in all of the logarithmic cases except that for 4,000 ohms. In order to analyze it carefully the extreme left record of Fig. 5E has been reproduced as the solid line in Fig. 14. The lower dotted line represents the theoretical curve resulting from equation (3) that should be expected from an arc-failure current sufficient to give the negative voltage peak that does occur. The oscillographic record follows the theoretical curve very closely until shortly after the negative peak is reached, then for a short time the potential between the electrodes decreases much more rapidly than it would according to circuit theory. This is indicated by the departure of the voltage trace from the lower line.

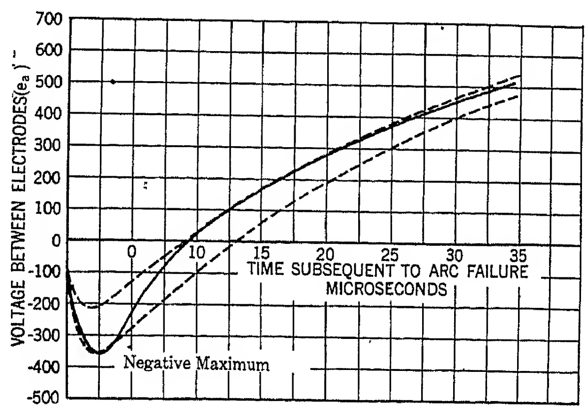


FIG. 14—ANALYSIS OF VOLTAGE CURVE FROM FIG. 5E TO SHOW EFFECT OF THE ARC-LEAKAGE CURRENT

As soon as zero voltage is reached it again follows a theoretical curve closely until reignition. The new curve has the same decrement and final asymptote as the original one, but differs in reference to initial conditions.

This indicates that for a few microseconds after the negative peak an appreciable "leak" of current takes place between the electrodes, without, however, resulting in a negative glow at its characteristic voltage. No such behavior as is illustrated in Fig. 14 appeared to take place in the capacity shunt observations.

Reignition Voltage as Related to Circuit Constants

One of the primary objectives of this investigation has been the determination of the nature of the relationship that exists between the value of voltage required for reignition and the time taken to reach that voltage. A program to meet this requirement necessitated modifying the circuit constants in such a way as to cause the voltage to rise at widely different rates. This was done by the use, first of resistance, then capacity, in parallel with the arc. The results of the resistance tests appear in Fig. 5, and those with the capacity in Fig. 12.

In making the tests for Figs. 5 and 12 considerable pains were taken to maintain the arc length, current, and power consumption constant. In spite of this quite a variation in reignition voltage appeared, especially in the case of slowly rising voltages. The results of all the tests that were made on the ¾-in. 25-ampere 660-volt circuit arc are combined in Figs. 15 and 16. Each point on these graphs represents a reignition voltage, plotted against time measured from the instant at which the arc current failed.

Referring back to Fig. 14, it is seen that after zero voltage the recovery curve with a resistance shunt follows a theoretical trace dependent on the logarithmic decrement determined by circuit constants, on the asymptote determined by line voltage, and on the time at which zero voltage occurs. The dotted lines in Fig. 15 are such curves calculated from equation (4).

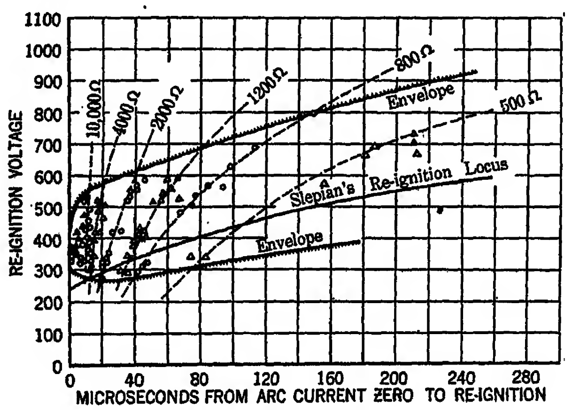


Fig. 15—Reignition Voltage—Time Values for Resistance Shunts

25-ampere, 660-volt, 3/4-in. arc between flat copper electrodes

In placing them the zero voltage time was selected so to make the curve lie as close as possible to all of the observed reignition points.

In Fig. 16, representing the capacity shunt condition, the dotted lines that appear for each value of capacity used are theoretical voltage rise curves calculated from equation (7). The values of arc failure currents used in the calculations were chosen to produce curves best fitting the observed reignition points. These currents may be regarded as representative arc-failure currents for each set of circuit conditions. The occasional points at some distance from the curves merely indicate arc-current failure at some value considerably different from the representative value chosen for the curve. For the 0.05, 0.2, 0.33 and 0.5 μf. capacity conditions the most representative arc-failure currents were so small, relative to other terms in the equation, that they had a negligible effect on the positions of the curves.

A small group of points to the extreme left of both figures, between 330 and 380 volts, and between one

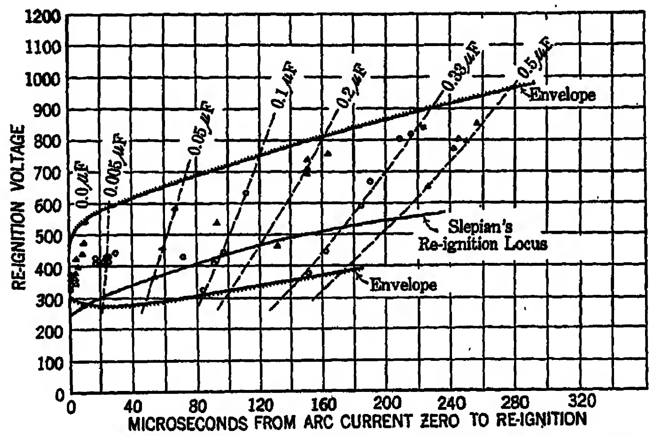


Fig. 16—Reignition Voltage—Time Values for Capacity Shunts

25-ampere, 660-volt, 3/4-in. arc between flat copper electrodes

and two microseconds, is taken from ignition points of the negative glow. These fit very well into the general appearance of the rest of the diagram; this tends to discount any supposition that would make the change in polarity of importance in determining the short-time re-striking voltage.

Closely related to the requirement of a generally higher re-striking voltage for smaller resistances or larger capacities there developed a consistently increasing difficulty in maintaining the arc for any great length of time. After only a few seconds of operation with small resistances or large capacities, and at some apparently accidentally determined half-cycle, the required re-striking voltage continued indefinitely to exceed the recovering voltage, hence the arc ceased.

The upper and lower envelope curves that appear on Figs. 15 and 16 provide a demarcation of the zone within which reignition occurred. It is interesting to note that the reignition locus obtained by Slepian using an indirect method is in general similar in shape to the upper envelope, but starts at a short-time value of 235 volts and is correspondingly low in value throughout its length. This locus is plotted on Figs. 15 and 16 for comparison.

The general features of these locus figures may be summed up as follows:

(1) For a very short time delay, the reignition voltage is quite definite in value, and is about the same as the glow voltage, but perhaps a trifle higher.

(2) As the voltage rise is delayed, the reignition voltage value varies between wide limits, the maximum rising considerably above the glow voltage, the minimum not changing greatly.

(3) There is no very marked difference between the reignition voltage required for a given time delay produced by resistance and that produced by capacity. The envelope curves in the two diagrams are identical.

It seems difficult to account for the great variation in late reignition voltages in terms of the various deionization processes that have been supposed to exist; especially is this true for the occasional very low restriking voltages. In the experiments leading to Figs. 15 and 16 the ends of the arc were not stationary; they traveled, relatively slowly, on the surfaces of the copper plates that served as electrodes. The motion, while not rapid, was certainly sufficient to expose the arc to operation on surfaces differing from one moment to the next due to local irregularities. It may be that the variations observed resulted from these surface irregularities.

In a glow discharge the electron stream from the cathode leaves the body of the metal as a result of a combination of several causes, among them being bombardment by positive ions and the existence of a potential gradient due to a space-charge adjacent to the metal surface. The severity of the ion bombardment is of course closely related to the potential gradient at the cathode and to the space-charge. It seems reasonable to suppose that when some critical gradient at the surface is reached, electrons are suddenly released freely enough to support a glow discharge. In this case the reignition voltage might be expected to depend somewhat on surface irregularities, surface temperature, and space-charge distribution in the arc path, all of these being variable elements.

Application to Power Systems

The facts that have been presented are of interest in a consideration of conditions under which circuit breakers on electric power systems must operate. Manufacturers have observed that circuit breaker test installations in which reactance coils are used to limit short-circuit current impose an actual circuit breaker duty more severe than that resulting from the same current interruption requirement in normal service on a power system. The difference is attributed to the introduction of a considerable capacity in parallel with the arc by the lines and apparatus that limit the short-circuit current, and to the production of a condition roughly equivalent to a parallel resistance by the presence of load circuits. Both situations delay the voltage rise toward reignition, thereby permitting a longer time for recovery of dielectric strength in the arc region. This of course favors an early extinguishing of the arc.

A single-phase short circuit is usually interrupted by an oil switch in which there are two breaks in series in each of the two line wires involved. There will then be four arcs in series in which reignition must occur if the arc is to persist after the end of any given half-cycle. Due to the variability of the reignition voltage, it is not to be supposed that all four arcs will reach re-striking voltage at exactly the same instant, but as soon as one breaks down the others will follow immediately due to the increased voltage imposed on them. If the electrostatic fields in the circuit breaker are such as to produce an approximately even distribution of the total voltage between the four breaks, the over-all reignition voltage will be approximately four times that for any one arc.

In oil circuit breakers the arc is drawn in oil. After a very short time it is playing in a bubble of gas formed by the decomposition of oil. The reignition and glow voltages cannot be expected to be the same in these gases as in air, but they should be of the same order of magnitude. The total reignition voltage can then be represented in the manner of Figs. 15 and 16 except with voltages in general about four times as great.

The calculation of the voltage rise curves for a transmission circuit is complicated by the necessity of considering distributed capacity and inductance in their relation to the time required for the propagation of an electrical impulse along a line. Thus it requires about 50 microseconds for an impulse to traverse 10 miles of line; therefore it would seem that only the capacity of that part of the line less than 10 miles from the circuit breaker can effect the voltage across the arc during the first 50 microseconds of voltage rise. Furthermore, the effect of the distributed capacity of a given section of line will depend on the inductance to be traversed by the charge in reaching it. The authors plan to present an analysis of this situation in the near future. For the present, however, a few general observations can be made as to what is likely to occur.

In general, power system voltages are much higher than those used in securing the results that have been described. Equations (1), (2), (3) and (4) all show that these higher voltages result in a tendency toward a correspondingly greater rate of voltage rise toward reignition. However, the values of inductance and capacity that apply to transmission circuits are also much larger than those used here. Rough estimates using characteristic values of circuit constants make it seem probable that the net result is a delay of reignition by periods of time of the order of magnitude of those illustrated in the oscillograms.

There will of course be great differences in the rates of voltage recovery according to the location of a circuit breaker in a transmission system. There seems no doubt, for example, but that the voltage rise in a circuit breaker just off a generating station bus will take place very much more rapidly than in one located at the load end of a 100-kv. line of considerable length.

The current density in an arc is usually approximately constant. Changes in total current result in an in-

crease or decrease in the cross sectional area of the arc stream and in the area of the cathode spot from which the electron stream emerges. As yet no reason is apparent for believing that the decreasing current in an arc of 1,000 amperes r. m. s. value will cease at a minimum value any higher than the 0.05 to 0.6 ampere found here to apply in the case of a 25-ampere arc. The larger arc current will produce a higher electrode temperature, and this might easily favor a lower arc-failure current. In an application to practical problems the arc-failure current is of much less importance on high voltage than on moderate voltage circuits so far as its effect on the rate of voltage recovery is concerned.

The arc studied in this investigation was ¾ in. long. When interrupting a short circuit of any considerable magnitude a circuit breaker arc will be maintained to a considerably greater length than this, and while it lasts each half-cycle will end with a considerably greater electrode spacing than did the preceding one.

The voltage required for reignition in the case of extremely rapid voltage recovery is approximately the glow voltage. With a moderately short electrode spacing this is much higher than the normal arc voltage that exists during the greater part of each half-cycle. Most of the potential difference between electrodes in the glow occurs in the cathode drop associated with the positive ion space-charge adjacent to the cathode. This cathode drop is of course not affected by increased electrode spacing. As the electrodes separate the glow voltage increases, but not nearly in proportion to the change in spacing, as the increase takes place entirely in the region between the cathode drop and the anode.

As the cathode drop in the normal arc is a much smaller fraction of the total than is that in the glow, the arc voltage may be expected to increase much more rapidly than the glow voltage as the electrodes spread. By the time the separation is several inches the two may be approximately equal. It is very unlikely, however, that the short-time reignition voltage will ever be less than the normal arc voltage. If it were to be so, a complete half-cycle of glow, at low current and glow voltage, would immediately precede final arc extinction. Such behavior in a circuit breaker would be readily observable in the voltage and current traces of an ordinary oscillographic record, and no tests indicating it have come to the authors' attention. Low-voltage arcs between copper rods have, however, been observed to extinguish immediately following a half-cycle at glow rather than at arc voltage, the extinction resulting from gradual electrode separation.

In any case, the short-time reignition voltage will be greater for each successive half-cycle as the electrodes separate. In addition to producing this change, the greater spacing permits readier access of relatively cold oil vapors and in this and other ways accelerates the deionization that is taking place in the arc region prior to reignition. This acceleration appears as an increase of slope in the reignition locus. Thus the reignition locus for each half-cycle should be steeper than the previous one as well as starting at a higher value.

In Fig. 17 an illustrative set of expected reignition loci for three successive half-cycles during a circuit breaker operation has been sketched qualitatively. Each one of these represents what must inevitably be true, that at every instant of time following each current zero there is some definite voltage that would cause reignition, and that this voltage increases with the passage of time. The reignition zones for a ¾-in. arc in air, bounded by envelope curves of Figs. 15 and 16, indicate the limits within which a reignition locus representing the behavior at the end of that particular half-cycle corresponding to a ¾-in. separation must lie. It will be observed that the reignition curves have been

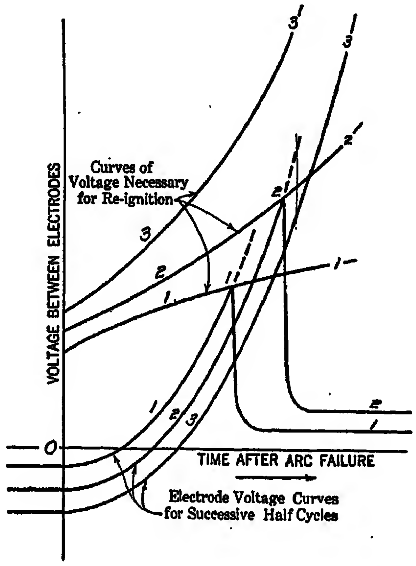


Fig. 17—Reignition Voltage—Time Values for Capacity Shunts with Increasing Electrode Spacing

represented in Fig. 17 as increasing both in short-time value and in steepness for successive half-cycles in accordance with the preceding discussion.

The lower family of curves of Fig. 17 represents typical voltage recovery curves, of the capacity shunt type, for successive half-cycles, numbered to correspond with the proper reignition locus. Each curve starts at a lower point on the graph than its predecessor because of the greater arc length and consequent greater normal arc voltage. The shapes of these curves are all alike, however, as they depend only on circuit constants external to the active breaker.

As soon as the electrode separation results in a reignition locus not intersected by the corresponding voltage rise curve, as illustrated by the No. 3 pair of Fig. 17, the circuit breaker duty is completed; the arc will not re-strike. It is evident that in any situation where the circuit constants result in a very rapid

voltage recovery, there must be compensation for this by a correspondingly rapid deionization in the region between the electrodes if the arc is not to be drawn out to an extreme length. Also, on the whole, a parallel resistance condition should be more favorable than a parallel capacity, as it would not demand ultimately so steep a slope of the reignition locus to result in a failure of the curves to intersect.

Summary

A series of oscillographic records of voltage and current in an a-c. arc between copper electrodes, taken near the moment of current reversal, has indicated the following points:

1. For a brief but measurable period at the end of each half-cycle the current between the electrodes is essentially zero.

2. The values of electrode potential difference during this time can be accurately predicted if the electrical constants of the circuit and the values of arc voltage and current at the moment the arc stops are known; or conversely, if the voltage trace is known, the arc-failure current can be obtained.

3. Before current can pass in the new direction the voltage between the electrodes must rise to a definite minimum value. For copper electrodes in air at atmospheric pressure this is about 300 volts, and is very little affected by electrode shape or placement up to spacings of at least one inch.

(4) Under certain conditions the electric current flow between the electrodes just before or after the current zero, or both, may take the form recognized as a glow discharge, the glow voltage being about the same as the minimum reignition potential.

(5) If either resistance, or capacity, or both, are connected in parallel with the arc, the rate of voltage rise in the new direction is materially decreased. Associated with this time delay in the approach to reignition potential an increase in the voltage required for reignition usually occurs. The change, however, is not consistent from one half-cycle to the next, due to variable factors whose nature is not yet definitely determined.

(6) As applied to circuit breakers on power transmission and distribution systems the equations for electrode voltage during the period of zero arc current involve consideration of distributed inductance and capacity, and perhaps more important, the time of propagation of the electrical impulses between points concerned on the circuit.

(7) The rate of voltage rise toward reignition in power system circuits may be expected to be greatly affected by the position of the circuit breaker and fault relative to the inductance and distributed capacity of the system.

(8) The presence of load circuits paralleled with the arc adjacent to its input side may be expected to act in the same general manner as a resistance in parallel with the arc, tending to cause a delay in the rise toward reignition, with a consequent lessening of the expected severity of circuit breaker duty.

(9) Increasing length of arc due to progressive electrode separation may reasonably be expected to result in a greater initial value, and greater steepness, of the reignition locus, and in a simultaneous lowering of the voltage available for reignition at any specified moment following the current zero. The arc extinguishes when at the end of some half cycle the voltage curve fails to cross the reignition locus.

Acknowledgment

The authors wish to acknowledge the kindness of Dr. J. Slepian for suggesting the desirability of this study, and the loan by the Detroit Edison Company of a cathode-ray oscillograph with which many of the oscillograms were taken. The remainder were taken with a General Electric Company cathode-ray oscillograph.

Appendix A

(See Fig. 1)

Arc Voltage During Arc Current Zero. (No Arc Shunt)

$$E = e_a + L\frac{d\,i_L}{d\,t} = e_a + \frac{q_c}{C_L}$$

$$i_L + i_c = 0$$

$$\frac{d\,i_L}{d\,t} = -\frac{d\,i_c}{d\,t}$$

$$\frac{q_c}{C_L} + L\frac{d^2\,q_c}{d\,t^2} = 0$$

$$q_c = A\cos\omega t + B\sin\omega t$$

$$\omega = \frac{1}{\sqrt{L\,C_L}}$$

$$e_a = E - \frac{1}{C_L}(A\cos\omega t + B\sin\omega t)$$

$$i_L = -i_c = -\frac{d\,q_c}{d\,t} = A\,\omega\sin\omega t - B\,\omega\cos\omega t$$

Initial Conditions:

At $t = 0$ $e_a = E_1$ and $i_L = I_1$

$$E_1 = E - \frac{1}{C_L}A \qquad A = C_L(E - E_1)$$

$$I_1 = -B\,\omega \qquad B = -\frac{I_1}{\omega}$$

Hence $e_a = E - (E - E_1)\cos\omega t + \dfrac{I_1}{\omega\,C_L}\sin\omega t$

$$e_a = E - E_m\cos(\omega t + \theta)$$

where $\omega = \dfrac{1}{\sqrt{L\,C_L}}$

E = the maximum of the supply voltage.

E_1 = the arc voltage when the arc fails.

I_1 = the arc current when the arc fails.
e_a = the arc voltage during the period $t = 0$ to $t = t_1$ (Fig. 2)

$$\theta = \tan^{-1} \frac{I_1/\omega C_L}{E - E_1}$$

$$E_m = \sqrt{(E-E_1)^2 + \left(\frac{I_1}{\omega C_L}\right)^2} \quad \Bigg\| \quad I_1 = [\sqrt{E_m^2 - (E-E_1)^2}]$$

$$\omega C_L \left(\frac{d e_a}{d t}\right)_{t=0} = \frac{I_1}{C_L}$$

Appendix B

(See Fig. 7A)

$$E = L \frac{d i_L}{d t} + R i = \frac{q_c}{C_L} + R i$$

$$i = i_L + \frac{d q_c}{d t} = \text{line current}$$

$$\frac{d i}{d t} = \frac{d i_L}{d t} + \frac{d^2 q_c}{d t^2}$$

$$E = \frac{q_c}{C_L} + R\left(i_L + \frac{d q_c}{d t}\right)$$

$$O = \frac{1}{C_L} \frac{d q_c}{d t} + R\left(\frac{d i_L}{d t} + \frac{d^2 q_c}{d t^2}\right)$$

$$O = q_c + \frac{L}{R} \frac{d q_c}{d t} + L C_L \frac{d^2 q_c}{d t^2}$$

If $q_c = e^{mt}$

$$1 + \frac{L}{R} m + L C_L m^2 = 0$$

$$m = -\frac{1}{2 R C_L} \pm \sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}$$

I. If $\frac{1}{4 R^2 C_L^2} < \frac{1}{L C_L} \quad \Big\| \quad R > R_c \quad \Big\| \quad R_c = \frac{1}{2} \sqrt{\frac{L}{C_L}}$

$$q_c = e^{-\frac{1}{2RC_L} t} [A \cos \omega t + B \sin \omega t]$$

where $\omega = \sqrt{\frac{1}{L C_L} - \frac{1}{4 R^2 C_L^2}}$

$$e_a = E - \frac{q_c}{C_L} = E - \frac{1}{C_L} e^{-\frac{1}{2RC_L} t} [A \cos \omega t + B \sin \omega t]$$

$$i_c = \frac{d q_c}{d t} = - C_L \frac{d e_a}{d t}$$

$$= e^{-\frac{1}{2RC_L} t} [- A \omega \sin \omega t + B \omega \cos \omega t]$$

$$- \frac{1}{2 R C_L} e^{-\frac{1}{2RC_L} t} [A \cos \omega t + B \sin \omega t]$$

Initial Conditions:

At $t = 0$ $\quad e_a = E_1$ and $i_c = - I_1$

$$E_1 = E - \frac{1}{C_L} A \qquad A = C_L (E - E_1)$$

$$- I_1 = B \omega - \frac{1}{2 R C_L} A = B \omega - \frac{E - E_1}{2 R}$$

$$B = \frac{1}{\omega} \left(- I_1 + \frac{E - E_1}{2 R}\right)$$

Hence

$$e_a = E - e^{-\frac{1}{2RC_L} t} \Bigg[(E - E_1) \cos \omega t + \frac{1}{\omega C_L} \left(- I_1 + \frac{E - E_1}{2 R}\right) \sin \omega t \Bigg]$$

This reduces to the value given for e_a in Appendix A when $R = \infty$. I_1 = the arc current at the breaking point.

$$\left(\frac{d e_a}{d t}\right)_{t=0} = \frac{I_1}{C_L}$$

II. If $\frac{1}{4 R^2 C_L^2} > \frac{1}{L C_L} \quad \Big\| \quad R < R_c \quad \Big\| \quad R_c = \frac{1}{2} \sqrt{\frac{L}{C_L}}$

Let $\beta = \sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}$

$$q_c = e^{-\frac{1}{2RC_L} t} [A \cosh \beta t + B \sinh \beta t]$$

$$e_a = E - \frac{q_c}{C_L}$$

$$= E - \frac{1}{C_L} e^{-\frac{1}{2RC_L} t} [A \cosh \beta t + B \sinh \beta t]$$

$$i_c = \frac{d q_c}{d t} = - C_L \frac{d e_a}{d t}$$

$$= e^{-\frac{1}{2RC_L} t} [A \beta \sinh \beta t + B \beta \cosh \beta t]$$

$$- \frac{1}{2 R C_L} e^{-\frac{1}{2RC_L} t} [A \cosh \beta t + B \sinh \beta t]$$

Initial Conditions:

At $t = 0$ $\quad e_a = E_1$ and $i_c = - I_1$

$$E_1 = E - \frac{1}{C_L} A \qquad A = C_L (E - E_1)$$

$$- I_1 = B \beta - \frac{1}{2 R C_L} C_L (E - E_1)$$

$$B = \frac{1}{\sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}} \left[- I_1 + \frac{E - E_1}{2 R} \right]$$

Hence

$$e_a = E - e^{-\frac{1}{2RC_L}t}\left[(E-E_1)\cosh\sqrt{\frac{1}{4\ R^2\ C_L^2} - \frac{1}{L\,C_L}}\ t + \frac{-I_1 + \frac{E-E_1}{2\,R}}{C_L\sqrt{\frac{1}{4\ R^2\ C_L^2} - \frac{1}{L\,C_L}}}\sinh\sqrt{\frac{1}{4\ R^2\ C_L^2} - \frac{1}{L\,C_L}}\ t\right]$$

$$\left(\frac{d\,e_a}{d\,t}\right)_{t=0} = \frac{I_1}{C_L}$$

Appendix C

(See Fig. 7B)

$$E = L\frac{d\,i}{d\,t} + R\,i + \frac{q}{C}$$

$$E = L\frac{d^2\,q}{d\,t^2} + R\frac{d\,q}{d\,t} + \frac{q}{C}$$

$$O = L\frac{d^3\,q}{d\,t^3} + R\frac{d^2\,q}{d\,t^2} + \frac{1}{C}\ \frac{d\,q}{d\,t}$$

If $q = e^{mt}$

$$O = L\,m^3 + R\,n^2 + \frac{1}{C}\ m$$

$$m = O$$

$$O = L\,m^2 + R\,m + \frac{1}{C}$$

$$m = -\frac{R}{2\,L} \pm \sqrt{\frac{R^2}{4\,L^2} - \frac{1}{L\,C}}$$

I. If $\frac{R^2}{4\,L^2} < \frac{1}{L\,C}$

$$q = C\,E + e^{-\frac{R}{2L}t}\,[A\cos\omega\,t + B\sin\omega\,t]$$

where $\omega = \sqrt{\frac{1}{L\,C} - \frac{R^2}{4\ L^2}}$

$$i = \frac{d\,q}{d\,t} = e^{-\frac{R}{2L}t}\left[\left(\omega\,B - \frac{R}{2\,L}A\right)\cos\omega\,t - \left(\omega\,A + \frac{R}{2\,L}B\right)\sin\omega\,t\right]$$

$$e_a = \frac{q}{c} = E + \frac{1}{C}\,e^{-\frac{R}{2L}t}\,[A\cos\omega\,t + B\sin\omega\,t]$$

Initial Conditions:

At $t = O$ $e_a = E_1$, $i = I_1$ and $q = C\,E_1$

$$C\,E_1 = C\,E + A \qquad A = -\,C\,(E - E_1)$$

$$I_1 = \omega\,B + \frac{R}{2\,L}\,C\,(E - E_1)$$

$$B = \frac{1}{\omega}\left[I_1 - \frac{R}{2\,L}\,C\,(E - E_1)\right]$$

Hence

$$e_a = E + e^{-\frac{R}{2L}t}\left[-\,(E - E_1)\cos\omega\,t + \frac{1}{\omega\ C}\left(I_1 - \frac{R}{2\,L}\,C\,(E - E_1)\right)\sin\omega\,t\right]$$

$$\left(\frac{d\,e_a}{d\,t}\right)_{t=0} = \frac{I_1}{C}$$

II. If $R = O$,

$$e_a = E - \left[(E - E_1)\cos\omega\,t - \frac{I_1}{\omega\,C}\sin\omega\,t\right]$$

$$i = I_1\cos\omega\,t + \omega\,C\,(E - E_1)\sin\omega\,t$$

$$\omega = \frac{1}{\sqrt{L\,C}}$$

$$e_a = E - E_m\cos(\omega\,t + \theta)$$

where $E_m = \sqrt{(E - E_1)^2 + \left(\frac{I_1}{\omega\,C}\right)^2}$

$$\tan\theta = \frac{I_1}{\omega\,C\,(E - E_1)}$$

$$I_1 = \omega\,C\sqrt{E_m^2 - (E - E_1)^2}$$

which is the same as the case of Appendix A.

$$\left(\frac{d\,e_a}{d\,t}\right)_{t=0} = \frac{I_1}{C}$$

Discussion

T. E. Browne, Jr.,: The investigation described in this paper is of extreme interest to those of us who are working on the problem of arc extinction, and a very welcome contribution to our knowledge of this subject. These oscillograms are, so far as I know, the first to be obtained showing directly on a time axis just what occurs to the current and voltage of a short a-c. arc during the elusive current zero period. Theories have, in the past, been only rough and approximate because of our lack of means for accurately checking them experimentally. The paper gives a commendably thorough interpretation and analysis of these important oscillograms, and of their relation to the problems of circuit interruption at moderate voltages.

Although many of the facts presented had previously been learned by less direct methods, there are several distinctly new contributions. Some of these are in confirmation of previous theoretical deductions, while others have not hitherto been suspected. In the first place, it is definitely shown that there does exist, even for the fastest practical circuits, a measurable period of time during which the current through the arc is sensibly zero and the voltage across the arc terminals is controlled almost entirely by the transient characteristics of the circuit in which the arc is playing. It is also shown, however, that during this period

there may be a small "leakage" current of sufficient magnitude to measurably alter the circuit transient. The existence of such a current has already been postulated in connection with the theory of the formation of a space charge sheath at the cathode.[1] It is also revealed that the circuit transient depends to some extent on the arc failure current, which is largely a random quantity, but is in turn partially determined by the circuit constants. The occurrence of a "negative glow" preceding reversal of arc voltage is also recorded for the first time.

It may be of interest to call attention to the points of agreement and also of apparent disagreement between the results of these experiments and the results of an earlier but somewhat similar investigation[2] using a cathode-ray oscilloscope. In the former case, photographic records of individual arc reignitions could not be obtained, but by photographing the figures produced on the fluorescent screen by a large number (900) of retracings of the dynamic volt-ampere characteristic (or "cyclogram") of a steadily running arc, records of the average type of voltage and current variation near current zero were obtained for short arcs under a variety of conditions. With plane copper electrodes, three distinct types of arcs having different restriking characteristics were observed. They are as follows: (1) the stationary arc, with terminals remaining in one position on the electrodes long enough to produce melting and boiling of the electrode surfaces, with resulting burring and pitting; (2) the "wandering" arc, which moves more or less discontinuously from one adjacent spot to the next, leaving a marked trail but not causing appreciable burning away of the electrode surfaces; and (3) the so-called "cold-cathode"[3] or Stolt[4] arc in which the arc terminals are moved over the electrodes with such velocity that no burning occurs, even with very large currents.

The first type of arc was very difficult to obtain with clean electrodes and currents less than 100 amperes. It is believed, however, that this is the normal type of copper-electrode arc for currents of several hundred amperes or more, where considerable melting and burning away of the electrodes is generally observed. Such an arc is therefore of primary practical interest since it is the most likely to occur in the operation of ordinary contactors and circuit breakers. In the earlier investigation referred to, its reignition voltages appeared to vary widely between values slightly in excess of the glow voltage and values very much less than this. No indication of the existence of an actual glow was observed.

The third type of arc can be obtained only by very rapidly rotating the electrodes mechanically or the arc magnetically. This is a special case. It has been shown[2] to possess the desirable characteristics of requiring comparatively high reignition voltages (more than three times glow voltage observed), even in the fastest circuits obtainable, and of normally breaking down to an unusually long-lived glow at constant voltage before transition to the arc.

It is clearly the second type, or wandering arc, which the authors of this paper have used in their experiments. The peculiar tendency of the voltage of the intermediate glow discharge to increase substantially with current immediately following its initial formation, as shown by the oscillograms and cyclograms of Figs. 3, 5, and 10, produce a repeated volt-ampere characteristic, which appeared to Mr. Todd and myself to defy any reasonable explanation at the time of the previous investigation. It is gratifying to see this puzzling case completely analyzed here, but the peculiar variation of the glow voltage still lacks explanation. The fact that the reignition and glow voltages measured by the authors are somewhat higher than those found in the earlier tests is probably due to the additional voltage required by the ¾-inch positive column, which cannot be neglected for the longer electrode separation. The non-occurrence of a "negative glow" in the former experiments with the wandering arc may be due to different circuit characteristics, which did not produce quite as high "negative dip" voltages, or to the higher arc current (92 amperes), which may have the same effect.

Perhaps the most interesting feature of the paper from a practical standpoint is the evidence presented in regard to the variation with time after current zero of the voltage required to reignite the arc. The most noticeable feature of Figs. 15 and 16, which show this relation, is the extreme variability of this voltage from one-half cycle to the next under the same arc and circuit conditions. This fact agrees with past experience and observation[2] though it is still not fully understood. On the basis of present theory,[1] it would seem to represent variation in the minimum thickness of the space charge sheath at the incoming cathode, which can be caused by varying contour and condition of the electrode surfaces, as suggested by the authors, and perhaps also by varying density of ionization next to the cathode due to turbulence of the arc gases. In consequence of this variation, the arc space obviously cannot be said to have, in general, a definite breakdown voltage at a given instant of time. Therefore, we can only say that there is a certain probability that a given voltage will cause reignition of the arc when applied after a given interval following a current zero. This probability may be expected to be a function, not only of the voltage and time coordinates of the graphs, but also of the length of time during which a voltage in excess of the minimum breakdown value is applied. Thus, a rapidly rising transient may be expected to reach a higher voltage before breakdown than will a transient passing through the region of high breakdown probability (bounded by the envelopes in the figures) at the same time but rising at a slower rate. Probability of breakdown with a given voltage applied after a given interval and for a given length of time may also be expected to vary somewhat with the area of the space charge sheath, and therefore with the arc current, since the likelihood of the random occurrence of a "thin spot," having a breakdown voltage near to the possible minimum, increases with the area of the sheath. From this it follows that lower and more consistent breakdown voltages are to be expected when using larger currents than the 25 amperes employed in these experiments; and this is borne out by experience in the laboratory.

In the "indirect" method by which the so-called "Slepian's focus," shown in the figures, was obtained, the rate of rise of circuit voltage to different peak values was also controlled by resistance shunts, (as in Fig. 15), but the final adjustment of these was such that at each circuit voltage the arc would just fail to reignite after one-half cycle. The transients corresponding to these critical circuit constants were plotted by means of an approximate equation, and the curve shown was drawn as their envelope. These applied voltage curves were probably quite flat as they approached the region of high probability of breakdown, and so would not be likely to rise much above the lower boundary of the region without causing restriking of the arc. Also, the currents used were of the order of hundreds of amperes. Consequently, their envelope might be expected to correspond very closely to the lower branch of the envelope determined by the authors of this paper. It is believed that such a curve gives the most accurate practical measure of the dielectric strength, or the minimum breakdown voltage, of an arc space obtainable, provided that the critical voltage recovery curves are accurately plotted. Unfortunately, the particular locus shown was obtained for an arc of the third type mentioned above (high-speed cold-cathode arc as used in the deion-air circuit breaker)[5] and so cannot be directly compared with the results given here. The above method applied to a 1/16-in., 300-ampere arc of the first, or stationary type (normal for large currents) yields a curve,

1. *Extinction of an A-C. Arc*, J. Slepian, TRANS. A. I. E. E., Vol. 47, 1928, p. 1398.

2. "Restriking of Short A-C. Arcs," F. C. Todd and T. E. Browne, Jr., *Phys. Rev.*, Aug. 15, 1930, p. 732.

3. J. Slepian, *Phys. Rev.*, 27, 1926, p. 407.

4. H. Stolt, *Ann. d. Physik*, 74, 1924, pp. 80-104.

5. *Theory of the Deion Circuit Breaker*, J. Slepian, JOURNAL OF THE A. I. E. E., Feb. 1929.

which, though also not directly comparable, is very similar in shape to the lower envelope of these figures, but lies about 150 volts below it.

Part of this discrepancy is very probably due to the distinctly different reignition characteristics already described and also to the difference in arc lengths in the two cases. However, it also seems from this work that our curves are too low, since in plotting the individual voltage rise curves of which it is the envelope, no account was taken of either the negative dip, due to finite arc failure current and arc voltage at current failure or of the reactor distributed capacity, both of which tend (as shown by the exact equations given) to cause the maximum value of the recovered voltage to exceed that given by the simple equation which was actually used. We may conclude, then, that the actual values of voltage which may be successfully withstood by a very short arc space between hot copper electrodes following the zero of a large current lie on a curve that probably falls somewhere between our curve, shown in Mr. Strom's paper, and the lower envelope of the region determined by Attwood, Dow, and Krausnick.

Before concluding, I wish to emphasize the need of caution in attempting to generalize about the behavior of short arcs from the results of these limited experiments. In our work in the laboratory, it has been found that the characteristics of short a-c. arcs, particularly with regard to extinction, may depend to a very large extent upon such conditions as current magnitude, arc length, electrode material, and gas medium,[6] as well as upon the conditions of motion or lack of motion mentioned above.

With regard to the attempt of the authors of this paper to apply their results to the action of circuit breakers in high-voltage power circuits, I believe a further work of caution is needed. It is undoubtedly true that the curves of their Fig. 17 do give a rough qualitative picture of the race between recovered voltage and recovered ability to withstand voltage, which, as Dr. Slepian[1] and others have already pointed out, determines the extinction or reignition of an a-c. arc in any practical circuit. However, it must be kept in mind that the few hundred volts which can be withstood by the space adjacent to the cathode in the length of time usually available is inadequate to account for the interruption of power circuits involving thousands of volts, even when four breaks in series are considered, and electrostatic unbalance[7] is neglected. It is clear, then, that a very large part of the high dielectric strength recovered by long a-c. arcs at current zero must reside in the main body of the arc path, where the mechanism of deionizatin, and therefore both the manner of recovery with time and the effect on this recovery of possible influencing factors may be entirely different from those relating to the region next to the cathode. This is, of course, especially true of oil circuit breaker arcs where both arc and interrupted gradients are many times those for an arc in air. Although in recent years much progress has been made in determining the factors that improve the interrupting ability of long a-c. arcs and in applying this knowledge to practical circuit-breakers, there is still no experimental information or exact theory with regard to the variation of dielectric strength of a long a-c. arc with time during the first few microseconds of the current zero period. Consequently, we should be careful to keep our minds open on this point until information is obtained that can be directly applied.

6. "Extinction of Short A-C. Arcs Between Brass Electrodes," T. E. Browne, Jr. and F. C. Todd, *Phys. Rev.*, Aug. 15, 1930, p. 726.

7. *Extinction of a Long A-C. Arc*, J. Slepian, JOURNAL OF THE A. I. E. E. April 1930, p. 310.

S. S. Attwood: The "leakage" current occurring during the interval which we described as the "current zero period" was only a simple fraction of an ampere. Had this current disturbed the circuit transient, our equations would hardly have held true. On the contrary the voltage equations proved to be correct to a surprising degree of accuracy. The slight slope of the current (so called current-zero) in Fig. 9A shows this leakage.

The arcs discussed in this paper were purely of the type which Mr. Browne describes as "wandering," inasmuch as they were moving slowly over the electrodes and the oscillograms were taken at any convenient current-zero period. However, the authors have since taken further oscillograms of arcs at the end of the first half cycle, when the arc life was perhaps only one-quarter cycle. The oscillograms are practically identical with those shown in the paper. In this short arc-life the wandering was very small.

Cathode-ray oscillograms recently taken of a 90-ampere "cold-cathode" arc moving at a considerable speed indicated a reignition voltage of about 1,000 volts and the existence of a prolonged glow at this voltage, which is about three times the glow values given in the paper, and is in entire agreement with Mr. Browne's observations. This high reignition and glow voltage value certainly distinguishes the cold cathode from the wandering arc.

It is suggested that reignition variability is due to variability of cathode space-charge sheath. Alternately, the authors would like to suggest the presence of electrode impurities as a likely cause of variability. It has been brought to our attention that the presence of certain foreign elements, purposely incorporated in the electrodes, tends to insure restriking of the arc. The wandering arc should be peculiarly sensitive to this influence. Impurities alone may be enough to account for the irregularity.

The Primary Network

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and

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Synopsis.—The primary network is similar to the low-voltage network in so far as the interconnection of the secondary mains is concerned. Two transformation steps are required in the primary network described here, and only one in the low-voltage network. Nevertheless, the primary network has economic possibilities which should be considered for areas of medium load density.

A preliminary study of the so-called primary network indicated that the principle had possibilities worthy of exploitation. It was, therefore, decided that such a network would be installed if at all feasible to obtain operating experience with the system.

The design work for this initial installation is complete and construction work has started.

It is recognized that there is a number of problems to be worked out more thoroughly. As experience in operation is obtained, and with intensive study of the requirements it is expected that improvements will be made and the designs materially simplified.

Even with the limitations existing at the time of writing it appears that the primary network offers important advantages over the radial system for certain types of load.

* * * * *

Introduction

THIS paper is presented to describe a system of electric distribution which offers great possibilities in the effort of the engineers of the power industry toward rendering better service to the consumer at a lower cost.

A network may be said to exist when the secondaries of two or more transformers not at the same location are connected in parallel to supply a load. This definition may be applied to low-voltage or high-voltage networks.

Low-voltage or secondary a-c. networks have been applied to areas of high-load density (10,000 kva. per sq. mi. and above) and the principles of design have been described in previous papers. The elimination of the substation is a major source of economy in this system. In the radial system two conversion steps were necessary, first, from the generator voltage (11-13 kv.) to distribution voltage (4 kv.) and second, from distribution voltage to a utilization value (110-120 volts). The secondary network permits this conversion to be made in one step (from 11-13 kv. to 110-120 volts) thus eliminating the substation.

The secondary network as applied to areas with load densities of 10,000 kva. per sq. mi. and above, are generally underground systems. Residential and commercial areas (6,000 kva. per sq. mi. and below) are usually fed by the overhead radial systems.

In 1927 there was suggested a method of distribution utilizing the general principles of the high-tension network. This scheme utilized a number of substations of moderate size (5,000 kva.) interconnected on the 4-kv. side forming a primary network.[1]

This principle of interconnecting 4-kv. mains, and certain principles used in the design of secondary networks have been combined in a new design of primary network fed at transmission voltage. This design is described below, together with an outline of the advantages of the system.

*Byllesby Engineering & Management Corp., Chicago, Ill.

1. "Serving a Medium Voltage Network," by D. K. Blake, *Electrical World*, March 5, 1927.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

The Primary Network Compared with the Radial System

A typical radial distribution system is shown in Fig. 1. There are four elements in this system (a) the transmission feeder from the source of supply to the substation, (b) the substation, (c) the distribution feeders, and (d) the mains. The feeders in Fig. 1 are shown terminating at feeder points, and mains are shown radiating from one feeder point.

The substation is located as nearly as possible to the load center of the area served, and each feeder terminates at the approximate load center of its own area, the mains radiating from these load centers.

Fig. 2 shows diagrammatically a primary network. The transformers are distributed over the area to be

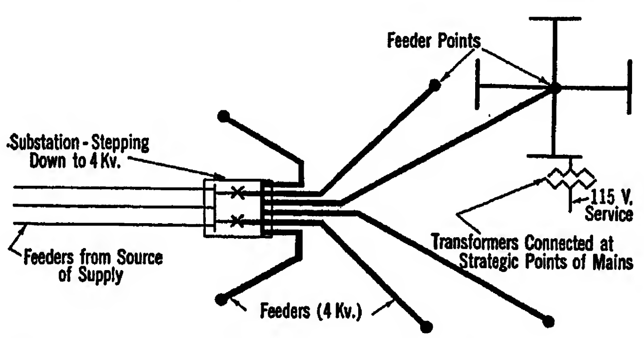


FIG. 1—TYPICAL DISTRIBUTION SYSTEM SHOWING HIGH-VOLTAGE SUPPLY FEEDERS AND MAINS WITH A DISTRIBUTION TRANSFORMER CONNECTED

served, each transformer being located at an intersection of primary mains with the transformer intersection constituting a unit. If regulators are required they will be installed between the transformer breaker and the breakers controlling the mains.

The area to be served is supplied from a number of these small and simple transformer and switching units rather than from a large substation. The transmission supply should preferably be by three or more feeders to reduce the spare capacity required when one transmission feeder is out of service. This is similar to the principle of design used in low-voltage networks.

There are, therefore, three elements in the primary network scheme, (a) the transmission feeders from the source of supply, (b) the transformers and switches,

and (c) the primary mains. Compared with the radial scheme the elements are the same except that the primary feeders have been eliminated. This feature, however, is only one of the items which permit a saving in initial cost over the radial system.

Cost Estimates of the Squirrel Hill Area

The initial study of a primary network was made for a section of Pittsburgh known as Squirrel Hill. This section embraces residences principally. There are a

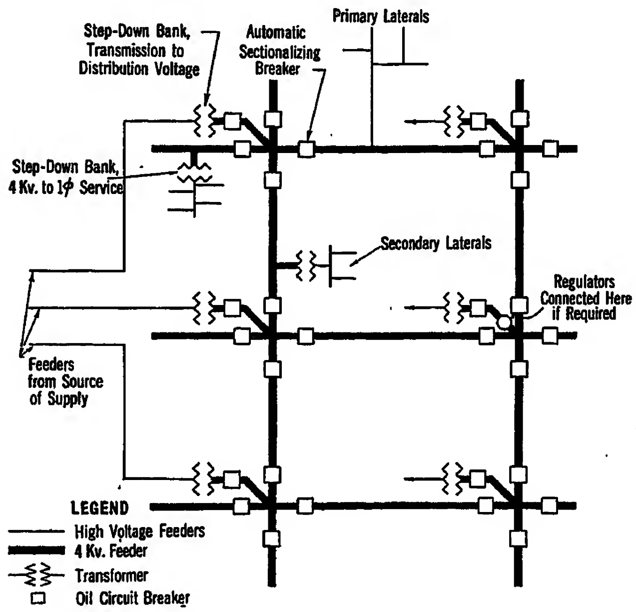


Fig. 2—Schematic Diagram of the Primary Network

small number of apartment houses, a small business district, but no manufacturing load. The area comprises about 2.0 sq. mi. with a winter peak of 4,200 kva., the load density being 2,100 kva. per sq. mi. The area is shown in Fig. 3. This area is at present fed by circuits from two adjacent distribution areas, Schenley Substation and East End Substation, and was chosen for the initial economic study; first, because the area had relatively definite boundaries, with no existing substation within its limits, and second, the growth of load indicated extension of existing radial system or adoption of a new system.

Three transmission feeders are available within the area. (See Fig. 4).

The estimates for the scheme of extension of a radial system are based on the standard practise for the Pittsburgh district. The substation location was chosen after an economic study of a number of available sites permitted by zoning ordinances and is reasonably near the load center of the area.

The substation proposed is a semi-outdoor type 22/4 kv. full automatic similar to those used on the Duquesne Light Company system.

The initial installation would consist of two 4,000-kva. transformer banks, a 22-kv. tie breaker, three 300-ampere 4-kv. regulated feeders and other necessary equipment. The building would provide space for an ultimate of two 6,000-kva. transformer banks and five 300-ampere regulated feeders.

The primary feeders would be run as shown in Fig. 3 in existing duct lines to their feeding points with sectionalizing switches as indicated.

The primary network includes the same area as covered by the radial system and is shown in Fig. 5.

The transformer and switching units of submersible type are to be in underground vaults located in the street.

Transformers are 1,500-kva. 22/4-kv. subway type with four 2½ per cent taps below and two 2½ per cent taps above on the high-voltage side, and a three-position oil immersed disconnecting and grounding switch on the 22-kv. side. The transformer oil circuit breaker is integral with the transformer. Lead-covered cable is used for all 4-kv. work in the vault.

The oil circuit breakers are 7,500 volt 600 amperes having a rupturing capacity of 14,000 amperes 2OCO

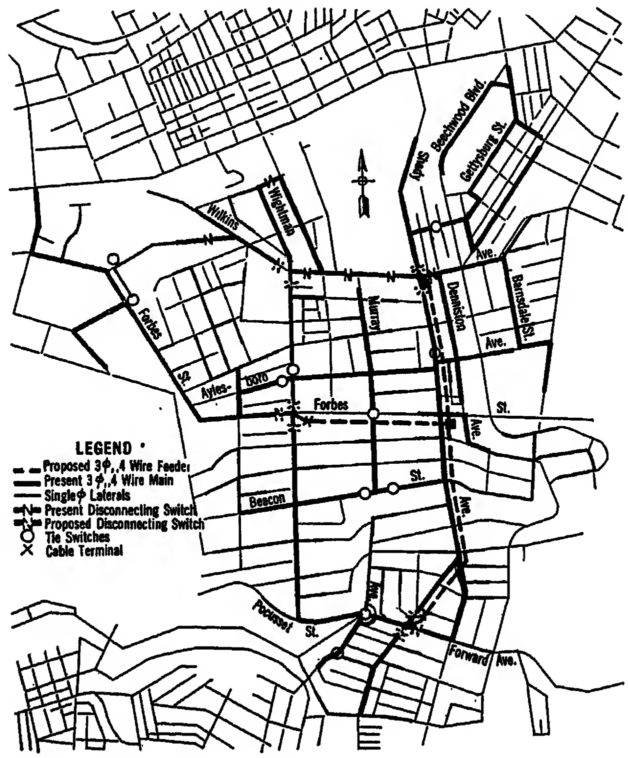


Fig. 3—Squirrel Hill Area

Radial primary distribution system which ordinarily would be used to supply area

at 4,000 volts. They will be arranged to reclose once on a fault, the breakers at each end closing at different times so that the equivalent of two reclosings is obtained. The breaker with its control mechanism and relays will be mounted in a submersible type box equipped with wiping sleeves for lead cable.

Duct connections are provided between existing manholes and the transformer vault. Cable is run in

pipe from the vault to the several poles at the intersection terminating in disconnecting potheads.

Table I shows comparative costs of radial and primary network schemes. The operating capacity of each scheme is 7,500 kva. If the cost units per kva. of capacity available for the radial scheme with substation are represented as 100 per cent, a corresponding

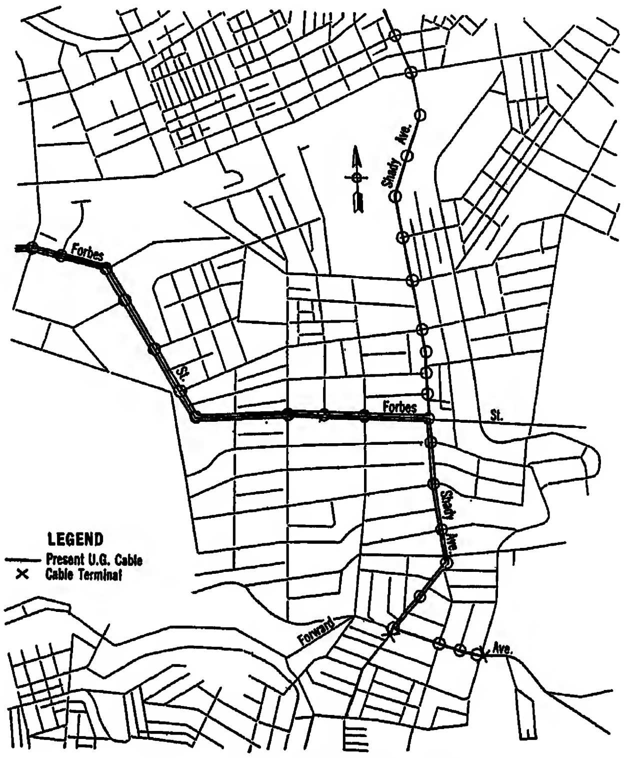


FIG. 4—SQUIRREL HILL AREA

Transmission feeders available

cost of the primary network is 56 per cent, or as expressed in the table the radial system is 78 per cent more costly than the network. If system voltage variation permits, regulators will, of course, be omitted in the network scheme because of the better inherent regulation due to the network.

Primary feeder and duct costs being common to both schemes they are omitted from the comparison.

Since cost data applying to one locality may not apply in other areas because of difference in labor costs, construction conditions and limitations, accounting procedure, etc. costs submitted are comparative and based upon experience in substation, transmission, and distribution construction in this territory.

The distribution capacity in the Squirrel Hill area is sufficient to carry the load for the present but since the study showed important savings, it was decided to make a similar study of the Verona-Oakmont district as immediate relief is needed for this district. Up to the present time this area has had adequate distribution and substation capacity. A brief description and comparative estimates for the two types of systems is given in the following section.

COST ESTIMATES OF THE VERONA-OAKMONT AREA

The Verona-Oakmont district shown in Fig. 6 is located about ten miles northeast of Pittsburgh. The area covered is 4.7 sq. mi. with a load of 2,000-kva. The load density is 425 kva. per sq. mi. The area is residential and light commercial load is supplied at present at 2,300 volts. The load in this area has only recently grown to an amount to justify a changeover to 4,000 volts. The present substation is to be abandoned for distribution purposes and the area converted to 4 kv.

For a radial system a new substation was proposed having two 3,000-kva. transformer banks. The station would be rated at 3,750 kva.[2] Two 4,000-volt regulated feeders are planned. Fig. 7 shows the proposed feeder runs.

In the primary network three 1,500-kva. transformer points are planned, located as shown in Fig. 8. With

TABLE I—SQUIRREL HILL COST COMPARISON
INITIAL STEP
Load Density 2,100 kva. per sq. mi.

	Radial	Network
Load	4,200 kva	4,200 kva.
Transf. capacity installed	8,000 kva	9,000 kva.
Transf. capacity with spare capacity out of service	5,000 kva	7,500 kva.
	Costs in per cent	
Real estate	18	....
Substations	21	26.7
Electrical equipment	67	56
Transmission	1.2	1.5
Distribution	18.4	3.4
Ducts	4.7	10.8
Changes in 4-kv. feeders & mains		1.1
Total initial step	130	100
SECOND STEP		
Load	7,500 kva	7,500 kva.
Transformer capacity installed	12,000 kva	9,000 kva.
Transformer capacity with spare capacity out of service	7,500 kva	7,500 kva.
	Costs in per cent	
Substation increase	28	None
Distribution increase	20.5	None
Total second step	178	100

Note: Radial system (initial step) will have two 4,000 kva., three-phase. transformers. With one out of service, the other will carry 25 per cent overload through peak or 5,000 kva., which is satisfactory for initial step and allowing for growth.

Network will have six 1,500-kva., three-phase banks and when one transmission line goes out, two 1,500 kva., banks go out also, leaving 6,000 kva. in service which, with 25 per cent overload capacity, gives 7,500 kva. capacity to carry 4,200 kva. initial load.

one feeder out of service the capacity is 3,750 kva. based upon 25 per cent overload on the remaining transformers.

The cost estimates are shown in Table II.

The network scheme has been estimated in three ways: (a) all equipment in underground vaults, (b) equipment in small buildings above ground, (c) outdoor

2. 125 per cent load on remaining bank with one bank out.

installation (equipment above ground on concrete pads). Calling method (c) 100 per cent, we find that the radial scheme is 35 per cent more costly, with the building type installation, and the underground vault at 7 and 18 per cent more costly respectively than the outdoor transformer installation. In areas of this type vaults will not ordinarily be necessary.

It was decided that one of each of the above types of

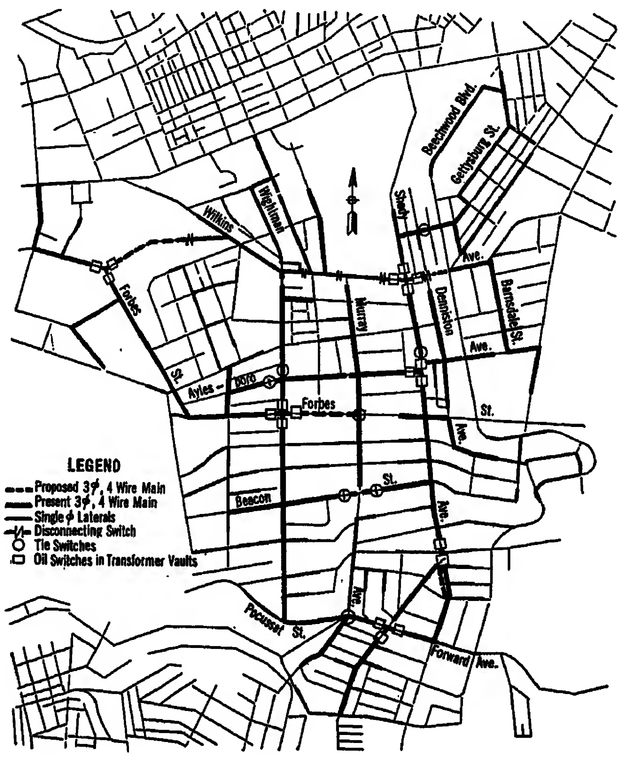


FIG. 5—SQUIRREL HILL AREA

Diagram of mains for primary network showing transformer points

installation be used in this initial primary network to form the three transformer points proposed for this area.

General Description of Transformer and Switching Unit

The transformer switching unit consists of (a) the transformer and (b) the oil circuit breakers and accessories. If system regulation is such as to make it necessary, a third element (c) the regulators, may be added.

These elements may be placed:

(a) In underground vault
(b) In small buildings
(c) On concrete pads surrounded by a fence.

Underground vaults may be used in congested areas where space above ground is expensive or cannot be obtained, or where zoning ordinances prohibit.

Small buildings may be used in the less congested and medium grade residential areas.

In lower grade areas the equipment may be placed on concrete pads and surrounded by a fence.

In the underground vault, it is essential that the equipment be subway type. Subway type transformers are now available, and regulators can readily be converted to subway type. Since there are no oil circuit breakers of the type desired that are submersible, it is proposed to mount standard breakers in welded steel submersible cubicles. Equipment used in buildings or on concrete pads may be standard for that type of service.

A proposed general arrangement of the equipment for a street vault is shown in Fig. 9. Regulators are shown, but will be installed only where necessary, thus reducing space required and the cost of the installation.

In one compartment will be installed the transformer and integral oil circuit breaker. A 22-kv. cable is brought in from ducts in the street to the pothead on the transformer. From the oil circuit breaker connected directly to the 4-kv. transformer leads, single-conductor 4-kv. lead cable connects to the regulators. From these regulators 4-kv. cables are carried to oil circuit breakers in the other compartment, and from these breakers the cable is carried to potheads on poles where connections are made to street mains.

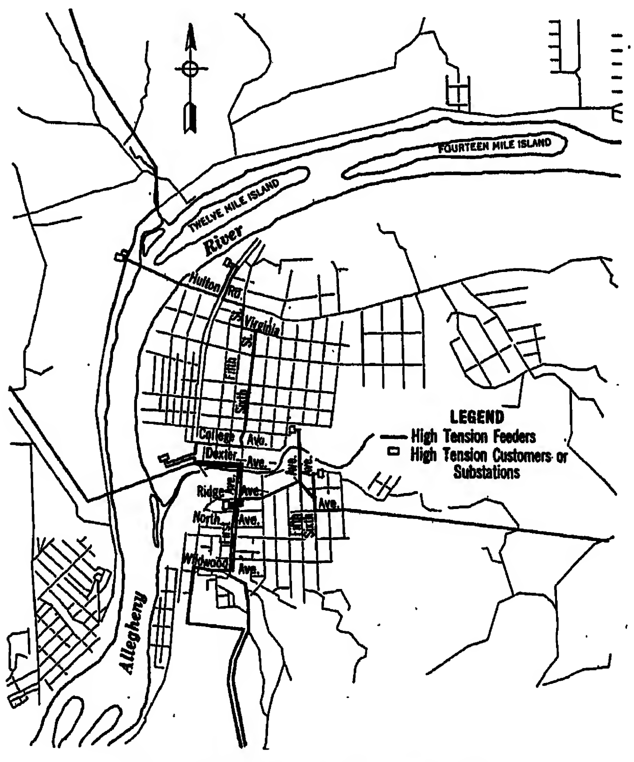


FIG. 6—VERONA—OAKMONT AREA

Transmission feeders available

It will be noted that the principle of barrier construction is used here whereby the transformer and switching equipment are separated. This principle was described in a paper by the authors as applied to the low-voltage network.[3]

3. *Low-Voltage A-C. Networks*, by Stanley and Sinclair, A. I. E. E. TRANS., January 1930, p. 265.

A removable hatch is provided over each compartment for installation and removal of equipment. Manhole covers are placed in each hatch for normal access.

A small power transformer bank (three 1½ kva.) is located in the transformer compartment for circuit breaker operation and for relay potential.

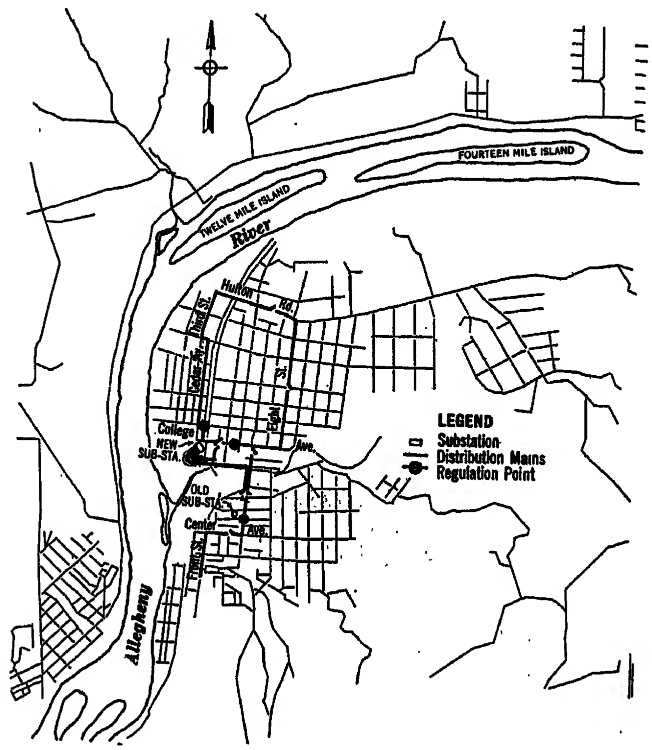


FIG. 7—VERONA—OAKMONT AREA

Radial primary distribution system which ordinarily would be used to supply area

Such a vault (1,500 kva.) capacity requires a floor space approximately 12 ft. by 30 ft., and a depth of 13 ft.

Relay Protection

When the primary network was first considered no scheme for protection had been studied, and it was felt that it would require a complicated combination of relays or the possible development of a new relay. However, when the matter was investigated it was found that a very simple protection scheme could be used and lend itself to ready expansion without the necessity of changing relay settings.

Transformer Protection. A network relay similar to the one used on the low-voltage a-c. network will be used to protect the transformer bank breakers. This relay would open the breaker on any type of high-tension fault, and also if the supply line is deenergized the magnetizing current for the bank in combination with the charging current of the line will be sufficient to open the breaker. It is also possible with this relay to have the breaker close when the high-tension line is reenergized. Therefore, the type of protection used on this breaker will be identical to that of the low-voltage network protector. In addition to this protection, consideration is being given to tripping this breaker by means of a contact-making

TABLE II—VERONA-OAKMONT COST COMPARISON
Load Density 425 kv-a. per sq. mi.

	Radial	Network		
		U. G. Vault	Bldg. type	Outdoor
Load	2,000 kv-a.	2,000 kv-a.	2,000 kv-a.	2,000 kv-a.
Proposed inst. capacity	6,000 kv-a.	4,500 kv-a.	4,500 kv-a.	4,500 kv-a.
Operating capacity*	3,750 kv-a.	3,750 kv-a.	3,750 kv-a.	3,750 kv-a.
	Costs in per cent			
Real estate	17.0		2.8	2.8
Substation bldg. or vault	4.2	24.5	13.0	5.8
Equipment	95.0	71.0	66.3	66.3
Transmission	2.5	3.3	4.4	4.4
Distribution	1.6	13.2	13.2	13.2
Duct connections	14.5	5.7	7.5	7.5
Total	135.	118.	107.	100.

*Based on 125 per cent load on transformers for five hours in case of feeder or transformer failure.

Note. Cost of regulators included in above estimates.

thermometer placed on the transformer. In case of excessive overload which would be likely to damage the transformer, the thermometer when it reaches a predetermined value would energize the trip circuit, thus

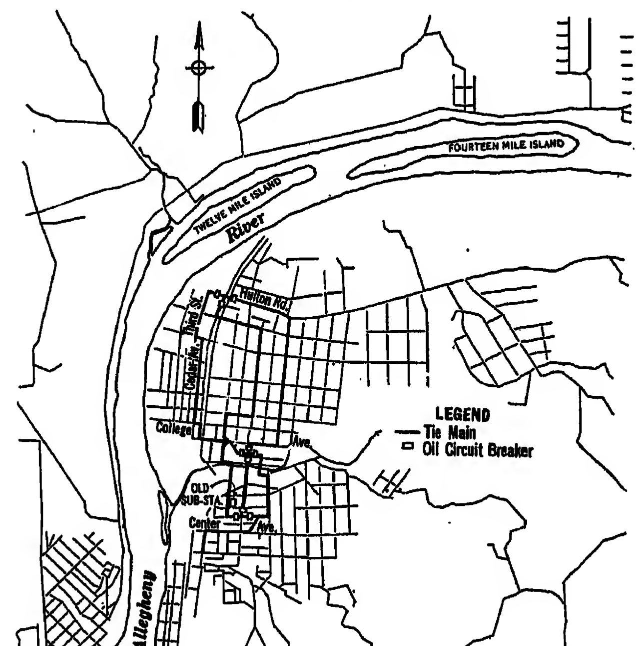


FIG. 8—VERONA—OAKMONT AREA

Diagram of mains for primary network showing transformer points

disconnecting the transformer from its load. This might be advisable in the event that two high-tension feeders should be out of service during a peak-load period, but would permit the one remaining bank to

carry the load for as long a period as possible without damaging the transformer. In the majority of cases relief could probably be given this one unit before the temperature would reach the tripping value. Under this scheme no other overload protection would be used for the transformer breaker.

Bus Protection. Bus differential will be used at each transformer installation to protect the bus within the intersection so that any type of fault will open all the breakers and lock them open. The overload relay for this protection will be installed in the transformer breaker cubicle.

Tie Main Protection. A detailed study has been made to determine the type of protection best suited to protect all tie mains. All breakers are arranged for automatic reclosing with the breakers on each end of a main set to close at different intervals, so that the main is reenergized two times. If a breaker on the faulty main closes and again opens, it is locked out and no further operations are possible.

Two schemes have been worked out which will give adequate protection, namely: (1) overload with definite time setting and (2) overload with inverse time characteristic.

1. Overload with Definite Time Setting. In this scheme the relays are set to operate at 150 to 200 per cent of the transformers full load current, namely 300 to 400 amperes. The time settings are staggered so that a breaker on one end of a main will open in 0.6 sec. and on the other end in 1.1 sec. This scheme has one disadvantage that breakers other than those connected to the faulty main open, although there is no interruption to service on other mains since none of the breakers set at 1.1 sec. have sufficient current after the 0.6 sec. breakers open to operate except the one directly connected to the fault.

2. Overload with Inverse Time Characteristic. This scheme employs a relay with a greater inverse time characteristic than is normally used. With a 400/5 current transformer and the 4-ampere relay setting, the relay will operate at 480 amperes in 9 seconds and will operate on a current as low as 320 amperes although the time would not be accurate. In this scheme only the two breakers connected to the faulty line operate. However, if corner transformers are installed in which tie mains are run in only two directions, proper selection cannot be made when this particular transformer is out of service. In general, it may not be advisable to use this design but rather run a third main to some adjacent intersection. This scheme appeared to have certain advantages over scheme (1) and was adopted.

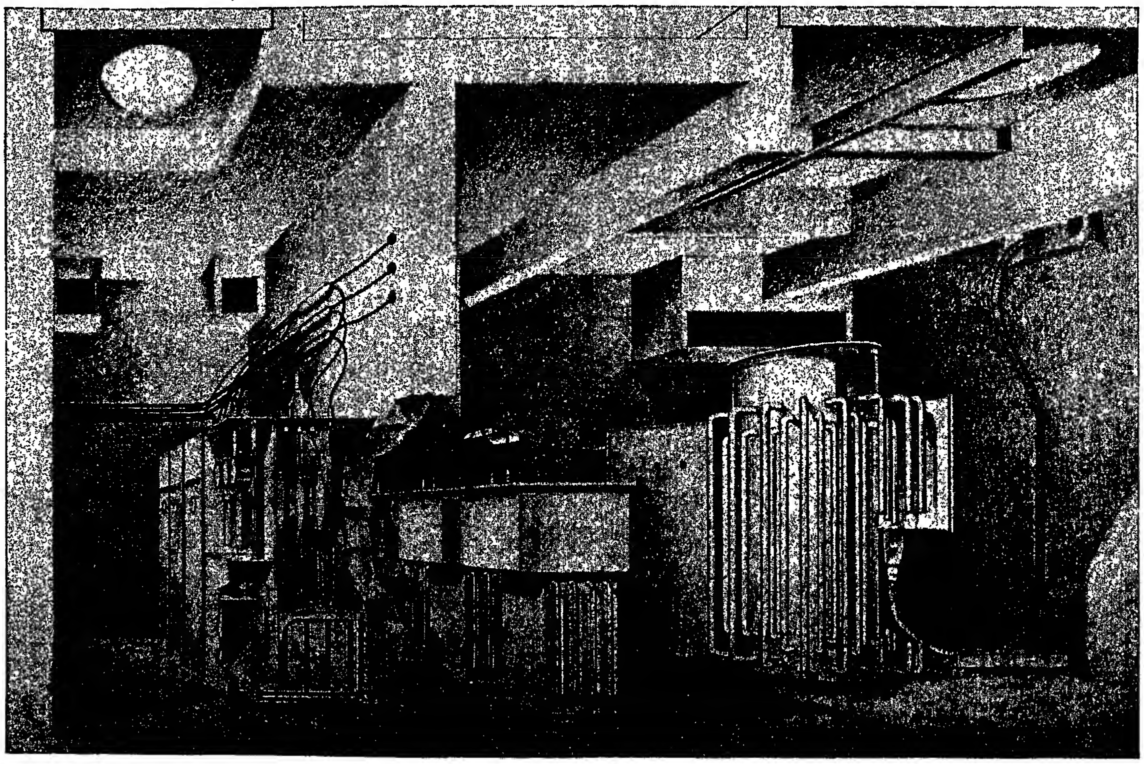


FIG. 9—TRANSFORMERS, REGULATORS, AND OIL CIRCUIT BREAKER CUBICLES IN AN UNDERGROUND VAULT

Note the barrier wall construction. The same general arrangement will be used in building jobs

APPLICATION

Short-circuit studies were made of the network layout for three-phase and phase-to-ground faults at all locations along the tie mains, with all transmission feeders in service and with each one out to determine the definite requirements of a relay.

Fig. 10 shows a typical example of the fault currents with a fault on one of the mains adjacent to the College Avenue and 6th St. intersection. In this case oil circuit breakers Nos. 1 and 5 must open to clear the fault. No. 5 has 3,960 amperes which from the curve will open in 0.5 sec. No. 1 has only 500 amperes and it would take 7.3 sec. for it to open while Nos. 6 and 8 are

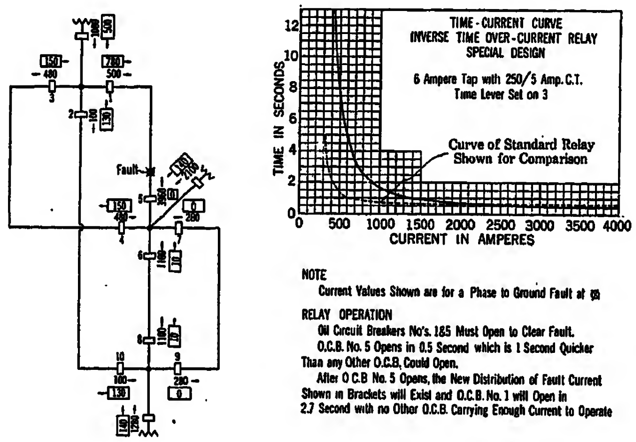


Fig. 10—Typical Analysis of Relay Operation on Tie Main Fault

carrying 1,100 amperes and would open in 1.5 sec. However, after No. 5 opens in 0.5 sec. which is 1.0 sec. sooner than Nos. 6 and 8 can open, a redistribution of current exists as is shown by the blocked figures. No. 1 is the only one now carrying enough current (780 amperes) to open and it will open in 2.7 sec. thus clearing the fault. Therefore, the fault is cleared without interruption to tie mains other than the one in fault, and the fault is cleared entirely in 3.2 sec. Faults at other locations show the same general conditions and in all cases only service on the faulty tie main is interrupted.

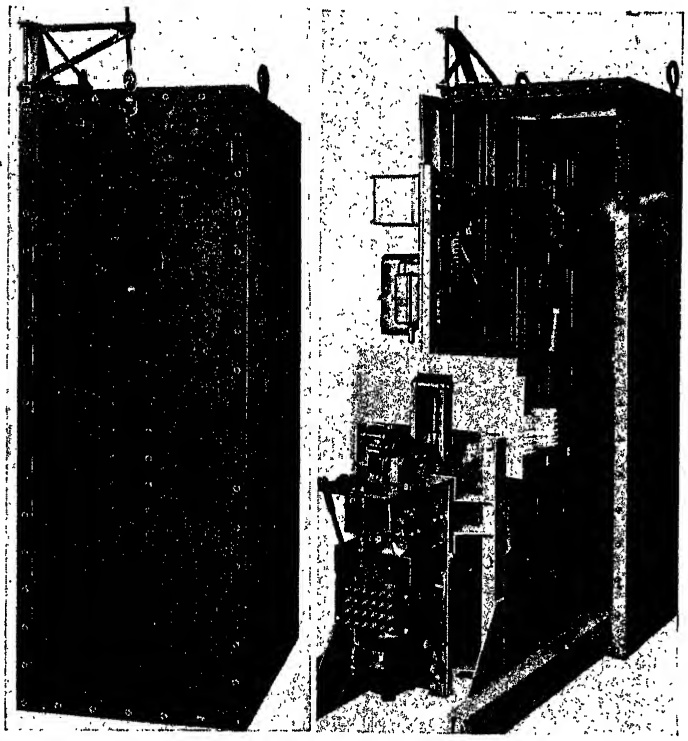
Similar studies have been made on layouts using various numbers of transformers, and with various length mains, and in all cases the results are comparable to the Verona-Oakmont study, so it is believed that no trouble should be experienced in relaying the primary network particularly if two-way feed transformer installations can be eliminated.

Transformer Details

The transformer will be 1,500-kva. three-phase subway type 22,000/4,000-volt Y with four 2½ per cent taps below and two 2½ per cent taps above 22,000 volts, and to have an inherent impedance of ten per cent. The 22,000-volt cable supplying the transformer will terminate in an oil tight pothead with a three-position switch so that the cable may be either disconnected from the transformer, connected or grounded, and is protected by an interlocking mechanism so if the transformer is energized from either side it will not be possible to operate the switch. This pothead or oil tight chamber is mounted on the transformer as an integral part of the case. On the 4,000-volt side the leads will be brought out through oil tight bushings so constructed that an oil circuit breaker cubicle can be bolted to the transformer case making a water-tight connection.

Oil Circuit Breaker Details

The oil circuit breakers are 600 amperes 7,500 volt and will rupture 14,000 amperes with the standard duty cycle. They are mounted on a small truck and placed in a small subway type cubicle in which all relaying equipment, current transformers, motor-operated mechanism, etc., are mounted. Space is not allowed for inspection of the breakers while in the cubicle but they are easily disconnected and rolled out. Each unit is self-contained, and at the time of installation it is only necessary to connect the cables to the wiping sleeves. A number of arrangements for the cubicle design has been worked out. Figs. 11 and 12 show the simplest design which requires cable connections on both sides, and this design will be used in the Verona-Oakmont installation. Fig. 13 shows the outline of this cubicle. Another design considered is shown in Fig. 14 using bus



Figs. 11 and 12—Oil Circuit Breaker Cubicle

Submersible type showing door removed, truck pulled out, relay panel swung open and barriers removed.
Only one current transformer is shown in place

connections between cubicles. A vertical truck type connection is shown in Fig. 15 in which the leads are disconnected when the oil switch is lowered.

Regulation

The percentage regulation for the network is very low so that in general it does not appear necessary to use regulators unless there is considerable variation in the

supply voltage of the transmission lines. To prevent the problems which arise with normal parallel regulator operation, it will be necessary to control each intersection by a contact-making voltmeter set so that each intersection will maintain the same constant voltage.

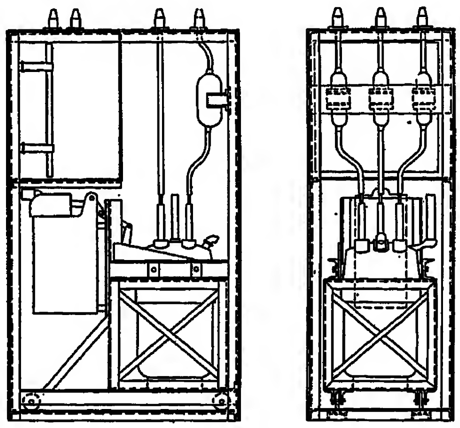


Fig. 13—Oil Circuit Breaker Cubicle

Outline of cubicle similar to Figs. 11 and 12 except arranged for different oil circuit breaker

Regulation Details. The regulators will be the standard single-phase oil immersed type, 200 amperes 2,300 volt, 5 per cent buck or boost and equipped with wiping sleeves. The motor, control, and transformer for the motor circuit will be mounted in a box on the side of the regulator, so that the unit is complete in itself, with all connections gasketed to make it subway type.

Conclusions

The primary network offers the following advantages over the usual radial system:

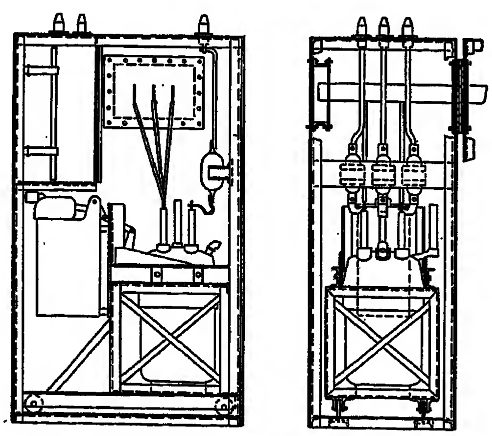


Fig. 14—Oil Circuit Breaker Cubicle

Outline showing bus connections between cubicles

1. *Material Reduction in Investment.* Table I. indicates that the radial system is 78 per cent more costly than the network. Or, expressing it in the reverse order, the network costs about 56 per cent of the radial system now in use. Even under adverse conditions such as in areas of load densities as low as 425 kva. per sq. mi., the cost runs as much as 35 per cent higher for the radial system.

2. *Greater Copper Economy.* Reduced copper sizes are possible on the mains because of the smaller currents and shorter distances of feed, the results of the network principle.

3. *Reduced Duty on Oil Circuit Breakers.* Short-circuit studies of the two areas studied and of theoretical systems indicate that the oil circuit breakers can be materially smaller than for the corresponding radial system.

4. *Better Regulation.* The network principle results in better regulation than on the radial system. Any radial system inherently has considerable voltage variation which regulators cannot compensate. It is believed from a theoretical voltage analysis made that a very uniform voltage will be obtained on the network.

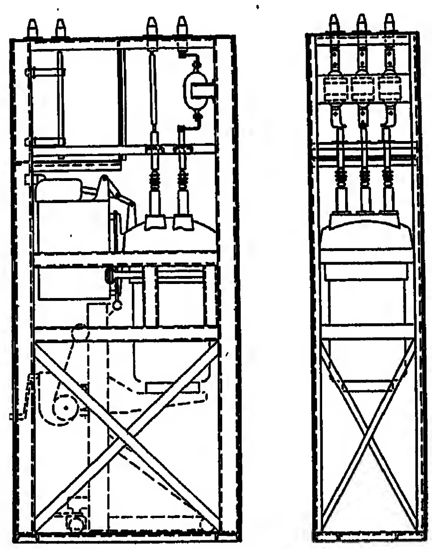


Fig. 15—Oil Circuit Breaker Cubicle

Outline showing vertical lift type trunk breaker

5. *Small and Standardized Transformer Units.* Transformer units (transformers, switches, regulators, if used, and auxiliary equipment), can be manufactured in units, stocked if necessary, and installed when required.

6. *Capacity may be Installed in Small Increments.* The radial system requires a substation and distribution system to be built well in advance of the load growth. Long range estimates are necessary for load growth, in both direction and magnitude. In the primary network transformer units are installed *where and when needed.* Only as the load appears is the system expanded. Long range predictions are unnecessary.

7. *Investment for Load Growth in Small Increments.* The investment is not made in large steps far in advance of the load growth. (See 6).

8. *Reduced System Losses.* Calculations indicate that the total losses in the primary network are less than for the radial system. One item of saving for example, is the complete elimination of feeder losses as there are no feeders.

9. Area of Outage Reduced. With the designs considered each section will have a capacity of 600-700 kva. The radial circuit has a capacity of 2,100 kva. A failure in the case of the primary network therefore, involves less than one-third the load of the radial system.

10. No 22-kv. breakers. The high-tension cables run directly to the network transformers and hence no breakers are needed.

Discussion

E. R. Hendrickson: Although the area affected by a failure on the type of system described by Mr. Stanley is somewhat reduced, still the length of interruption to any individual or group of individual customers would not be materially different from that of the usual radial system with proper emergency switching centers.

Because of the increasing dependence being placed upon continuity of service by the average medium class residential customer who is now using electrically operated oil burners, clocks, etc., it is becoming of very much more importance to give him continuous service.

In order to provide this type of service, it appears that some form of low-voltage secondary network must eventually be used for at least the better class of residential and apartment house loads. I would therefore like to ask whether or not, in general, it is considered that the primary network as described would represent the correct solution for service to a territory of low or medium load density, where because of the importance of continuity of service or future growth in load, a secondary network would be required in the future, and if so, how would the secondary network be superimposed on the primary network system?

In many cases in high-class residential territory it is not possible to install overhead transmission feeders to the various transformer stations. Have any studies been made which would indicate that the primary network is the most economical solution if underground transmission feeders are to be installed to each transformer station instead of only extending them to a central substation?

H. Richter: The primary network system that is described in the 1927 *Electrical World* article mentioned in this paper has been publicly recommended to the industry on several occasions. That system utilizes substations each of about 5,000 kva. capacity. Westinghouse engineers worked on such a scheme for two mid-western cities in 1924 and 1925, and again in 1927 and 1928 for two cities in the east. The 1924, 1925, and 1927 projects were broached primarily to improve service in medium load density and high-class residential areas, but the 1927 study had in view mainly a possible saving due to eliminating the usual type of step-down substation.

In every case the method of relaying using the inverse time limit overload relays thus far available, and advocated in the *Electrical World* article, was dismissed as not likely to give sufficient continuity of service from a 4-kv. network. It was early realized that this particular type of system would certainly not be less expensive than the radial system and might become quite complicated to operate. For these reasons publicity did not seem warranted. Up to the present time there has been no known application of the scheme.

The primary network system evolved here in Pittsburgh, on the other hand, reduces the transformer installations to relatively smaller sizes. This permits the equipment to be installed in manholes, where there is no real-estate item, or on small lots away from the more expensive locations and less likely to be affected by zoning ordinances. A considerable reduction in rupturing capacity of the oil-circuit breakers is another important result, and a fault on a 4-kv. tie line shuts down service to a much smaller number of consumers. The Pittsburgh type primary network is thus practical, and the comparisons in the paper show distinct savings over the radial system for two specific installations.

With regard to the method of relaying, it will be observed from Fig. 10 that the characteristic curve of the standard inverse time over-current relay becomes quite flat beyond 1,000 amperes and that the selectivity therefore is impaired beyond that point. It was only when the improved characteristic shown in Fig. 10 was suggested for the Pittsburgh primary network that satisfactory relaying appeared likely. Even this curve flattens beyond 1,500 amperes, and the useful part, which is the inverse portion, is still fairly restricted. Calculations on an average system of this type indicate that short-circuit currents falling outside of the useful portion of this curve may be encountered. This is especially true when the system is enlarged or the unit transformer capacity increased. To furnish the better selectivity at these higher currents the relay should have the inverse time characteristic over a greater range.

This more ideal relay characteristic, however, will give no better performance at corner transformers where tie mains branch in only two directions than the relay of Fig. 10. The method using over-current relays with definite time setting was devised to avoid the need of bringing in a third tie at such corners and of depending on the inverse time characteristic for proper selectivity throughout the system.

The two 1,500-kva. three-phase transformers manufactured in Sharon for the Verona application are the largest subway type transformers thus far produced. The regulators are likewise of special interest in that they constitute the first subway type in this country as far as is known. Stable operation of the regulators at different transformer installations can be assured by the simple method of using a bucking characteristic with increase of load.

Prior to the work on the Pittsburgh type primary network the only form of oil circuit breaker of relatively high rupture capacity in use in manholes was a strictly subway type breaker. Several years ago, in conjunction with proposed underground loop feeders to supply low-voltage networks in four cities, standard indoor type B-16 and B-20 oil circuit breakers in submersible welded steel cubicles were offered. This appeared to be the best way to combine accessibility, safety against water penetration, and interchangeability of parts, with reasonable cost. The B-13 breakers with type CFO motor-operated mechanism, in subway type cubicles as shown in Fig. 12, comprise the first application of this idea.

If no regulators are required all of the oil circuit breakers associated with a transformer might be mounted on the transformer tank. This would make a complete factory-built set, conserve space and cable connections, and permit the handling and installation of the apparatus as a single unit. This construction would be particularly applicable to outdoor or indoor installations, but might also be feasible for manholes under some circumstances.

W. R. Bullard: The paper by Messrs. Stanley and Sinclair emphasizes certain economies and advantages of a particular type of overhead distribution system. This type of system has a place in the art for particular conditions, such for instance, as those encountered in the particular situation described in the paper. One of the particular conditions referred to is the presence, in the area considered, of a main supply operating at 22 kv. Another is the requirement for an unusually high grade of service.

I feel, however, that the picture is incomplete without further mention of another much simpler type of system which I believe to be basically more economical than the one described. This is the type utilizing generating station or transmission terminal substation voltage (6.6 kv., 11 kv., 13.2 kv. and even higher) as the primary voltage for supplying the distribution transformers

directly. This system avoids entirely the use of the intermediate transformation and thus usually effects a very considerable saving in investment and energy losses over any system using an intermediate voltage for the distribution primary. Indeed, it has been shown repeatedly that the elimination of the intermediate transformation usually provides considerably greater possibilities of economy than any of the other plans for reducing investment in distribution systems which have recently been studied.

To date, systems of this type serving overhead districts have had radial primary and secondary circuits. Reasonable reliability of service can be provided by other means than the duplication of supply units. One method is the use of higher grades of construction in order to reduce the number of system faults. The use of the higher primary distribution voltages usually carries with it, for other reasons, the use of higher safety factors and better quality of construction. Therefore, radial systems of the type mentioned, while possibly not having quite the high grade of service reliability of the 4-kv. network, are in successful operation and are giving entirely satisfactory service under average conditions encountered in certain urban, suburban, and rural districts where overhead service is normally required.

Under conditions where greater service reliability may be required, it appears that the network principle could be applied to the higher voltage primaries in much the same manner as described in this paper, except for the absence of the intermediate transformation. However, the cost and complexity of sectionalizing devices would be greater in the case of the higher voltages, and a much simpler solution presents itself. This is the probability of the development in the near future of an overhead type of low-voltage a-c. network. Thus the installation of radial systems utilizing the higher voltage primaries is a natural step toward the development of the most economical type of system giving highest quality of service, viz., the low-voltage a-c. network with radial primaries utilizing generating station or transmission terminal substation voltage.

There are certain difficulties encountered in connection with the installation of overhead, open-wire primaries utilizing the higher voltages, such as governmental objections to overhead high-voltage lines in the streets, clearance required from shady trees, etc., but these difficulties are being in many cases entirely overcome, and since such systems are in line with the present positive drift to higher primary voltages, I confidently expect that they will find a large use in the future. I also am confident that such systems offer appreciably greater possibilities for economy than those involved in the system described by Messrs. Stanley and Sinclair.

A. H. Sweetnam: In 1930, The Edison Electric Illuminating Company of Boston was faced with the problem of relieving its a-c. substation in Allston of approximately 3,400 kva. of load before the fall of 1931, as the capacity of this station with the largest transformer bank out of service, was only 13,600 kva., and it was estimated that the 1931 peak load would be about 17,000 kva.

A preliminary study indicated that by building a new substation in Brookline, it would be possible to pick up about 4,500 kva. of load in an area of approximately 4.4 sq. mi. (load density 1,020 kva. per sq. mi.) by installing short 4,000-volt radial feeders out to the load centers of several of the existing circuits in that immediate vicinity.

It was proposed, therefore, to install initially two 7,500-kva. transformer banks in the new station and supply them by taps from two existing transmission lines in the vicinity.

A location for the new substation was chosen, and a careful study was made for the purpose of obtaining the cost of this station and of the necessary conduit and cable work in the street to bring the transmission lines into the station and to take the new 4,000-volt feeders out to their respective load centers.

About the time that the preliminary plans for the new substation was completed, attention was directed to the possibility of serving this same area by means of a 4,000-volt a-c. network, and, accordingly, with the assistance of Mr. D. K. Blake of the General Electric Company, a preliminary study of the existing conditions was made which indicated such an appreciable saving in initial and ultimate investment, by the installation of an a-c. network in this territory, not to mention the other inherent advantages offered by this system of distribution, that it was decided to obtain a detailed estimate of cost for such an installation.

Very careful estimates were made of the relative costs of supplying the area in Brookline by means of the 4,000-volt network and by a new substation. This economic study was extended to cover the ultimate load requirements of the territory to be served, and it was very clearly indicated that the capital investment showed a reduction of at least 20 per cent through the installation of the network system. Corresponding figures for the initial installation showed even greater savings.

The reduction in the ultimate investment did not include savings in favor of the network obtained through being able to add relatively small increments of capacity, thus closely following the requirements of the load, as against large increment additions inherent in the development of a substation.

In general, the principle of the 4,000-volt a-c. network is to distribute the transformer capacity in relatively small units over the area to be supplied, rather than to concentrate it in one central location, as in the case of a substation.

With this general idea of the network as a guide, the particular problem consisted of creating this new system of distribution from the existing 4,000-volt radial circuits.

The problem resolved itself into three major parts:

1. The location and equipment of the vaults.
2. The transmission line extensions to the vaults.
3. The cutting over of the existing radial circuits to form a 4-wire network between vaults.

After careful consideration of this immediate problem and of the larger problem of the more general application of this network system, it was decided to proceed with the network installation and to standardize on the 1,500-kva. unit and its equipment installed in underground vaults approximately 30 ft. long, 10 ft. wide, and 10 ft. deep. Each vault will contain one 1,500-kva., three-phase, self-cooled transformer rated 13,800 volts primary and 4,330 volts secondary, with one full capacity tap for 5 per cent above normal brought out to a ratio adjuster. The transformer will also have the necessary equipment for load ratio control, designed to automatically change the ratio in eight equal steps over a 10 per cent range, two steps being above the tap in use and six steps below. Provision will also be made so that blower equipment may be added to increase the rating of these transformers to 2,000 kva.

All apparatus will be entirely metal clad making it impervious to moisture and permitting actual flooding of the vault without damage.

The primary and secondary windings of the transformer will both be Y-connected with the neutral of the 4-kv. winding solidly grounded, and the neutral of the 13.8-kv. winding arranged so that it may be operated either solidly grounded or isolated.

The transformer and submersible metal-clad switching equipment will be designed for assembly and operation as a unit, thereby standardizing the vault construction and simplifying and hastening the installation of new equipment when needed.

It was not necessary to do much rephasing on the network because the single-phase loads on the existing radial circuits were already balanced between the three phases.

Lightning arresters are to be installed on all standpipe poles where the 4-wire mains change from overhead to underground,

and on all overhead taps of any considerable length which connect directly into an underground main.

Voltage regulation will be ideal due to the fact that vaults will be located at load centers and will be relatively close together.

All circuit breakers being used in the network vaults are of ample rating (36,000 amperes at 4,000 volts) to handle any short-circuit currents which may be developed by the network, up to an installation of capacity sufficient to supply load of a maximum density of 20,000 kva. per square mile.

To those who are faced with the problem of keeping pace with the ever increasing short-circuit duties on substation circuit breakers, this feature of the network will have a particularly strong appeal.

A study of the above factors shows quite definitely such a decided superiority of the network over the radial system of distribution with regard to economy, simplicity, continuity of service, voltage regulation, standardization of equipment, provision for load growth, et cetera, and as it is so readily adaptable to areas of any load density, that whenever it becomes necessary to make any extensive changes in the existing distribution system, or plan a new system, careful consideration should be given to the possibility of installing a 4,000-volt network.

L. G. Smith: I desire to stress one point of economic consideration, of which I am sure the authors are aware, but which, in my opinion, is not sufficiently emphasized in the paper by virtue of the fact that the relative economics of a conventional radial system as compared with a primary network system may depend upon this one point.

In making an economic comparison of a primary network with a conventional design of radial system undoubtedly the cost of transmission to the substations will be somewhat higher in the primary network system. Of course, this depends to a considerable extent upon local conditions including the capacity per transmission circuit and the general layout of the system. The substation costs, moreover, will probably be somewhat higher for the primary network system due to the fact that submersible equipment for manhole installations will be more expensive and since the transformer capacities are smaller, a higher cost per kva. of transformer capacity should result. Naturally, there is one feature of saving in the substations with the primary network systems, namely, the saving in feeder regulators. In the primary network system, the distribution costs from the substation to the feeder point are entirely eliminated because the substations are placed at the feeder points. Other items of cost, such as primary mains, distribution transformers, secondaries, and customers' services, will be the same in either type of system. With the conventional radial type of system, assuming the cost of transmission, substation and distribution to the feeder points at 100 per cent, this cost will be divided approximately as follows:

Transmission (underground system)	33 per cent
Substations	40 per cent
Distribution feeders	27 per cent

The savings obtained by eliminating the distribution feeders may be sufficient to overbalance the increased cost of transmission and substations. From this it can be seen that the balance of economy may not be very great in either direction and as a matter of fact, local conditions may have a very material effect upon any economic studies.

In addition to the above points the primary network system presents another distinct economic advantage over the conventional radial system, that is, in the cost of spare capacity. With a primary network due to the fact that substation units are added in very small capacity steps, it is possible to follow the load growth very closely and maintain a reasonably small percentage of spare capacity. With a radial system this is not true. If, for example, a 20,000-kva. substation is shown by study to be of economical ultimate size, the initial step may consist of two 10,000-kva. transformer banks and four 13.2-kv. cables giving a sustained capacity of 10,000 kva. In view of the fact that the initial load may be well under 10,000 kva. the spare capacity during the initial life of the substation may be well over 50 per cent. This materially increases the cost per kva. of transmission and substations during the early stages of the substation development. As a matter of fact the costs do not decrease to the ideal values until the ultimate development is reached, at which time a new substation has to be started. Therefore, the cost per kva. for substations and transmission capacity cannot be determined for the ideal basis in an economic study but an average cost must be used which covers the average cost of sustained peak capacity per kva. over the period of the growth of the substation from its initial construction to its ultimate development.

In view of the fact that the cost per kva. of substation and transmission capacity may be 4 or 5 times the cost per kva. for the ultimate development, it can be seen that the characteristic of the primary network system of being able to closely follow the load with installed capacity may be the feature which determines its economic advantage. As I see it, one of the most important features in the design of a power system is the maintaining of adequate reliability to service by means of spare capacity at a minimum cost.

R. T. Henry: Some of the claims made in this paper for the primary network scheme are very optimistic. There is danger in attempting to draw general conclusions from comparisons in specific cases which involve such unusual conditions as the two cases described. It should be noted that both of these cases involve very small areas where transmission circuits already existed and that practically no expense is included for real estate for the network scheme. In such comparisons the investment already made in transmission circuits should be recognized as a part of the total investment if not of the immediate investment required.

In considering the primary network for a larger area, it should be noted that the ideal scheme would provide a different transmission circuit for each network unit. Obviously, this cannot be realized in any ordinary case and it becomes necessary to connect two or more units to each transmission circuit. Two units connected to the same transmission circuit must not be adjacent nor even alternate units if satisfactory diversity is to be obtained. They must be separated in every direction by at least two units connected to different transmission circuits. It can readily be seen that, when this idea is applied to a large area, it results in a perfect maze of transmission circuits and becomes very difficult and costly.

In addition to the difficulty involved in the transmission circuits, as described above, the problem of relaying an extensive primary network is rather difficult. The inverse time relays may operate satisfactorily on feeder faults if each unit has four feeders connected and all transformers are in service. However, with a transformer out of service there is very real danger of undesired operations, particularly if less than four feeders are connected to the unit whose transformer is out of service. This is still further complicated by inequalities in the length of different feeders which cannot be avoided in practical cases.

About two years ago the Buffalo General Electric Company was faced with the problem of designing and building a distribution system covering a territory of about 65 square miles. The load involved was in excess of 100,000 kw. In this case it was necessary to provide transmission circuits to serve the new distribution system. The primary network scheme was very carefully and thoroughly studied at that time in comparison with several other schemes. These studies resulted in the adoption of a radial scheme, a description of which will probably be published in the near future. This scheme has practically all of the advantages of the primary network scheme without its disadvantages and careful comparative estimates indicated that it would cost at least 10 per cent less than the primary network scheme for the territory involved.

Referring to the conclusions in the paper in the same order in which they are presented, we have the following discussion to offer:

1. Material Reduction in Investment. It is very doubtful if network substation locations can be secured in every case, nor even in most cases, without any cost for real estate. Certainly this is too much to expect in cities.

The figures presented indicated a considerable saving in the cost of electrical equipment in favor of the primary network scheme. This should not and need not be the case in a properly designed radial substation. In many cases, if not in most cases, the cost of transmission is a considerable portion of the total cost and should certainly be recognized in making such comparisons.

The studies made in Buffalo covering a larger territory and including transmission indicate that the total cost of the primary network scheme would be greater by at least 10 per cent than the cost of the radial scheme adopted.

2. Greater Copper Economy. For the conditions encountered in Buffalo, there would have been practically no reduction in the cost of distribution feeders in the network scheme over the cost for the radial scheme adopted. This is largely due to the irregularities encountered and to the fact that a large proportion of the investment in feeders is in the branches which are the same in either case.

3. Reduced Duty on Oil Circuit Breakers. It is easily possible in a properly designed radial system to keep the duty on oil circuit breakers as low or even considerably lower than in the primary network scheme. In the radial scheme adopted in Buffalo the maximum short-circuit current at the 4-kv. bus is 65,000 kva. This value will not be exceeded even with the ultimate development of the scheme.

4. Better Regulation. While the voltage level in the network scheme may be more uniform than in a radial scheme with long feeders, it should be noted that in the primary network scheme, it is practically impossible to apply load compensation without involving instability between regulators. Since the voltage drop in distribution transformers is as great or greater than the total voltage drop in the primary circuits, this is a very serious objection. In the radial system adopted in Buffalo the feeders are very short and load compensation is provided to overcome the voltage drop in the distribution transformers as well as in the primary circuits.

5. Small and Standardized Transformer Units. A properly designed radial system permits of standardization at least as readily as the primary network scheme or any other scheme. The initial installation in the new distribution system in Buffalo consists of 24 standard substations, each containing three standardized units making a total of 72 standardized units. This entire installation is covered by one set of drawings.

6. Capacity May be Installed in Small Increments. The radial system adopted in Buffalo provides for the addition of capacity in small increments "where and when needed." Long range predictions of load growth are quite as unnecessary in this scheme as in the primary network scheme.

7. Investment for Load Growth in Small Increments. Same as No. 6.

8. Reduced System Losses. The claim of "complete elimination of feeder losses as there are no feeders" is hardly justified as the feeders are not eliminated but simply called "tie lines." Here, too, the principal losses are in the branches rather than in the main part of the feeder. Studies indicated that there was practically no difference in the losses in the radial system adopted in Buffalo from those in a primary network system for the same territory.

9. Area of Outage Reduced. The area affected by an outage is no greater in the radial system adopted in Buffalo than in the primary network system described in the paper.

10. No 22 Kv. Breakers. The high-tension cables are connected direct to the substation transformers without high-tension breakers in the radial system adopted in Buffalo as well as in the network system described in the paper.

H. L. Wallau: The relative cost submitted by the authors of this paper is the reverse of that obtained by me some six years ago when I had occasion to make a study along similar lines.

The comparison made by me was between a 12,000-kva. radial substation and four 3,000-kva. substations with 4,600-volt lines networked. The equipment estimate was, of course, based on the types available at that date. The study, however, included the total transmission required from the generating plant to the stations to be served, land, buildings, substation, and distribution equipment for a load of 9,000 kva., 3,000 kva. of spare transformer capacity being available in each case. Transmission was at 11 kv. underground and regulation was by means of feeder, not bus regulators. Under these conditions the relative investments arrived at were 100 per cent for the radial type *vs.* 106 to 118 per cent for the networked type.

The figures obtained by the authors are considerably lower for the primary network. This is accounted for in part by the use of existing facilities, and in part by the use of bus regulation. The installation now being made seems entirely justified for the conditions existing. Whether or not it can be laid down as a general principle that this type of distribution for territories of the load densities under consideraton will always be justified is open to question.

In Table II there appears a figure which seems doubtful. The relative investment in real estate is given as 17 per cent for a radial substation and as only 2.8 per cent for the networked stations of the surface type. This latter figure is but 16 per cent of the former and to be comparable must cover the cost of three distinct sites. Is it not possible that the real estate investment in but one such site was included through an oversight?

Assuming the correctness of the figures as given could not the three independent surface installations of each type be concentrated at one location? Using the same types of construction and equipment, exclusive of network interconnections, relative investments of the following order, on the basis of three independent primary feeds, would seem indicated:

	Bldg. type	Outdoor type
Real estate	2.8	2.8
Structures	13.0	5.8
Equipment	66.3	66.3
Transmission*	3.8	3.8
Distribution	1.6	1.6
Duct Connections	14.5	14.5
	102.0	94.8

*Increased 50 per cent for a third primary feeder.

If these assumptions are correct, they would indicate that a modification in the radial type substation design might yield economies in excess of those shown by the authors, for the primary network type, in the Verona-Oakmont district.

The development of a-c. networks is progressing. Nevertheless, many d-c. networks still exist involving large investments which it may not be advisable to retire. If the d-c. substation and its numerous low-voltage feeders could be eliminated, being replaced by high-voltage a-c. feeders and relatively inexpensive d-c. conversion sources of moderate capacities for installation at points corresponding to those at which network transformers are now installed, the picture would be quite different.

With the continued development of the thyratron, it seems not unreasonable to visualize a three-wire d-c. network fed at street intersections from manholes or vaults through tubes. The supply transformers would be single-phase high voltage, *e. g.*, 11- to 33-kv., two pairs of tubes, one on each side of the

three-wire circuit, smoothing reactances, and an over-current, reverse-current d-c. circuit breaker, completing the main installation. Grid current at suitable voltage would be obtained from an extra winding on the transformers. Individual installations would be balanced across the three phases of the primary supply cables and these in turn so interlaced that the loss of one or more primary cables (dependent upon the extent and capacity of the network) would in no way interrupt service to the consumers supplied therefrom.

It is possible that this development may be realized in the comparatively near future. Three years ago Mr. A. W. Hill, of the General Electric Company, in a paper presented before the New York Section, stated that "cathodes with a normal emission of 10,000 amperes appear quite practical."

G. M. Miller: From the load densities found in the Verona-Oakmont District it seems questionable whether the network class of service is necessary. Should the primary network prove to be as practical and satisfactory as seems entirely possible it may be that the customers in the lower density areas will receive better service than those in the older heavily-loaded areas served by the radial feeder and large transformer substation scheme.

The use of small transformers for stepping from the transmission to the distribution voltage is a decided advantage and permits the installation of additional transformer stations at locations where the load is increasing. Possible mistakes in location are avoided by this scheme whereas with large banks of transformers or large capacity substations it is difficult to select the right location due to the shifting of load centers. In some cases the trend of load growth changes and large investments cannot be recovered. Improved regulation and copper economies are possible with a primary network which have not been accomplished in any other way.

Experience with relaying schemes and the general arrangement of the network will no doubt lead to improvements which will reduce cost of equipment. Improvement in design and methods of installation may further reduce the over-all costs so that the primary network will be much less expensive than the present radial scheme and the class of service will be superior to that given at present.

C. T. Sinclair: Mr. Hendrickson raises the question as to whether or not the primary network is the correct solution for areas of low and medium load density where a high standard of service is required and where a low-voltage network will be needed in the future. In the present system, the radial distribution system has been considered adequate to fit these areas. In most low and medium load density areas it possibly will be many years before a low-voltage network will be economically justifiable. It is our thought that in these areas a higher standard of service may be rendered by the primary network than by the usual radial system. The primary network is less costly than the present radial systems for the Pittsburgh area and our experience thus far indicates that it may be expected to render a higher grade of service.

In converting the usual radial system to a network there is quite frequently a serious economical handicap in scrapping or at least removing equipment already in operation. With the primary network it does not appear that a conversion would be so difficult. The first step would obviously be to network the secondaries, using the present 4-kv. mains as primaries. The additional capacity beyond this point would be provided by feeders into the secondary grid supplied by the base supply voltage (22 kv. in the Pittsburgh case).

In starting the low-voltage network a minimum of three feeders available for supply is very desirable in order to reduce the percentage of spare capacity in the system. By way of illustration, if only two feeders were used 100 per cent spare capacity must be provided in the system for the system must be designed on the assumption that the one feeder is out. With the two-feeder system either feeder with transformer banks must be capable of supplying the entire load. With the three-feeder system 50 per cent spare capacity is required on the same assumption, and with four feeders 25 per cent. Thus it is apparent that with more feeders available the percentage spare capacity is reduced. The primary network in the conversion scheme mentioned above has certain advantages in this respect in so far as each sectionalized primary main is essentially a feeder and a number of these mains will be available by the very nature of the network. This will especially be true when a given area has reached the load density at which a low-voltage network appears economically feasible.

We are in complete accord with Mr. Bullard's statement as to the desirability of using generating station or transmission terminal substation voltage where possible. We are confident that at this time it would be impractical to operate an 11-, 13-, or 22-kv. system for our distribution areas. The necessity to work this voltage under congested conditions is extremely difficult. In some of the areas underground construction would make it impossible to operate such a distribution system. As Mr. Bullard has pointed out, there are certain governmental objections and in certain areas it is practically impossible to spread this higher voltage throughout the entire district. The extensive joint use system introduces another factor difficult of solution.

For certain types of distribution areas, for example rural distribution, the higher voltages discussed by Mr. Bullard certainly offer material advantages. The primary network, however, is not offered as a solution to this problem.

It is very interesting to note that Mr. Sweetnam has likewise found material economies in the adoption of the primary network and his discussion needs no elaboration here. The important conclusion to be drawn from his discussion is that the primary network is readily adaptable to areas differing in type from Pittsburgh.

Mr. Smith refers to the reasonably small percentage of spare capacity. This advantage is true in both the primary network and the low-voltage network. It is possible to add load in small increments when and where desired. We would be inclined to term this extra capacity needed "over capacity" rather than "spare capacity," as he terms it. Spare capacity is always provided to take care of emergency and abnormal conditions whereas this over capacity is required in the usual system by the very nature of the large units involved in the usual radial system. As Mr. Smith points out, the initial load on a 10,000 kva. bank may be during the initial life of the substation well under 50 per cent (or, as he expresses it, the spare capacity may be well over 50 per cent). As he points out, the cost per kva. cannot be determined on an ideal basis in an economic study but average costs must be used considering the period of the growth of the substation.

The difficulties of relaying and regulation, mentioned by Mr. Henry, have not been met with thus far in the Pittsburgh network after five months of operation and several series of tests. The fact that this time is too short to make any definite statements is recognized but our studies combined with the little operating experience have led us to believe that these problems are no more difficult than those met with in other cases in the electric light and power industry. We feel sure that the scheme described by Mr. Henry is very satisfactory and will give a good account of itself.

Mr. Wallau raises the question as to the investment figure of 17 per cent given in our table and notes that it is high compared to the cost for the network stations. The 17 per cent represents the actual value of the property now in our possession which was to be utilized for the construction of the substation and is not an estimated figure but actual value. Radial substation locations are chosen after an economic study which, speaking generally, balances feeder costs against property costs. In other

words, it is possible to locate a station away from the load center and buy cheap property. This, however, means a longer average feeder length and each substation location may be chosen after this economic study is made, taking into consideration all factors involved. The substation location chosen in this report for a radial substation is the best location for the radial system considered. The reason that the real estate item in the network analysis is low is because of the fact that this substation can be located either underground entirely, where no property is purchased or on a small and inexpensive lot.

We feel confident that in our particular case the three independent units could not be located at one spot as suggested by Mr. Wallau as this would result in costs largely as described for the radial type system. The use of the thyratron for supplying d-c. networks seems to have certain possibilities. It is also interesting to note that the application of d-c. transmission to the primary network appears to have possibilities. This, however, is looking rather far in to the future.

It is interesting to note that Mr. Miller feels the importance of being able to add transformer capacity when and where the load is increasing. He points out that this largely eliminates the possibility of mistakes in choosing large substation locations.

The authors of this paper do not contend that primary network should be applied to all overhead distribution systems. It is felt, however, that this system provides economies and operating possibilities for the areas described that are superior to other systems now in use. Our operating experience thus far has justified our analysis. Additional studies made indicate that the scheme can very likely be applied to a large proportion of our load.

The Philadelphia A-C. Network System

Development and Operation of a Network System Having Fused Secondary Mains and 2,300-Volt Loop Circuit Supply

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Synopsis.—*The object of this paper is to outline the engineering plan of the low-voltage a-c. network system in Philadelphia, Pa. and to summarize operating experience over a five-year period. The development of this system and the operating experience will be of interest, since several basic features, which contribute toward the successful results, are different from the basic plan in other types of network systems.*

The system consists of sectionalized primary loop feeders, supplying a fused secondary network. At present there are twenty-four feeders in operation, with a total normal operating capacity of 32,540 kva.; the coincident feeder peak to date being 27,600 kva.

The conclusions in the paper are essentially as follows:

1. The system plan initially adopted has been followed without fundamental change, and has proven amply flexible to meet changed load conditions as the area develops.

2. Operating experience has been very satisfactory, under all conditions, including those at time of faults on the network or elsewhere on the system.

Introduction

TEN years ago many of the central station companies were experiencing considerable difficulty in determining upon a distribution system which would best provide for load growth in areas of higher load densities. A number of individual companies inaugurated studies to determine the practicability of a secondary a-c. network system to supplement or supplant the d-c. network and radial a-c. distribution systems. Although the principle of multiple primary feeders supplying a secondary network underlies all network systems, the methods and apparatus actually used have varied to such an extent, due to local conditions and standards, that a number of different types of network systems is in use.

The system described herein was found by a study of all types of secondary network systems to be best suited to conditions in the downtown business district of Philadelphia from the standpoints of reliability, simplicity, flexibility, and economy.

General Description

The first network was established in the early part of 1926 as an integral part of a ten-year program of changing the downtown business district from direct current to alternating current.

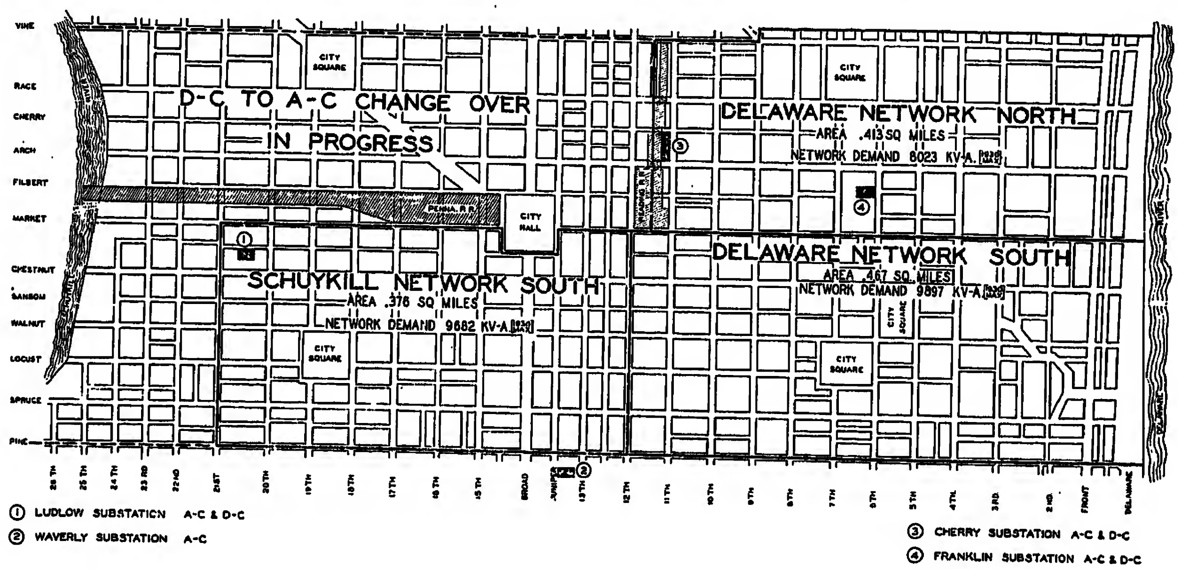

FIG. 1—PHILADELPHIA DOWNTOWN BUSINESS DISTRICT SHOWING A-C. NETWORK AREAS

The downtown business district, covering an area of 1.6 sq. mi. (Fig. 1), consists largely of office buildings, banking institutions, hotels, apartment houses, theatres, railway terminals, high-class merchandizing establishments, etc.

The 1930 peak demand of this area was 72,820 kva.

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Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

of which 27,600 kva. was carried on the a-c. network system and 6,000 kw. on the d-c. system. The remaining load is generally in such large blocks and of such a character as to generally warrant service at a higher voltage and is usually supplied from dual radial nominally 2,300-volt or 13,200-volt feeders.

The entire network and the greater part of the radial feeder load in this area, as well as some of the radial feeder load in areas adjacent thereto, is supplied from two generating stations through four substations located as shown on Fig. 1.

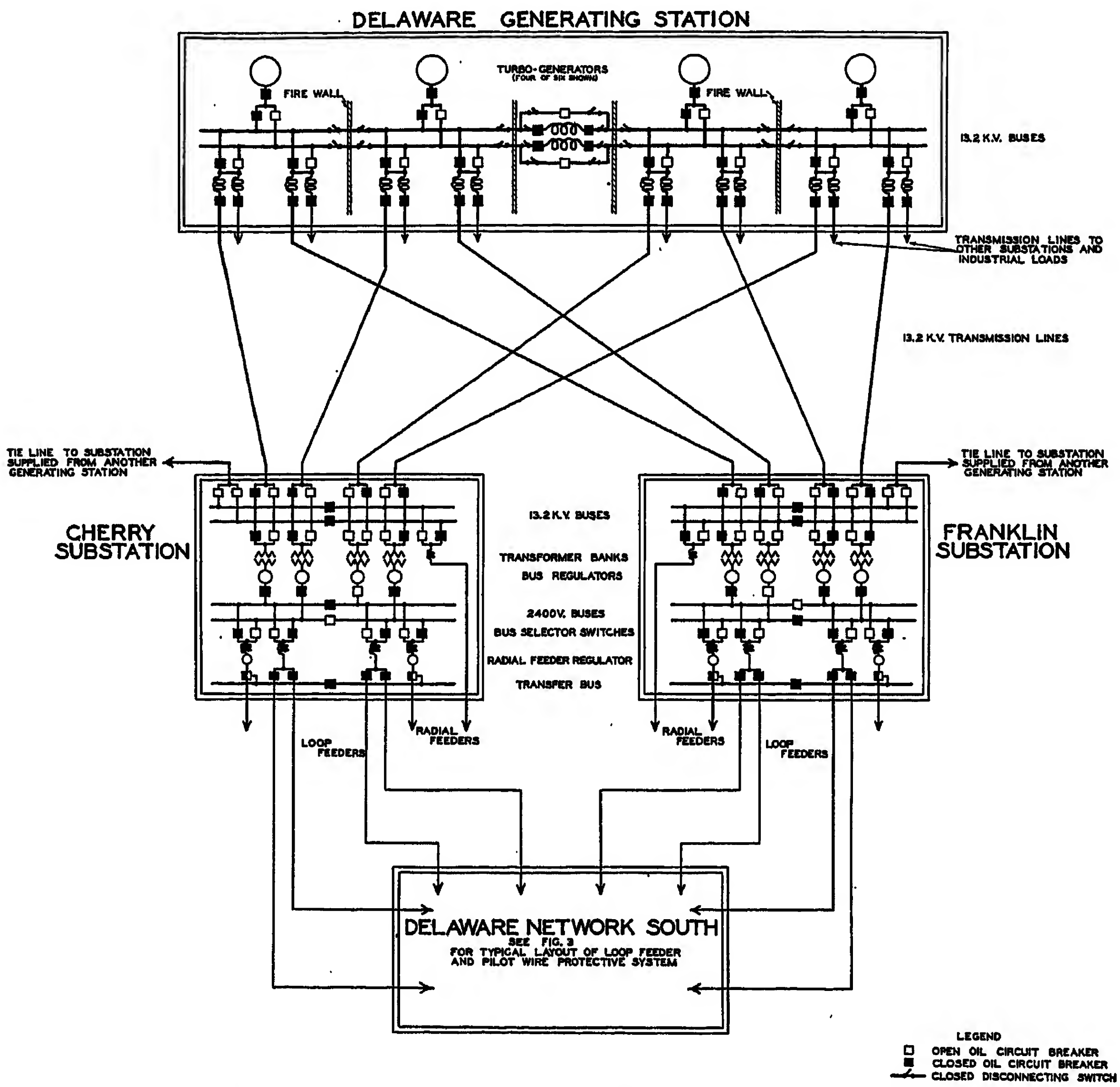

Fig. 2—Schematic Diagram Showing Typical Method of Supply to Network Area

The present a-c. network system comprising an area of 1.26 sq. mi., consists of three separate networks, each having a load of approximately 9,000 kva. Each network area is supplied by eight 2,300-volt loop feeders from two substations. Each such pair of substations is fed from the same generating station, by a number of 13,200-volt feeders. The feeders to the same substation are connected to different generating station bus sections which are separated from each other by barrier walls, as illustrated in Fig. 2. The generating stations, Schuylkill and Delaware, are located approximately one and one-half miles from the network area.

Voltage regulation on the network system is secured by means of induction type regulators on the low-voltage side of the substation transformer banks or on the individual loop feeders. The use of bus regulation is gradually being extended to all substations feeding the network system.

No auto transformers are required on the secondary network because the system conforms to accepted voltage standards for both light and power service; therefore, all parts of the system can be paralleled without the use of translating devices.

Statistics pertaining to the network load at the time

of the 1930 peak, number of loop feeders, length of feeders, number of transformers and oil circuit breakers are presented by areas in Table I.

The secondary a-c. network system is a gridiron of secondary mains, two-phase, five-wire 115/230 volts,[3] fed from transformer banks located at each main street intersection and which are supplied from a number of interlaced loop primary feeders.

The secondary mains and transformer leads are provided with copper-link fuses for automatically sectionalizing parts of the system in case of trouble.

The loop primary feeders consist of several unit sections which are connected by means of 15-kv. automatic oil circuit breakers operated by means of a balanced pilot wire control system. Practically all the cable comprising the loop feeders, together with the majority of the stepdown transformers have been purchased and installed for possible ultimate 13.2-kv. operation. Oil circuit breakers are rated at 400 amperes and 15 kv.

TABLE I—DATA PERTAINING TO THE PHILADELPHIA A-C. NETWORK SYSTEM

		Delaware north	Delaware south	Schuylkill south	Totals December 1930
1.	Area in square miles operating network	0.413	0.467	0.376	1.256
2.	Maximum demand in kva. (network only)	8,023	9,897	9,682	27,602
3.	Number primary loop feeders	8	8	8	24
4.	Total normal capacity of loop feeders	10,620 kva.	9,600 kva.	12,320 kva.	32,540 kva.
5.	Average normal load per loop feeder	1,003 kva.	1,237 kva.	1,210 kva.	1,150
6.	Total length of loop feeder cable	70,400 ft.	99,800 ft.	92,400 ft.	262,600 ft.
7.	Average length per loop feeder	8,800 ft.	12,475 ft.	11,550 ft.	
8.	Total length of loop feeder cable between substation and first transformer bank	17,160 ft.	36,698 ft.	30,240 ft.	84,098 ft.
9.	Total length of loop feeder cable between substation and boundary of network area		21,292 ft.	6,805 ft.	28,097 ft.
10.	Number of unit sections on loops	57	70	72	199
11.	Average loop section length in feet	1,230 ft.	1,430 ft.	1,280 ft.	1,320 ft.
12.	Number of primary oil circuit breakers	49	62	64	175
13.	Installed transformer capacity kva	12,500	13,600	15,400	41,500
14.	Number of transformers (100 kva.)	125	136	154	415
15.	Number of transformer manholes	41	54	56	151
16.	Ratio normal loop feeder capacity to transformer capacity	85%	71%	80%	78%
17.	Ratio maximum coincident feeder demand to transformer capacity	64%	73%	63%	66%

Operating Results

Operating experience with this network system over a five-year period has been satisfactory.

To date there have been two major cases of system disturbances which affected the a-c. network system; in one instance service to one entire network area was interrupted for a short time and in the other case the continuity of the network service was unaffected.

Case 1. Due to trouble in one of the generating stations, power supply to the two substations feeding a 5,000-kva. network load was lost. One of the two substations involved had "tie lines" to a third substation which was fed from another generating station. By closing the tie line oil circuit breakers, this substation picked up the entire load of the network and carried it until conditions were restored to normal.

Case 2. Two substations *A* and *B* were jointly carrying the load of one of the network areas totaling approximately 6,000 kva. The 13.2-kv. feeders to substation *A* dropped out successively, so that the entire transmission supply was lost. The remaining substation *B* continued to feed all of its regular load, and all the network load without noticeable drop in voltage. In addition, the feed back through the network and the loop feeders energized the 2,300-volt bus in substation *A* which in turn picked up a part of the load on a limited number of important outgoing radial 2,300-volt feeders.

Twenty-two primary cable and three transformer failures have occurred, and in every case the oil circuit breakers and fuses have isolated the faulty section without interruption to service to any other unit section or impairing service to any customer.

The operation of secondary fuses in well-established network areas has demonstrated that faults on secondary mains are quickly and selectively isolated, consequently with a minimum amount of damage.

In two cases of trouble on the secondary transformer leads the faults have burned clear within a few inches of point of origin.

An inspection is made every three months by a group of five men in order to determine the condition of the pilot wire control system, oil circuit breakers, transformers, sectionalizing boxes, etc. It is expected that the rate of inspection may be reduced. The periodic field inspection consists of:

1. Testing transformers, oil circuit breakers and sectionalizing fuse boxes for water tightness. Work involved in making the equipment water-tight consumes the major part of the inspection time.

2. Testing the pilot wire system (conductors, current transformer, and trip coils) for (a) grounds, (b) open and short circuits.

3. Inspection of trip coil settings and operating mechanism for sensitivity.

4. Examination of sectionalizing boxes for—(a) blown fuses, (b) open secondary cable mains.

3. *Two-Phase, Five-Wire Distribution*, by P. H. Chase, A. I. E. E. Trans., Vol. XLIV, p. 737.

5. Minor repairs and adjustments.

Open breakers on the loop circuits are checked in two ways. By recording the hourly ammeter readings on each loop feeder, and by alternately opening the loop feeder ends, twice each week, and checking the approximate total currents under these test conditions. The current readings on each loop end often indicate if a unit section is open and its approximate location.

Basis of System Design

Reliability, flexibility, simplicity, and economy were the four outstanding requisites upon which the system was designed and the basis upon which extensions have been made.

The detail engineering plan was based upon the requirements:

1. That the network system as a whole shall have sufficient spare capacity to assure against overloading any equipment during period of unit section or transformer outage.

2. That adequate network voltage shall be maintained under all conditions, including short-time (approximately 30 minutes) loss of one of the two substations supplying a given area. With the loss of one substation feeding a network area, it is possible to carry the entire load on the remaining substation by taking advantage of short-time overload rating on all substation and network equipment.

Development of Detail System Design

With the method of protection employed and the interlacing of loop feeders, operating experience has shown that it is possible to load the network transformers to 80 per cent of the manufacturer's nameplate rating with the assurance that there will be sufficient capacity available in the event of a failure occurring in a loop unit section at time of the network peak.

Loop feeders are normally operated at 80 per cent of the rated equipment capacity. The loading on some of the individual loop feeders may exceed the 80 per cent value for periods of relatively short duration, *i. e.*, at the time of the network peak. Conditions of this nature are infrequent and are permissible in view of the available reserve capacity in all parts of the system. With the loss of a unit section of loop feeder, the feeder continues to function, though the load on the respective ends of the loop feeder may be unequal, depending upon the location of the unit section isolated by the operation of the pilot-wire protection system. In the event of a failure in either the first or the last unit section, the remaining operating end carries approximately 75 per cent of the total load carried on the respective ends prior to the fault on the open end. The load dropped by the faulty loop, is absorbed by the adjacent interlaced loop feeders.

Coincident with the studies undertaken to determine the possibilities of establishing an a-c. network system, it was recognized that an "a-c. calculating board" upon which actual alternating-current conditions of operation could be simulated, would prove of material assistance in securing the desired information with the maximum degree of accuracy, in the minimum space of time. Accordingly a board was designed and constructed by the Philadelphia Electric Co.,[4] upon which various conditions of operation, kinds of faults, possible changes in the fundamental plan, etc., were studied. This proved to be a most valuable aid in the solution of problems involving the magnitude of currents flowing into faults of various kinds; the amount of circulating current caused by phase angle or voltage differences between substations, the effect of motor starting currents, together with the permissible loading of loop feeders under given fault conditions.

Having determined upon the operating conditions that would be encountered, complete laboratory tests were conducted to determine the ability of the protective scheme to function properly at time of a fault.

Primary Feeders

The primary supply to the network system is in the form of three-conductor, 350,000 cir. mils, paper-insulated, lead-covered cables. Each loop feeder extends from a substation through a number of intersections throughout the network area and returns to the same substation. Each feeder is routed through widely separated parts of the network area and interlaced with other feeders.

The primary feeders at present are operating at 2,300 volts, three-wire, two-phase.

Since the initial operation of the network system in 1926, there have been 22 primary cable failures approximating one failure per 34 unit sections per year, or 12 failures per 100 miles of operating cable per year. The relatively high rate of cable failures has been largely due to external mechanical injury.

Balanced Pilot Wire System of Protection

Each primary loop feeder is composed of a series of unit sections, each of the unit sections being provided with sectionalizing oil circuit breakers. The present average length of unit sections is approximately 1,300 ft. The arrangement of substation equipment, composition of unit sections and protective scheme is shown in schematic form in Fig. 3. The balanced pilot wire system is fully described in an article by Mr. P. H. Chase.[4]

Each unit section is protected by a pilot wire control circuit connecting current transformers and trip coils in the circuit breakers at each end of the unit section. Current transformers in the secondary leads of the distribution transformers are also connected into the control circuit. The distribution transformers are thus included within the protected section of the feeder.

Each unit section of feeder may be considered as

4. "An Alternating-Current Network," by P. H. Chase, *Electrical World*, Vol. 88, No. 13, Sept. 25, 1926.

three branches around the point where the distribution transformer bank is connected to the feeder. Current transformers in each of these three branches are connected through the pilot wire to obtain a three-way current balance, corresponding to the current balance in the three branches of the unit section. Trip coils in the oil circuit breakers are connected across the pilot wire system at each end of the unit section.

Under normal operating conditions, the relative polarity of the current transformers is always such as to cause a circulating current to flow in the pilot wire circuits, with zero or negligible current in the trip coils. In the event of a fault anywhere in a unit section, the relative polarity of the associated current transformers change, so that current flows through the trip coils and the oil circuit breakers at each end of the faulty unit section are tripped open. The back feed from the low-voltage network blows the 1,500-ampere fuses in the transformer secondary leads and the unit section is completely isolated.

The protective system for each unit feeder section is responsive only to fault conditions within the section. There may be reversals of current in other unit sections, but the relative polarities of the current transformers are always such that a current balance in the control circuit is maintained. There may also be a heavy current flow in other unit sections, but the impedance of the trip coils is sufficiently high, relative to that of the pilot cable, to prevent current of operating value from flowing through them.

It will be observed that only one transformer bank and associated primary cable is isolated in case of a fault on the primary loop feeder. The other transformer banks remain in service, being supplied from the two parts of the loop feeder which have been separated by the isolation of the faulty unit section. Due to this feature the occurrence of a fault causes a minimum of disturbance to the remainder of the system.

The pilot cable which parallels the primary loop feeder is a four-conductor No. 4, rubber-insulated, and lead-covered cable. All branches of the pilot wire control system are connected through manhole type junction boxes provided with test clips to facilitate testing and measuring load on transformer banks.

Field inspections to date have disclosed three cases of failure in the four-conductor cable of the pilot wire system; two cases of incorrect cable connections in the junction boxes and a defective current transformer. In no case did the six irregularities cause operation of the loop feeder oil circuit breakers.

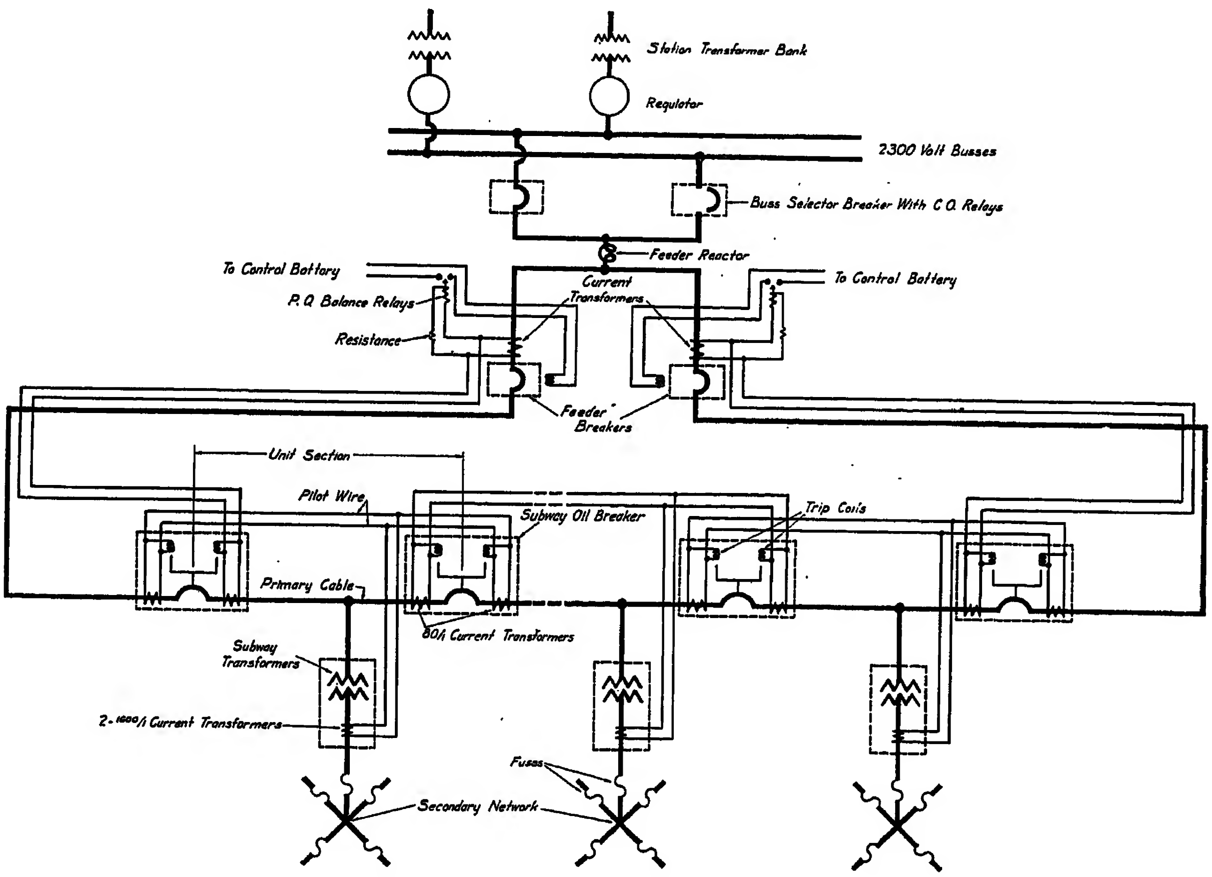

Fig. 3—Single Line Schematic Diagram Pilot Wire Protective System

Primary Oil Circuit Breakers

The oil circuit breakers used for sectionalizing the primary loop feeders are of the subway type, rated at

15,000 volts and 400 amperes, with an interrupting capacity of 30,000 amperes at 2,300 volts on standard 2 OCO basis. Each oil circuit breaker contains six bushing type current transformers, and six 5-ampere trip coils. Connections are made to the oil circuit breakers with single-conductor cables which in turn are spliced to the three-conductor primary cables. Wiping bushings are provided for extension of current transformer and trip coil leads, in lead-covered cables, for connection to the balanced pilot wire protective system.

On the first consignment of oil circuit breakers, difficulties were experienced with tripping out due to vibration. Since the proper corrections have been made to these breakers, no such operations have occurred. In other respects breakers have given satisfactory results and have required only minimum maintenance except where one was replaced on account of entrance of water.

Network Distribution Transformers

The distribution transformers used in the Philadelphia network system are all 100-kva. single-phase subway type units of standard impedance.

The majority of the transformers are constructed with the high-voltage winding in six sections, connected in parallel for present operation at 2,300 volts and arranged for series connection for operation at 13,200 volts, if desired in the future.

There have been three network transformer failures which were due to admission of water and breakdown of windings.

Transformer Manholes

All network transformers on the Philadelphia system are of the subway type and are installed in transformer manholes usually located in the highway under the sidewalk. The majority of the manholes are 15 ft. by 9 ft. with 8 ft. head room, and are located near the street intersection. There are two removable slab openings, one of which has an open grating lid and two openings for the ventilating chimneys, which extend from the street surface to the bottom of the manhole at each end.

Each transformer manhole usually contains from one to four 100-kva. transformers, one primary oil circuit breaker, two pilot cable junction boxes and four low-voltage sectionalizing boxes, illustrated in Fig. 4. Barrier walls are not used to isolate the primary oil circuit breakers from the transformer but all cables, except the control cables, are fireproofed.

Secondary Network

The cables on the 115/230-volt, five-wire, two-phase secondary network are 350,000 cir. mils, 600 volt paper-insulated and lead-covered. Single-conductor cables from the single polarity five-way fused sectionalizing boxes which are installed at every main street intersection may be spliced to single or multiple conductor secondary mains. Three-conductor cable mains of the former d-c. network were used in many instances for one phase, and two single-conductor cables installed to complete the two-phase main system. The present practise is to install single-conductor cable for the phase wires and a bare No. 2/0 neutral wire. Both lighting and power loads are supplied from the same set of secondary mains.

Two-phase motors of various sizes up to 40-hp. are taken on any part of the secondary network system when well established, without requiring special starting current limiting devices.

Secondary Fuse Protection

In the original investigation the magnitude of current which could be expected under fault conditions was determined from the calculating board. Tests were then conducted by means of a spare generator and transformer bank to determine the current-time characteris-

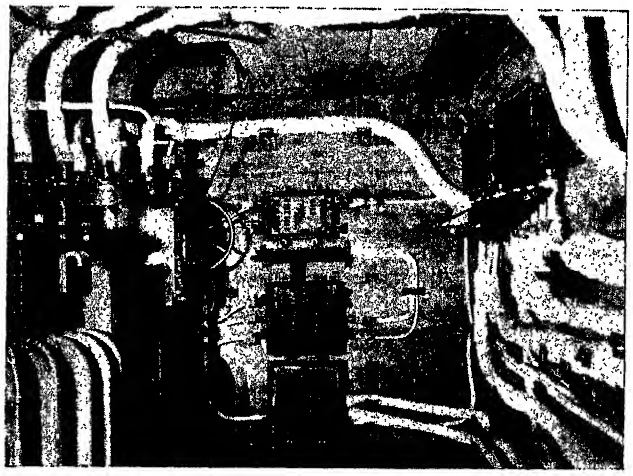

Fig. 4—Typical Arrangement of Equipment in Transformer Manhole

Left to right—transformer, oil circuit breaker, pilot cable junction boxes, secondary sectionalizing boxes

tics of various types and sizes of copper-link fuses for 115-volt duty. Oscillograph measurement of the current and time of blowing of the fuses mounted in different types of boxes, with fuse contact blocks of different sizes and under different temperature conditions indicated that the fuse sizes determined upon gave the necessary selectivity.

These copper-link fuses are installed in five-way single polarity sectionalizing boxes, to which the four secondary mains and the transformer leads are connected. The mains are protected with 1,250- and the transformer leads with 1,500-ampere rated capacity fuses.

Operating experience with a fused a-c. secondary network system in well-established areas has indicated that fusing of mains and secondary transformer leads is desirable and that the fuse sizes used are satisfactory. In establishing the secondary network, old d-c. mains of various types and sizes were used. The different condi-

tions existing on the secondary system and in the supply to the network during the construction periods make it impossible to definitely analyze or summarize the types of secondary failures and the operation of fuses at times of faults. There have been no cases of fuses blowing when a fault has not existed on the particular section. When fuses have not blown the faults have cleared themselves without any damage other than destruction of the cable immediately adjacent to the point of failure.

Discussion

H. Richter: The paper states that there is a number of types of a-c. low-voltage network systems in use for underground distribution in this country. There are really only two types, the Philadelphia system, which uses loop feeders and fuses for protection on the network side of the transformers, and that which employs radial feeders and automatic network protectors for protection. There are four variations of the automatic protector scheme, but no differences fundamentally. Whether the secondaries are two-phase or three-phase is incidental.

In 1924, when the two types of systems were in their infancy, a few individuals predicted that both would be made to give satisfactory service, despite dire misgivings from many quarters. This paper, together with the extensive literature on the automatic protector system, proves that the *predictions* were correct.

As of January 1930, the two systems served peak loads of 28,000 kva. and 330,000 kva., respectively. In terms of operating companies that employ the two systems as the present standard, the ratio is 1 to about 55. Loads that are among the most important and exacting in the world, as in the skyscrapers in New York City, depend entirely on the automatic protector system. This system is being installed where direct current with storage battery was the rule.

The network system described in the paper seems to have a disadvantage as regards length of time for the transformer secondary fuses to clear primary faults. These fuses take at least twice as long to clear as would automatic protectors, and frequently much more than this. Primary cable and transformer faults predominate, and it is very desirable to isolate every fault as quickly as possible to prevent the trouble spreading by overstressing the other parts of the system. The automatic protector, I believe, answers this requirement much better than the transformer secondary fuses.

One object of the sectionalized loop feeder scheme with one transformer bank per feeder section is to keep down the spare transformer capacity required to carry the network after a primary fault is isolated. The paper states that this permits loading the transformers to 80 per cent of their rating at time of peak load. In the automatic network protector system, 16 parallel radial feeders would correspond, with respect to the approximate total length, to the 8 loop feeders in each of the three network divisions in the Philadelphia network. Under the same assumption of a single primary fault, tripping out an entire radial feeder would require no more total spare transformer capacity than with the sectionalized loop scheme, and possibly less might be tolerated. On the other hand, more copper is needed in the loops.

The two-phase five-wire secondary network utilizes accepted voltage standards for power as well as light. But it is also true that the fear of having to use considerable booster auto-transformer capacity for the prevalent three-phase system has long since faded away. Extensive experience has proven that auto-transformers constitute but a small part of the total expense of a well-designed automatic protector system employing 120/208- or 115/199-volt secondaries. The minor difficulties in fitting standard devices to the non-standard voltages in this type of secondary system comprise just one of the routine matters encountered in underground distribution practise. The effort to spread the use of translators for 115/230-volt three-phase networks has likewise fallen down, so there is no need to compare the two-phase network with that scheme.

To sum up, it appears that the two types of a-c. underground network system are, in general, giving reliable service; but the automatic protector system is probably cheaper in first cost and maintenance and conserves the investment by more promptly isolating primary faults.

H. S. Davis and W. R. Ross: Mr. Richter points out the difference in the two prevailing types of a-c. network systems and indicates the relative amount of load connected to each. Admittedly, there are a predominance of a-c. network systems which employ the low-voltage automatic a-c. network protector. However, in spite of the larger number of systems of this type, little or no thorough-giving information has been placed at the disposal of the industry as to actual operating performance. The object of our paper was to present operating data secured over a five-year period of actual operation of the Philadelphia a-c. networks. This operating experience has fully demonstrated the soundness of the sectionalized loop-feeder plan. We believe the highly satisfactory performance has resulted largely from the simplicity of the equipment utilized.

Of the various devices available to interrupt circuits in the 115/230-volt class, it is believed that the link fuse is one that can be depended upon reliably to interrupt the circuit. Further, it is our opinion that the slight additional time required for the fuse, as compared with the network protector, to interrupt fault currents of small magnitude, does not nearly outweigh the advantage of having this dependable simple interrupting device.

Regarding the matter of speed of isolation of primary faults, Mr. Richter overlooks the fact that under the loop scheme the primary sectionalizing breakers operate in about ten cycles and thus isolate the faulty section from the remainder of the primary feeder, then the final isolation of the fault, from the network, is accomplished by the fuses interrupting only the fault current fed back through one network transformer bank.

To meet conditions such as the complete outage of a feeder or all the feeders due to the loss of a substation, the parallel radial feeders scheme requires as high a degree of interlacing as does the section sectionalized loop-feeder plan. Unless this is done, an equivalent grade of service cannot be afforded without materially increasing the total feeder or transformer capacity, or both.

No attempt was made to stress the relative merits of the five-wire, 115/230-volt, two-phase, network system. Mr. Richter concedes that minor difficulties are experienced from time to time in fitting standard devices to operate satisfactorily on non-standard voltage systems. However, to quote Mr. Richter, "this has become more or less one of the routine matters encountered in underground distribution practise." It is obvious that standard voltages have very definite advantages and afford greater ease of operation from a regulation standpoint than can be considered where the secondary network consists of four-wire, three-phase, 120/208- or 115/199-volt secondary mains.

In summing up his remarks, Mr. Richter makes the general statement that the automatic network low-voltage protector system is probably cheaper in first cost and maintenance. Exception is taken to these very general statements in that information at hand does not bear out this contention. Comparisons of operating maintenance costs on systems using the network protectors and the maintenance cost of operating loop-sectionalizing devices, such as those in operation in Philadelphia, indicate that there is little, if any, saving in one system over the other.

Present Day Practise in Grounding of Transmission Systems

Second Report of Subject Committee on Grounding*

Introduction

IN 1923 the Protective Devices Committee through its Subcommittee on Grounding issued a report on the grounding practise of that day. Last year it was felt that there had probably been sufficient change since that time to justify a second report of the same nature, and the Joint Interconnection Subcommittee of the Power Generation, Protective Devices, and Transmission and Distribution Committees undertook to work up the report which is submitted herewith.

In preparing this report the general scheme of the first report was followed, the information received from the participating utilities being given without any attempt to draw conclusions as to preferred practise.

Some of the salient points of comparison between ungrounded, solidly grounded, resistance grounded, reactance grounded, and tuned reactance grounded systems, such as drop in voltage, kilowatt demand, steady-state voltage stresses and apparatus cost can be fairly well calculated, but there is still too much uncertainty regarding other factors, such as transient voltage stresses, possibilities of double faults, effects on relaying, to allow of any clear-cut preference for any one scheme of grounding over the others. However, in order to assist the reader in applying the data to his own conditions, the committee felt that a short review of the fundamentals of grounding might not be out of place.

Definitions

For the purposes of discussion the Committee has set up the following definitions:

A. Solidly Grounded

A station is considered solidly grounded when the neutral of every generator or transformer bank in that station is connected solidly and permanently to earth through a low resistance connection. A system is solidly grounded when all of its major generating stations or transmission substations are solidly grounded.

B. Resistance Grounded

A system is considered resistance grounded when a resistor is interposed between the neutral and earth.

C. Reactance Grounded

A system is considered reactance grounded when an air core or iron core reactor is interposed between the neutral and earth; if grounded through a grounding transformer, or if only a portion of the generator or transformer capacity at any given location is solidly grounded.

D. Tuned Reactance Grounded

A system is considered tuned reactance grounded when the aggregate reactive current passed by the neutral reactors or their equivalent substantially equals the aggregate charging current to ground of the system when one conductor is grounded, as with the Petersen coil, Bauch transformer, and the dissonance coil.

E. Ungrounded

A system is considered ungrounded when there are no conductive paths between the neutral and earth, except perhaps through potential transformers.

Selection of Grounding Method

There are several factors which influence the choice of grounding method, and the most important of them are listed below:

1. Apparatus insulation.
2. Fault-to-ground current.
3. Stability.
4. Relaying.
5. Arcing grounds.
6. Double faults.
7. Lightning protection.
8. Inductive coordination.
9. Continuity of service.
10. Adaptability to interconnection.
11. Operating procedure.
12. Equipment cost.

1. Apparatus Insulation

Transmission line insulation is principally determined on the basis of keeping down flashovers due to lightning and there is practically no difference in this insulation regardless of method of grounding. On transformers having voltage ratings lower than 115 kv., both ends of the high-voltage winding are insulated according to the rated voltage of the transformer, hence there is no difference in transformer costs regardless of how the neutral is grounded. For rated circuit voltages, 115 kv. and above, a saving in the cost of the transformers can be effected by the use of transformers having the insulation graded to various extents. The maximum saving is reached on transformers for solidly grounded neutral service, particularly where the rated circuit voltage is

*Interconnection Subcommittee of the Power Generation, Protective Devices and the Transmission and Distribution Committees—F. C. Hanker, Chairman. Subject Committee on Grounding:

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M. T. Crawford, W. W. Lewis, H. H. Spencer
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H. J. Scholz

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

high. Smaller savings are effected if the transformer is only partially graded as when reactance or resistance grounding is employed. No savings are available on oil circuit breakers except for 230-kv. service where under favorable conditions a special breaker with reduced insulation may be used for solidly grounded neutral service. While the level of insulation will be determined largely by lightning surges, undoubtedly switching surges and surges resulting from system disturbances contribute to insulation failures so that maintenance charges should be less over a long period with a grounding system giving fewer and smaller surges.

In cable systems, it is the practise of some companies when using multi-conductor cables to provide insulation between conductor and sheath in accordance with the type of grounding used, that is, the maximum insulation is used on isolated-neutral systems and the minimum on solidly-grounded systems. For the higher voltages, type *H* cable is now quite commonly used. On account of its fundamentally correct design, for any given voltage, type *H* cable with a total insulation equal to only the conductor insulation of belted cable is generally considered to have an equal factor of safety. However, the present practise is to use a slightly higher value of insulation than this in such cable.

2. Fault-to-Ground Current

The fault-to-ground current varies very widely, depending upon the type of ground connection used. The maximum current is obtained when the neutral is solidly grounded and the minimum current when a tuned reactance is used. In general, there are no cost differences for the different methods of grounding because equipment must be applied on the basis of the worst possible conditions, so that circuit breaker interrupting capacities are usually fixed by line-to-line short circuits, and this is also true of generator and transformer bracing. The heavier short-circuit currents cause greater and more widespread drops in system voltage, possibilities of system instability and greater damage to equipment at the point of fault with consequently increased time of outage. However, the practical development of the high-speed, high-voltage breaker and associated relay systems gives promise of relieving this situation.

3. Stability

The introduction of high-speed breakers and relays has made it possible to meet practically all stability problems under conditions of single-phase faults to ground. The fact, however, that high-speed clearing of faults to ground is sometimes necessary for the maintenance of stability with a solidly grounded neutral, should not be held as a reason for penalizing this method of grounding with the extra cost of high-speed breakers. Due to the fact that line-to-line faults are sufficiently numerous that they must be taken into account, high-speed breakers would be installed in any case, irrespective of the method of grounding. Where resistance grounding is used, assistance in maintaining stability is afforded by the power loss in the resistor (provided it is located at the sending end), but this assistance decreases with decreasing breaker and relay time. It is of no value for line-to-line faults but may be of considerable help in the case of a two-conductor-to-ground fault.

4. Relaying

Relaying for ground faults is simplest on a system grounded at one point because the residual currents flow only in one direction. On systems grounded through a tuned reactance, the fault current itself is practically zero although there is considerable current in the reactor itself, depending upon the extent of the system. The distribution of fault current, however, is not affected by the location of the fault. The relaying when necessary is accomplished by short-circuiting the tuned reactor and then relaying in the usual manner. The total current is then comparable to that obtained with a system grounded at one point. Relaying is extremely difficult on ungrounded neutral systems, as the charging current and its distribution change radically when the line connections are changed.

5. Arcing Grounds

By arcing grounds is meant the tendency for the creation of a high-frequency oscillation at the point of fault and building up the voltage to several times normal to ground on successive restriking. This condition is apparently caused by the action of the line capacity to ground in maintaining a direct-current potential between successive arcs.

6. Double Faults

Faults to ground on different phases may occur on any system, but since an excessive dynamic voltage to an upper limit of 1.73 times normal appears on two of the three conductors all over the system in the case of a single-phase ground on the systems grounded through high impedance, the possibility of a second fault appearing as a result of the first fault is increased.

7. Lightning Protection

Lightning arresters have been developed to the point where it is now possible to obtain a protective ratio of approximately 2.5 to 1, that is, the maximum voltage during surges can be limited to 2.5 times the normal dynamic voltage. However, the arrester must be applied on the basis of withstanding the maximum dynamic voltage which may exist from line to ground and in the case of ungrounded systems, or systems grounded through a high impedance, the arresters must be applied on the basis of withstanding 1.73 times normal phase-to-ground voltage without the flow of power current. This results in a decrease in the protective value of the arrester and an increase in its cost. The stress on the neutral end of the transformer winding due to a surge is higher when the neutral is grounded through a reactor than when solidly grounded or grounded through an ideal resistor. This requires that

the neutral end of the transformer and the reactor be correspondingly strengthened or some form of protective device, such as a lightning arrester or capacitor, be placed across the reactor.

8. Inductive Coordination

Under normal operating condition, power line phase currents and voltages are usually balanced as far as practicable. However, it is impossible to obtain an absolute balance of these factors. Any residual current or voltage, however small, will operate through the earth when the power system neutral is grounded. These residuals may be manifested in adjacent communication circuits as noise or troublesome induced voltage. When the system neutral is grounded, it is, therefore, necessary to maintain closer balances of both lines and equipment than would otherwise be necessary. It may be necessary also to reduce generated harmonics of the fundamental frequency or render them innocuous by means of suitable filters.

The magnitude of voltage induced into a communication circuit from a ground fault on a power circuit is dependent upon the coupling between the power and the communication circuit and the magnitude of the ground fault current. Under certain conditions there may be fundamental frequency voltage induced in the communication circuit from the unbalanced voltages on the power circuit at the time of fault. These are usually unimportant compared to the effects induced from the currents. Manifestations in the communication circuit are dependent both upon the magnitude and duration of the abnormal voltage. On grounded neutral systems large fault-to-ground currents are most likely to occur if the system is solidly grounded. The introduction of any impedance in the neutral lessens the magnitude of this current, and the higher the impedance naturally the lower the magnitude of the current. The use of high neutral impedance puts serious limitations on relaying, both as to selectivity and as to the total time required for clearing the fault. This, of course, may have adverse reaction from the inductive coordination standpoint, as would also any tendency to increase the frequency of occurrence of double faults.

If double faults occur on a system with isolated or high impedance neutral the ground fault current may be greater than the ground fault current for a single fault on a solidly-grounded neutral system. However, the probability of double faults occurring at such points as to lead to the largest induced voltages is less than the similar probability of a single fault on a solidly-grounded neutral circuit.

9. Continuity of Service

With any type of system grounding except ungrounded systems of small extent, or systems grounded through tuned reactance, a momentary flash to ground on one conductor requires that the circuit be isolated in order to clear the fault. This means that where continuity of service is essential a second circuit must be available to maintain the supply. However, even on systems where a flashover to ground is not accompanied by power-arc, a second circuit would probably be required to take care of faults due to causes other than flashovers.

10. Adaptability to Interconnection

Systems grounded solidly, through a resistance or through a reactance are usually adaptable to interconnection. A system grounded through a tuned reactance, however, cannot be tied in conductively with systems using another method of grounding, because the tuning is destroyed and arc suppression cannot be obtained. Ungrounded neutral systems are subject to the same difficulty, in that connection to a grounded system destroys their characteristic.

11. Operating Procedure

The operating procedure for solidly grounded, resistance grounded, and reactance grounded systems is the procedure which is well known in this country today, that is, automatic isolation of lines and switching in and out of lines at will. With the system grounded through tuned reactance, when lines are switched in and out of service, the taps on the coils must be changed so as to compensate for the charging current to ground. This need not be done for each change in transmission line set-up, but the compensation must be maintained within reasonable limits.

12. Equipment Cost

The solidly-grounded system requires no neutral impedance device and therefore effects a saving in this regard. In addition there may be a saving in apparatus insulation. In the case of 230-kv. transformers, the saving resulting from the use of graded insulation is considerable with smaller savings down to 115 kv. For lower voltages than this, there is no saving. The resistance and reactance grounded systems are probably next lowest in cost. In the case of high-voltage systems a partial grading may be possible with these methods of grounding with some saving in cost. The tuned reactor coil may under certain circumstances cost less than a resistor or reactor, but grading of the transformer insulation is not possible. The lightning arrester costs on the resistance, reactance, and ungrounded systems will be more than on a solidly-grounded system of equal voltage.

Summary of Grounding Practise

For the purpose of obtaining a cross-section of present day practise in grounding system neutrals a questionnaire was submitted to thirty-nine representative utilities throughout the States and thirty-two replies were received. A list of the participating companies is given at the end of the paper. The reference numbers used in the tables and figures do not correspond to this list.

The Committee deemed it desirable to analyze and

condense the data received rather than publish them in their entirety, and this has been done as far as possible in the form of tables and figures. In these by a "system" is meant a certain number of circuit miles conductively connected, that is, connected by metallic ties. One company, therefore, may operate several systems, and on the other hand a system may overlap two or more properties. This is necessary because in the problem of grounding the mileage of the conductively connected transmission lines is of importance and not the total mileage owned by any one operating company.

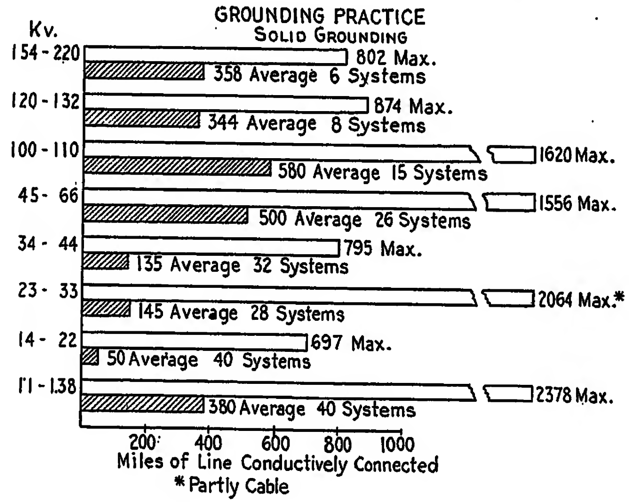

FIG. 1

METHODS OF GROUNDING

The methods of grounding adopted by the companies reporting are given in Figs. 1 to 4, which show the longest single system and the average length of all the systems. Where there are only two or three systems of one type, they have been reported in full, since an average would not be representative. In these charts the Committee accepted the reporting companies' classifications of their grounding methods, which may not agree with the definitions set up in the first part of this report.

The information contained in the figures has also been given in Table I, which shows the reported total mileage of transmission lines in the various voltage classes.

It will be noticed from the charts that 93 per cent of the reporting systems of 110 kv. and above are classified as solidly grounded, showing a marked preference for this type of grounding. One company reports solidly-grounded neutrals on 132- and 66-kv. systems except at three substations, where from 53 to 75 per cent of the transformer capacity is grounded. These three sub-

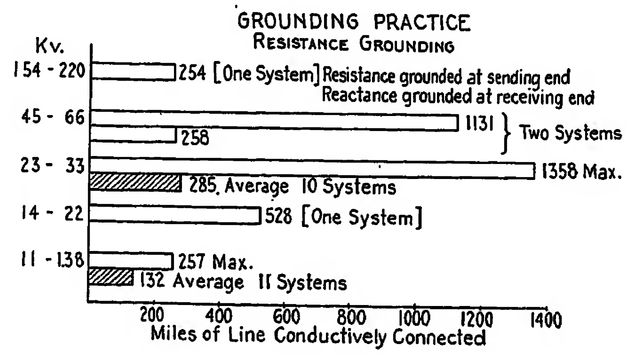

FIG. 2

stations are by definition reactance grounded, but because of the preponderance of solidly-grounded substations on the systems they are reported as solidly grounded. Another company grounds only one-third of the transformer capacity at one station, but the entire bank capacity at other grounding locations.

One company reports using a resistance at the gen-

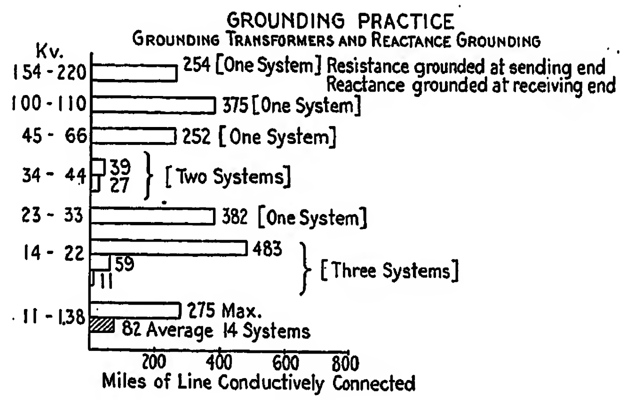

FIG. 3

erating end of its 220-kv. system and a reactance at the receiving end. This arrangement tends to decrease the angular swing between the two ends in case of a fault to ground and thus improves the stability of the system. Another company is using an impedance

TABLE I

System kv.	Miles reported	Solidly grounded		Resistance grounded		Reactance grounded		Ungrounded	
		Miles	Per cent	Miles	Per cent	Miles	Per cent	Miles	Per cent
154–220	2,402	2,148	89.4	127	5.3	127	5.3	None reported	
120–132	2,752	2,752	100	None reported		None reported		None reported	
100–110	9,075	8,700	96.0	None reported		375	4.0	None reported	
45–66	17,265	13,000	75.3	1389	8.0	252	1.5	2624	15.2
34–44	4,556	4,320	94.9	None reported		66	1.4	170	3.7
23–33	8,022	4,060	50.6	2850	35.5	382	4.8	730	9.1
14–22	3,796	2,000	52.7	528	13.9	553	14.6	715	18.8
11–13.8	8,991	5,320	59.2	1452	16.2	1148	12.8	1071	11.8
Total all systems	56,859	42,300	74.5	6346	11.1	2903	5.1	5310	9.3

grounding device in its most recent 220-kv. transformer installation. There appears to be a tendency in very high-voltage systems to limit the ground current by some kind of impedance—resistance, reactance or partial grounding of the total transformer capacity.

On systems of 66 kv. and below, the solid grounding practise is less pronounced. It will be noted that in this voltage range 51 per cent of the systems are solidly

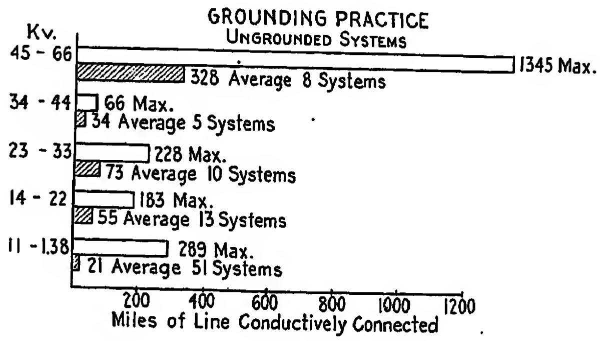

Fig. 4

grounded, 9 per cent are grounded through resistance, 8 per cent are grounded through either reactors or grounding transformers, 32 per cent have free neutrals.

Two companies report cases of reactance grounded substations in parallel with solidly-grounded substations. This is done to obtain a desired distribution of fault current through the two or more grounding locations. It will be readily seen that grounding a small transformer solidly may be the equivalent of grounding a large transformer bank through an impedance as far as distribution of ground current is concerned.

Another company reports the use of a Petersen coil. The report shows quite a marked reduction in the number of interruptions caused by lightning during the year in which the Petersen coil has been in service on this line.

In the case of the 11-kv. to 14-kv. systems we find that it is almost universal practise to ground only one generator on each bus section. Four companies report grounding the generator bus sections through grounding transformers, and two use grounding reactors.

It has been generally accepted as a result of experience that on ungrounded systems the possibility of obtaining dangerous voltages becomes more prevalent as mileage and voltage increase. This was universally recognized and led to different methods of grounding. In America solid grounding was resorted to in the majority of cases, and resistance was inserted in the neutral only where the fault current was likely to be excessive.

Regarding the value of resistance used no very definite practise was followed except to keep it high enough to achieve the desired result in limiting current and low enough to pass adequate current for relaying. For the same degree of protection it may be assumed that the grounding resistance should vary inversely as the charging current. A formula based on this premise was published in the First Committee Report, *viz.*:

$$K = (L_a + 5 + 25\, L_b) \times f \times R$$

Where K = Constant
L_a = Miles of overhead lines
L_b = Miles of underground cable
f = Frequency
R = Neutral resistance

It will be interesting to make the comparison of the value of K in 1923 with that of today. Table II shows this comparison. In the generated voltage class the K values of today tend to fall in all cases below those of

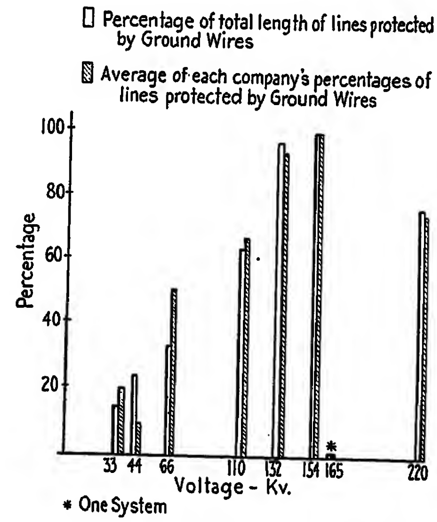

Fig. 5

1923, while in the higher voltages they have remained about the same except as regards the maximum. It is significant that this maximum value of 3.4×10^6 occurs on a system in which the company is contemplating reducing the value of neutral resistance. Even so, it is well within the 6.2×10^6 value, which according to the first report, Petersen states will prevent arcing grounds.

TABLE II

Note	1923 Values				1930 Values			
	No. of systems	Value of $K/10^6$			No. of systems	Value of $K/10^6$		
		Min.	Average	Max.		Min.	Average	Max.
A	8	0.008	1.04	3.55	13	0.0069	0.116	0.93
B	7	0.022	0.52	2.00	12	0.0134	0.495	3.4

Note A = Systems at generated voltage.
Note B = Systems above generated voltage

Ground Wire Protection

Twenty-nine companies reported their practise as regards the use of ground wires, and this information is given in Table III.

TABLE III—GROUND WIRE PROTECTION

Ref. No.	Volts	Total miles	Ground wires: Miles protected	No.	Diam. inches	Mat.	% of lines with G. W.	Remarks
1	132	278	278	1	1/2	*C W*	100	*C W* = Copper-weld
2	132	218	218	1	1/2	*C W*	100	
	66	22	22	1	3/8	*C W*	100	
	66	143	143	2	5/16	*G S*	100	*G S* = Galvanized steel
3	66	258	258	1	7/16	*C W*	100	
4	110	375	0				0	
	66	252	0				0	
	33	382	0				0	
5	66	294	294	1	11/32	*C W*	100	
6	110	26	0				0	
	66	130	0				0	
7	154	97	97	1	3/8	*G S*	100	
	110	418	418	1	3/8	*G S*	100	
	66	217	150	1	3/8	*G S*	69	
	44	605	60	1	3/8	*G S*	10	
8	44	302	0				0	
9	110	1,190	734	1 & 2	3/8	*G S*	62	
	66	370	292	1 & 2	3/8	*G S*	79	
	44	865	0				0	
	33	495	0				0	
10	110	122	122	2	1/2	*G S*	100	
	44	379	0				0	
	33	19	0				0	
11	154	88	88	2	7/16	*G S*	100	
	110	1,620	1,620	2	1/2	*G S*	100	
	44	1,781	949	1 & 2	3/8	*G S*	53	
12	66	1,177	94	1	1/4	*G S*	8	
	33	359	40	1	1/4	*G S*	11	Located below conductors
13	110	456	449	2	3/8	*C W*	98	
	66	1,130	845	1 & 2	3/8	*C W*	75	
	33	194	0				0	
14	120	560	560	1	7/16	*C u*	100	*C u* = copper
	40	8		1	5/16	*C u*		3,000 ft. from each end
15	132	116	116	1 & 2	7/16	*C u*	100	
	110	287	287	1	5/8	*C S*	100	
	66	197	197	1	5/16	*C W*	100	
	33	497	58	1	5/16	*G S*	12	
16	66	100	100	1			100	
17	110	282	282	1	3/8	*G S*	100	
	66	1,254	1,254	1	3/8	*G S*	100	
	33	228	228	1	3/8	*G S*	100	
18	132	397	397	2	3/8	*C W*	100	
19	220	352	352	2	1/2	*A 1*	100	*A 1* = Aluminum
	132	165	165	2	5/8	*C W*	100	
	66	1,143	168	2	9/16	*C W*	15	
	33	369	58	1	3/8	*G S*	16	
20	66	433	433	1	3/8	*A 1*	100	
	33	140	140	1	3/8	*G S*	100	
21	132	145	66	2	3/8	*G S*	45	
	66	1,935	1,053	1	3/8	*G S*	55	
	33	100	24	1	3/8	*G S*	24	
22	110	383	0				0	
	66	830	0				0	
	44	63	0				0	
	33	97	0				0	
23	110	628	377	2	5/16	*G S*	60	
	66	566	0				0	
	33	150	0				0	
24	100	810	810	2	3/8	*G S*	100	
	66	1,662	0				0	
25	220	254	254	2	1/2	*G S*	100	
	110	481	481	2	7/16	*G S*	100	
	66	694	492			*G S*	71	
	33	129	30				23	
26	220	420	0				0	
	165	398	0				0	
	110	1,968	348	1	3/8	*G S*	18	
	60	2,602	0				0	
	44	168	0				0	
27	220	802	802			*G S*	100	
	66	1,838	0				0	
	33	111	0				0	
29	132	874	874	1 & 2	7/16	*A 1*	100	
	33	1,379	0				0	

In order to show the over-all practise in the use of ground wires for the protection of transmission lines in a more concise manner, Fig. 5 has been prepared. This figure has been worked up on two percentage bases. In the first case the total length of all lines reported in a given voltage class was taken as 100 per cent and the mileage of lines protected by ground wires was drawn as a percentage of that total. This gives the practise of the country as a whole. In the second case, the total mileage in a given voltage class of each individual company was taken as 100 per cent, and the mileage of protected lines was expressed as a percentage of this total (eighth column in Table III). The average of these percentages in any voltage class is the value given in the chart. This tends to show the opinion of transmission engineers better than the first value perhaps, because the short systems are given the same weight as the long systems.

Tower Footing Resistance

In protecting a line against direct strokes of lightning the performance of the ground wire depends on its receiving the direct stroke and conducting the energy to ground without causing a flashover to the line conductors. This depends upon the ability of the ground wires to remain at a low potential, and this again is dependent to a large extent on the value of the tower footing resistance.

From the replies to the question regarding tower footing resistance it is apparent that there is very little accurate data available, and that at this time no average figures of any value can be given, either on the basis of general territory or nature of soil. One company reports a range of 1 ohm to 58 ohms over a very restricted territory. The extremes reported by the seven companies replying to this question are 0.05 ohm and 300 ohms and the average 22.5 ohms.

Simultaneous Faults at Different Locations

In reply to the question: "Do you have evidence of simultaneous faults at different locations?" twenty-five replies were received. Of these sixteen companies reported no evidence; two reported less than 1 per cent; three reported 1 per cent; one reported 5 per cent, and three gave replies as noted below. In all cases the evidence was taken from relay operations or from automatic oscillographs.

One company had trouble on a 300-mile 66-kv. system until it was grounded. The neutral was found to shift to 80 per cent of the line-to-ground voltage at the middle of the section when one conductor was grounded.

One company reports no evidence of simultaneous faults on its 132-kv. and 220-kv. systems. One 66-kv. and two 22-kv. systems show 1 per cent or less; one 66-kv. system 15 per cent, eight 26-kv. and three 13-kv. systems (mostly overhead) show 33 per cent, as indicated by relay operations. The last two percentages include double faults at the same physical location as well as double faults at different locations, the former very largely predominating.

One company operating an ungrounded system reports conclusive evidence of arcing grounds without stating any figure for this type of fault.

Types of Faults

There are four possible types of faults on a system:

a. One conductor to ground.
b. Two conductors to ground.
c. Phase-to-phase short circuit.
d. Three-phase short circuit.

In reply to the question regarding the division of the faults into these four classes twenty-seven replies were received. These are given in Table IV as a percentage of the total number of faults. It must be borne in mind that these figures include both overhead and underground lines.

TABLE IV

	One cond. to ground	Two cond. to ground	Ph. to Ph. sh. circuit	Three-ph. sh. circuit
Maximum	97	45	54	36
Average	69	14	11	6
Minimum	30	0	0	0

Sufficiency of Ground Current

Regarding the sufficiency of ground current for relaying purposes, all companies except two report sufficient current for relaying purposes. These two exceptions encounter trouble at times of light load in the matter of selectivity.

The loss of synchronous load caused by ground faults is rare. Seven companies report having this occasionally. One company is making changes in its relay equipment to rectify this condition.

Recent or Contemplated Changes in Method of Grounding

In reply to this question, some companies have described certain changes in their system layouts which they are contemplating or have recently completed to correct faults which they have been experiencing. The changes which seem pertinent to the subject under report are given below.

Company No. 1. The neutral resistor on the 11-kv. underground system has been reduced from three ohms to one and one-half ohms to prevent occurrence of simultaneous faults on this system.

Company No. 7. This company formerly operated certain lines ungrounded, but in order to eliminate arcing grounds and simultaneous faults, it is now grounding everything solidly including the 11-kv. circuits.

Companies Nos. 9 and 11. The tendency of these two companies has been to reduce the number of grounding points on their systems so as to decrease the magnitude

of the ground currents. This is to reduce the duty on equipment and the possibility of telephone interference.

Company No. 12. The 33-kv. lines of this company were originally fed through delta-connected transformers. These have been replaced by auto transformers with grounded neutrals. This change has resulted in eliminating about 60 per cent of the trouble on these lines.

Company No. 18. The resistance in the 25-kv. neutral has been decreased from 28.8 ohms to 19 ohms and at one of the stations to 14 ohms. This was made possible by the installation of higher capacity breakers.

Company No. 19. One of the original 66-kv. lines of about 30 circuit miles was delta connected and ungrounded. In extending this system it was necessary to use underground cable, which was not suitable for ungrounded operation, and this fact, together with the advantages of the grounded system in eliminating arcing grounds and limiting transient voltages, led to a decision to change the entire system to star connection with the neutral grounded.

The original 26-kv. systems were solidly grounded through the wye-connected transformers at the supply end, but resistors of 75 ohms have been installed as an inductive coordination measure. However, secondary advantages have been obtained as a result of lower fault currents, which produce less general disturbance to the system and are less destructive at the point of fault and to apparatus. The relay system is somewhat more expensive and complicated but is proving quite reliable in operation.

The only other change in method of grounding was the adoption of the policy of grounding one piece of equipment on each side of certain bus sectionalizing breakers. The object of this change is to permit retention of a fixed neutral in the good section of a bus should sectionalizing breaker open. It is expected that this will result in fewer insulator failures on the operating bus section at times of fault, but to date little operating experience has been secured.

Company No. 23. Grounding transformers have been installed recently on a formerly ungrounded 60-kv. delta-connected system to relieve telephone interference and arcing grounds resulting from line-to-ground faults. Results have been satisfactory.

Company No. 25. In the generating stations, the neutral grounding resistances are being reduced as the growth of the low-tension distribution systems require. The value of resistance is selected on the basis of minimum allowable relay currents, and the maximum allowable resistance which will eliminate arcing grounds.

Company No. 29. This company's 33-kv. system was formerly solidly grounded, but now five points are grounded through 50 ohms resistance and the other points are left ungrounded. This step was taken to eliminate telephone interference.

Company No. 31 This company had the 13.2-kv. system neutral grounded through resistors which would limit the ground current to 1,000 amperes. The method of operation was changed so that the sources of generation were segregated and synchronized at the load. This permitted short-circuiting the neutral resistors and going over to solid grounding, the only tie between generator bus sections being at remote locations. Only one generator neutral on any bus section is grounded. The new method of operation has resulted in a greatly reduced number of simultaneous faults.

Participating Companies

Alabama Power Company and Associates.
Buffalo, Niagara and Eastern Power Corporation.
Cleveland Electric Illuminating Company.
Commonwealth Edison Company.
Connecticut Light and Power Company.
Consolidated Gas, Electric Light and Power Company of Baltimore.
Detroit Edison Company.
Duquesne Light Company.
Georgia Power Company.
Houston Lighting and Power Company.
Memphis Power and Light Company.
Metropolitan Edison Company and Associates.
Mississippi Power Company.
Montana Power Company.
New York Edison Company and Associates.
New England Power Association.
Northern States Power Company.
Ohio Edison Company and Associates.
Oklahoma Gas and Electric Company.
Pacific Gas and Electric Company and Associates.
Pennsylvania Power and Light Company.
Philadelphia Electric Company.
Public Service Electric and Gas Company.
Public Service Company of Northern Illinois and Associates.
Puget Sound Power and Light Company.
Southern California Edison Company.
South Carolina Power Company.
Tennessee Electric Power Company.
Texas Power and Light Company.
Union Electric Light and Power Company and Associates.
Washington Water Power Company.
West Penn Power Company.

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Discussion

E. C. Stone: A comparison of the first report on grounding published about eight years ago with the present report shows no radical changes in practise, but indicates some interesting trends. This report covers 57,000 miles of transmission line as compared with 31,000 miles covered in the previous report.

In the following table is a comparative summary of the mileages of lines grounded by the various methods:

Method of grounding	Present report Mileage	Present report Per cent of total	1923 report Mileage	1923 report Per cent of total
Ungrounded	5,310	9.3	6,385	20.4
Solidly grounded	42,300	74.5	18,375	58.5
Resistance grounded	6,346	11.1	6,448	20.8
Reactance grounded	2,903	5.1	100	0.3
Total	56,859	100.0	31,308	100.0

It is interesting to note that with the exception of approximately 5 per cent of the total mileage which is reactance grounded, all of the increase in mileage covered by the second report is solidly grounded.

The present report clearly indicates a better understanding of the principles of system neutral grounding, of the fields for application of the various methods and of the results to be expected. Thus it has been found that the ground current in cases of a system fault must be large enough to prevent arcing grounds and to provide for positive selective relay operation. The points at which the neutral is grounded must be sufficient in number and so located to permit of adequate current flow wherever on the system a fault may be located. On the other hand the neutral ground current must be limited to values which will not cause undue damage at the point of fault and which will not produce system instability. The limit is of course a function of the maximum time a fault current is permitted to remain on the system. Larger currents are permissible where high-speed circuit breaker tripping is employed.

Effects on communication circuits are a function both of the wave form and the magnitude of the fault current.

It seems to the writer that the present report should definitely differentiate between systems operated at generated voltage and those operated at higher than generated voltage, just as the first report did. Analysis of the data presented in the present report shows that of the 36,000 miles of line reported on as operating at 34,000 volts or above, approximately 34,000 miles or 90 per cent is solidly grounded, while for the 21,000 miles operated below 34,000 volts only 55 per cent is solidly grounded. There are definite reasons for this difference in practise. On the high-voltage lines solid grounding gives greater benefits from the insulation standpoint while at the same time the ground currents in relation to system capacity are relatively low and those systems are in major part outdoor overhead systems so that the physical damage from ground fault currents generally is reduced. On the other hand on the lower voltage systems the fault ground currents are relatively large and the insulation benefits from solid grounding comparatively small. These systems consist largely of underground cable and indoor substations, so that damage from large fault ground current becomes more serious.

As stated above the present report shows lower average impedance in the grounding circuit for systems operated at generated voltage. It is believed that further analysis of the data on these systems will show that while the over-all grounded neutral impedance may be lower than before, ground fault currents are still limited in value by the sectionalizing of the generating station buses so that a larger element of line impedance than before is introduced into the ground fault circuit.

The problem of double simultaneous faults at different parts of a system is of particular importance because of the difficulties in many cases of providing relaying which will isolate such faults without interruption to service. In an ungrounded system simultaneous faults are naturally charged to arcing grounds. In a properly grounded system arcing grounds should be impossible so that simultaneous faults may be assumed to be the result of the rise in voltage on two-phase when the third-phase is grounded. It seems fair to raise the question, particularly on moderate voltage systems, whether the relatively small lowering of fault voltages by reduction in value of ground impedance will correct the situation and whether the insulation required for normal operation should not be sufficient to withstand these excess voltages resulting from ground currents for the short periods of time required for relay operation. The conclusion is drawn that the solution is to be found not in reducing the impedance of the neutral ground circuit but rather in raising the system insulation to a level which will permit it to withstand the overvoltages prescribed by our standard A. I. E. E. tests.

Another subject discussed in the report which is of major importance is the impedance of grounds which are used in connection with lightning protection. The evidence of operating experience seems to point very clearly to the fact that much lightning protective apparatus has had its effectiveness much reduced or destroyed by the unsatisfactory grounds used in connection with it. With the latest types of lightning arrester now available it is beginning to look as though the neck of the bottle in lightning protection lay in the impedance of the grounding circuits of the protective equipment.

The problem of system grounding is only one phase of the greater general problem of reducing insulation failures which is facing power engineers today. We may be optimistic on this subject, however, because of the extensive research in progress and the definite advances being made.

C. A. Powel: Mr. Stone's comparison of the present report with the first report is particularly valuable and raises some points which might well be taken into consideration in future reports. It is hoped that the next report will include comparisons based on more precise methods of defining the degree of grounding so as to facilitate the classification of the various systems.

Reactance of Transmission Lines With Ground Return

BY J. E. CLEM[1]

Member, A. I. E. E.

Synopsis.—In this paper there is presented an analysis of the circuit consisting of any number of line conductors in parallel with a return through the earth and any number of ground wires in parallel. In the development of this analysis certain assumptions have been made which tests indicate do not detract at all from the engineering usefulness of the result but which simplify the formulas very much. The general formulas resulting from this analysis are condensed in Table A-1.

For the application of this method to the calculation of ground faults it is necessary to know what value of earth conductivity applies. Considerable data in regard to this for various localities have been collected and are given in Table C-2. For convenience in making the calculations curves are given for various values of the earth conductivity over the range as indicated by test and experience. These curves are given as Figs. C-2 and C-3. Occasionally preliminary calculations are made in which very approximate reactance values are used. In such cases a line reactance of 0.8 ohms per mi. is used, and a zero phase sequence reactance of 2.6 ohms per mi. could be used with the same degree of accuracy. With one ground wire the zero phase sequence reactance would be about 2.0 ohms per mi. and with two ground wires about 1.6 ohms per mi.

In the application of this method it should be noted that the values of a total impedance should be used and that it is necessary to know the contact resistance between conductor and earth at each end of the circuit. When a fault occurs somewhere along the line the contact resistance may be very high especially if no ground wire is used. When a ground wire is used the effective fault resistance will be relatively low even though the contact resistance of the tower footing to ground at that particular tower is high, since the fault current can pass to earth at several towers.

INTRODUCTION

As electric power systems continue to increase in size and as demands for more nearly perfect service become more insistent, increasing attention must be given to the provision of arrangements whereby faulty portions of the system may be promptly isolated. Predictions concerning the stability of systems are largely based upon a knowledge of the flow of synchronizing power between machines during fault conditions, and the application of suitable relays is determined largely upon knowledge of current flow during fault conditions. The faults which must be considered are generally those involving ground. Calculations involving unbalanced faults are greatly simplified by the use of the symmetrical component method introduced by Fortescue[2] and applied by MacKerras[3] and others. The successful use of this method requires a more accurate knowledge of the zero phase sequence impedance of transmission lines than has heretofore been available.

The zero phase sequence reactance of transmission lines is the reactance of the three-line conductors in parallel with ground return. In 1927 the writer developed equations for the impedance of a circuit consisting of any number of line wires in parallel and a return path through ground or through ground and any number of ground wires in parallel. The application of these equations is contingent upon a knowledge of the impedance of a single conductor with ground return. The impedance of such a circuit varies with several factors, chief of which is the distribution of the current in the return path below the surface of the earth. As this is largely dependent upon soil conditions, it will obviously vary with the locality and terrain. Hence, field tests have been made to obtain experimental data for use in predicting ground impedance, and to check the theoretical analysis previously prepared. This paper presents not only the analytical study, but also its confirmation by field test data, plus a suggested method of predicting the impedance of the fundamental circuit of conductor with ground return.

A. CIRCUIT ANALYSIS

The analysis of the circuit is very simple although it involves such factors as the self inductance of the earth path and the mutual inductance between a wire and the earth which have usually been considered as fictitious quantities. However, by proper manipulation the equations can be made to take on a real and definite form. For the analysis as given here certain simplifying assumptions are made. It is assumed that the line wires are transposed, so that the same current flows in each of the conductors. It is assumed also that the spread of the current in the earth is such that its mutual effect on all the line conductors or ground wires is the same within the usual horizontal spread of line conductors. The tests herein reported show that the error from this source is small and these assumptions, therefore, do not detract from the engineering usefulness of the results.

1. Central Station Engg. Dept., General Electric Co., Schenectady, N. Y.

2. *Method of Symmetrical Components Applied to the Solution of Polyphase Networks*, C. L. Fortescue, A. I. E. E. TRANS., 1918, Vol. 37, p. 1027.

3. "Calculation of Single-Phase Short Circuits by the Method of Symmetrical Components," A. P. MacKerras, *G. E. Review*, 1926, Vol. 29, pp. 218 and 468.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

In Fig. A-1 let

m = number of line wires.
n = number of ground wires.
g = subscript indicating ground.
o = subscript indicating ground wire.
w = subscript indicating line wire.
t = subscript indicating auxiliary test wire.
L = $j\,w\,\mathfrak{L}$ self inductance, as indicated by subscripts.
M = $j\,w\,\mathfrak{M}$ mutual inductance, as indicated by subscripts.
r = resistance, as indicated by subscripts.
E_{mn} = circuit voltage.
E_t = voltage induced on test wire.
I = current as indicated by subscripts—lack of subscript indicates total current.

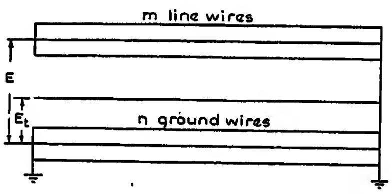

FIG. A-1

Impedance of Circuit. The impedance of the circuit in this analysis is defined as the ratio of the applied voltage, E_{mn}, to the total current I. The current flows from the generator out along the line wires to the point of fault, and flows back to the generator through the ground and ground wire in parallel. The effect of the current in the ground is simulated by a pseudo conductor embedded in the earth as shown in Fig. A-2.

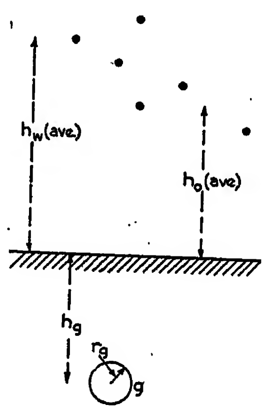

FIG. A-2

This artificial earth conductor is assumed to have a self inductance of its own and to have mutual reactance with the real conductors of the circuit. The depth of this earth conductor below the surface of the earth and its effective diameter are not of interest at the present time.

Starting from the generator the voltage of the circuit is made up of the voltage drop from the generator through the ground to the point of fault and the voltage drop along one of the line conductors from the point of fault back to the generator. The voltage drop in any conductor of a circuit is made up of the voltage drop produced by the current in the conductor and its own self inductance and the voltage drop induced in the conductor by the currents in other conductors through the mutual inductance. Writing this in the form of an equation we get:

$$E_{mn} = I_g\,(r_g + L_g) + I_o\,M_{og} - I\,M_{wg} + \frac{I}{m}\,(r_w + L_w) + \frac{m-1}{m}\,I\,M_{ww} - I_o\,M_{ow} - I_g\,M_{wg} \quad (1)$$

But $I_g = I - I_o$ so that this expression becomes

$$E_{mn} = I\,(r_w + L_w + r_g + L_g - 2\,M_{wg}) - \frac{m-1}{m}\,I\,(r_w + L_w - M_{ww}) - I_o\,(r_g + L_g - M_{wg} - M_{og} + M_{ow}) \quad (2)$$

In this expression the self inductance involving the earth is usually said to be impossible of separate determination. If it is combined with the mutual inductance between the earth and the line wire, or ground wire, reactance quantities are obtained which are real and can be determined by measurement. The self inductance of a conductor is a characteristic of the conductor itself and does not depend in any way upon other conductors in the neighborhood. For mutual inductance another conductor is required. If the current returns through the other conductor the voltage drop in the first conductor is $I\,(L - M)$. This quantity $(L - M)$ is that which is usually known as reactance. It can usually be determined by test or calculations either for a one way conductor of a circuit alone or for both in series.

If $(r_g + L_g)$ is combined with M_{wg} a measurable reactance results and the introduction of $(r_o + L_o) - (r_o + L_o)$ to combine with $- M_{og} + M_{ow}$ will give measureable reactances; or if $(r_g + L_g)$ is combined with M_{og} another measurable reactance results and the introduction of $(r_w + L_w) - (r_w + L_w)$ will combined with $- M_{wg} + M_{ow}$ to give other measurable reactances. In the form of an equation this can be expressed by

$$r_g + L_g\,M_{wg}\,M_{og} + M_{ow}.$$

$$= \frac{r_g + L_g}{2} - M_{wg} + \frac{r_w + L_w}{2}$$

$$+ \frac{r_o + L_o}{2} - M_{og} + \frac{r_g + L_g}{2}$$

$$- \frac{r_o + L_o}{2} + M_{ow} - \frac{r_w + L_w}{2}$$

$$= \frac{Z_{wg} + Z_{og} - Z_{ow}}{2} \quad (3)$$

Let

$$r_w + L_w - 2\,M_{wg} + L_g + r_g = Z_{wg}$$

$$r_w + L_w - M_{ww} = \frac{Z_{ww}}{2}$$

When these expressions together with that derived in (3) are introduced into equation (2) it becomes:

$$E_{mn} = I\,Z_{wg} - I_o\,\frac{Z_{wg} + Z_{og} - Z_{ow}}{2} - \frac{m-1}{2\,m}\;I\;Z_{ww} \tag{4}$$

Let $$M_{ow} = \frac{Z_{wg} + Z_{og} - Z_{ow}}{2} \tag{5}$$

and then the total circuit impedance is given by

$$\frac{E_{mn}}{I} = Z_{mn} = Z_{wg} - \frac{I_o}{I}\,M_{ow} - \frac{m-1}{2\,m}\,Z_{ww} \tag{6}$$

The indulgence of the reader is asked in keeping the terminology of M as it is defined in (5), as a form of mutual impedance, and appears in (6) distinct from that appearing in equation (1). This is an easy matter because the meaning of M as defined by (5) is the only one used in the final equations and in the calculations. The other meaning of M, which is the mutual inductance between two conductors, is only used in the course of the development.

Current in Ground Wires. The over-all voltage drop from the generator through the ground to the point of fault outside of the earth and along the ground wire to the point of fault must be equal so that

$$I_g = (r_g + L_g) - I\,M_{wg} + I_o\,M_{og}$$
$$= \frac{I_o}{n}\,(r_o + L_o) + \frac{n-1}{n}\,I_o\,M_{oo} - I\,M_{ow} + I_g\,M_{og} \tag{7}$$

As before $I_g = I - I_o$ so that

$$I\,(r_g + L_g - M_{wg} - M_{og} + M_{ow})$$
$$= I_o\,(r_o + L_o + r_g + L_g - 2\,M_{og})$$
$$- \frac{n-1}{n}\,I_o\,(r_o + L_o - M_{oo}) \tag{8}$$

From which in the same manner as before and letting

$$r_o + L_o - M_{oo} = \frac{Z_{oo}}{2}$$

$$I\,\frac{(Z_{wg} + Z_{og} - Z_{ow})}{2} = I_o\,Z_{og} - \frac{n-1}{2\,n}\,I_o\,Z_{oo} \tag{9}$$

which gives

$$\frac{I_o}{I} = \frac{M_{ow}}{Z_{og} - \dfrac{n-1}{2\,n}\,Z_{oo}} \tag{10}$$

Voltage Induced on Test Wire. In Fig. A-1 the voltage to ground E_t which can be measured on the test wire must be equal to the voltage drop along the ground plus the voltage induced along the test wire. This test wire was introduced to obtain additional data for checking the analysis and it can also be considered to simulate a communication circuit on the same supports as the power line.

$$E_t = I_g\,(r_g + L_g) + I_o\,M_{og} - I\,M_{wg}$$
$$+ I\,M_{wt} - I_o\,M_{to} - I_g\,M_{tg}$$

As before $I_g = I - I_o$ so that

$$E_t = I\,(r_g + L_g - M_{wg} - M_{tg} + M_{wt})$$
$$- I_o\,(r_g + L_g - M_{og} - M_{tg} + M_{ot}) \tag{12}$$

Which expression reduces to

$$E_t = I\,M_{wt} - I_o\,M_{ot} \tag{13}$$

In Table A-1 the equations which apply to 1, 2, or 3 line wires in parallel and 0, 1, 2, 3 ground wires are given. Numerical subscripts refer to number of line wires and number of ground wires respectively, and in the order given. Thus $Z_{3\cdot2}$ means the impedance of the circuit with ground return when three line wires are in parallel and there are two ground wires. The impedances then are defined as follows:

I_o = total return current in ground wires.
I = total fault current.
Z_{wg} = average impedance, line wire with ground return.
Z_{og} = average impedance, ground wire with ground return.
Z_{tg} = average impedance, auxiliary wire with ground return.
Z_{ww} = average impedance, line wire with line wire return.
Z_{wo} = average impedance, line wire with ground wire return.
Z_{oo} = average impedance, ground wire with ground wire return.
Z_{tw} = average impedance, auxiliary wire with line wire return.
Z_{to} = average impedance, auxiliary wire with ground wire return.

The zero phase sequence impedance enters into the calculations of ground fault currents as a quantity which operates on the zero phase sequence current. This current is equal and in the same direction in all three line wires and is one-third of the ground fault current. Accordingly the zero phase sequence impedance of a transmission line is the quantity obtained by multiplying by 3 the value of impedance given in column 3 of Table A-1 depending upon the number of round wires. This is indicated in column 4 of Table A-1.

B. Tests

Tests have been made on four power systems to check this analysis. Design data on these systems are given in Table B-8.

On three of the systems tests were made before and after a ground wire was installed; and on the fourth, a line was tested which had ground wires over part of its length. The test data are given in Tables B-3 to

TABLE A-1—IMPEDANCE OF CIRCUIT WITH GROUND RETURN CURRENT IN GROUND WIRE RETURN

No. gd. wires	1 One line wire	2 Two line wires	3 Three line wires	4 Zero phase sequence impedance
0	$Z_{10} = Z_{wg}$	$Z_{20} = Z_{wg} - \frac{Z_{ww}}{4}$	$Z_{30} = Z_{wg} - \frac{Z_{ww}}{3}$	$3 \times Z_{30}$
1	$Z_{11} = Z_{wg} - \frac{I_o'}{I} M_{ow}$	$Z_{21} = Z_{11} - \frac{Z_{ww}}{4}$	$Z_{31} = Z_{11} - \frac{Z_{ww}}{3}$	$3 \times Z_{31}$
	$\frac{I_o'}{I} = \frac{M_{ow}}{Z_{og}}$	Same as Col. 1	Same as Col. 1	....
2	$Z_{12} = Z_{wg} - \frac{I_o''}{I} M_{ow}$	$Z_{22} = Z_{12} - \frac{Z_{ww}}{4}$	$Z_{32} = Z_{12} - \frac{Z_{ww}}{3}$	$3 \times Z_{32}$
	$\frac{I_o''}{I} = \frac{M_{ow}}{Z_{og} - \frac{Z_{oo}}{4}}$	Same as Col. 1	Same as Col. 1	....
3	$Z_{13} = Z_{wg} - \frac{I_o''}{I} M_{ow}$	$Z_{23} = Z_{13} - \frac{Z_{ww}}{4}$	$Z_{33} = Z_{13} - \frac{Z_{ww}}{3}$	$3 \times Z_{33}$
	$\frac{I_o''}{I} = \frac{M_{ow}}{Z_{og} - \frac{Z_{oo}}{3}}$	Same as Col. 1	Same as Col. 1	....

B-7. The tests are as follows. (See Fig. A-1)

Test No. 1 One wire out—ground return
Test No. 2 Two wires out—ground return
Test No. 3 Three wires out—ground return
Test No. 4 One wire out—ground and one wire return
Test No. 5 One wire out—ground and two wires return
Test No. 5A Two wires out—ground and one wire return
Test No. 6 One wire out—one wire return
Test No. 7 One wire out—two wires return

In tests 4-5-5A one or more line wires were used as a ground wire.

In Table B-1 there is given a comparison of the calculated and tested results for tests made before the installation of the ground wire and in Table B-2 the same comparison is given for the tests made after the installation of the ground wire.

For Table B-1 the known values are considered to be the impedance of one conductor with ground return, indicated as Z_{wg}, and obtained from tests 1-2-3; and also the impedance of one conductor with line wire return, indicated as Z_{ww}, and obtained from tests 6-7.

In Table B-2 the known values are Z_{wg}, obtained from the tests without the ground wire, and Z_{ww} obtained from tests 6-7. Similar known impedances with the ground wire Z_{og}, Z_{oo}, and Z_{ow} are found by calculation to obtain Z_{oo} and Z_{ow}, and by a combination of calculation and reference to the tested values of Z_{wg} to obtain Z_{og}.

For the calculation of those tests when one of the line wires is in parallel with the real ground wires Z_{og}, Z_{oo}, and Z_{ow} must be modified. The modified values as used in the calculation are not given in the tabulation but may be obtained easily. For instance, if there was one real ground wire (Z_{og}) and one line wire used as a ground wire (Z_{wg}) then the effective ground wire impedance (Z_{og}') used in the calculation would be

$$Z_{og}' = \frac{Z_{og} + Z_{wg}}{2}$$

and the other necessary quantities are obtained in the same manner.

During the tests after the ground wire was installed on the Turner-Logan line a lineman was stationed on one of the towers near the middle of the line to read the current in the ground wire. The results indicated that the current in the ground wire was the same at the middle of the line as at the end. Data are given in Table B-9. In tests 4, 5, and 5-A one or more of the line wires used as an additional ground wire was in parallel with the actual ground wire so that in Table B-5 the ground wire current given is the sum total, while in Table B-9 the current in the actual ground wire itself is recorded.

C. Predetermination of Ground Return Impedance

The main problem in the calculation of ground fault currents and the division of the return current between the ground wires and ground is the predetermination of the impedance of the fundamental circuit consisting of the conductor with ground return.

It seems that for many years it was customary to consider the earth as a perfectly conducting plane. All the reactance was assigned to the conductor and the corresponding distance to the "neutral plane" was

TABLE B-1—COMPARISON OF TESTS AND CALCULATIONS
Tests Made Before Ground Wire Installed

Test		Pa. Pr. & Lt. Co. 220 Kv. Wallenpaupack—Siegfried	Appa. Elec. Pr. Co. 132 Kv. Turner—Logan	Alabama Pr. Co. 110 Kv. North Auburn—Newann	Consumers Power Co. 140 Kv. Saginaw—Flint
1-2-3	By test, Z_{wg} =	17.00 + j 95.53	14.00 + j 57.55	25.55 + j 120.3	16.42 + j 60.13
6-7	By test, Z_{ww} =	16.2 + j 107.0	22.29 + j 66.36	30.91 + j 114.7	26.65 + j 73.8
4	Calculated, I_0 =	44.7	41.5	51.7	76.4
	Tested =	44.9	41.2	51.6	74.3
	Calculated, Z =	13.08 + j 76.75	12.84 + j 47.45	20.33 + j 87.76	15.27 + j 51.39
	Tested =	12.84 + j 77.88	14.14 + j 47.40	22.20 + j 86.50	16.9 + j 51.6
	Calculated, E_t =	23.78	14.37	30.80	
	Tested =	23.75	14.35	30.70	
5	Calculated, I_0 =	61.4	58.7	68.3	55.9
	Tested =	61.9	58.2	66.5	53.4
	Calculated, Z =	11.57 + j 69.80	12.62 + j 43.25	18.65 + j 77.30	14.66 + j 46.98
	Tested =	10.86 + j 69.48	13.83 + j 43.23	19.65 + j 77.00	16.51 + j 47.3
5A	Calculated, I_0 =	44.7	41.5	51.7	76.4
	Tested =	44.9	41.3	51.4	74.8
	Calculated, Z =	9.03 + j 50.50	7.27 + j 30.96	12.60 + j 59.08	8.61 + 32.94
	Tested =	8.93 + j 49.70	8.71 + j 30.77	15.23 + j 57.80	9.43 + 33.3

Calculations are made on the basis of the data obtained in Tests 1-2-3 and Tests 6-7.

TABLE B-2—COMPARISON OF TESTS AND CALCULATIONS
Tests Made After Ground Wire Installed

Test No.		Pa. Pr. & Lt. Co. 220 Kv. Wallenpaupack	Appa. Elec. Pr. Co. 132 Kv. Turner—Logan	Alabama Pr. Co. 110 Kv. Martin Dam—North Auburn	Consumers Pr. Co. 140 Kv. Saginaw—Flint
	By test and calc. Z_{wg} =	17.09 + 95.53	14.00 + j 57.55	14.88 + j 45.30	22.2 + j 60.13
	By test and calc. Z_{ww} =	16.2 + j 107.0	22.11 + j 66.78	12.76 + j 45.32	31.6 + j 73.2
	" " " " Z_{og} =	43.78 + j 102.2	21.31 + j 58.54	32.53 + j 48.05	77.4 + j 64.5
	" " " " Z_{oo} =	70.0 + j 119.6		48.06 + j 50.18	
	" " " " Z_{ow} =	43.1 + j 107.1	29.31 + j 72.05	30.41 + j 44.63	86.8 + j 81.7
1-2-3	Calculated, I_0 =	60.1	35.7		43.9
	Tested =	58.0	41.6		42-38-32
	Calculated Z =	15.13 + j 85.54	12.93 + j 49.69	9.53 + j 32.97	19.38 + j 50.67
	Tested =	15.14 + j 86.85	14.28 + j 49.57	13.53 + j 33.10	21.2 + j 55.3
	Calculated Z_t =		16.40		
	Tested =		16.49		
4	Calculated I_0 =	74.7	55.7		
	Tested =	74.6	57.9		
	Calculated Z =	12.05 + j 70.86	12.62 + j 44.76	8.71 + j 28.16	
	Tested =	11.83 + j 73.60	13.04 + j 44.75	10.37 + j 29.87	
	Calculated Z_t =		11.47		
	Tested =		11.71		
5	Calculated I_0 =	93.9	66.3		
	Tested =	94.3	62.2		
	Calculated Z =	9.67 + j 62.56	12.03 + j 42.00	10.41 + j 26.20	
	Test =	10.67 + j 67.52	12.81 + j 42.35	8.33 + j 27.27	
5A	Calculated I_0 =	74.7	55.7		
	Tested =	85.5	60.6		
	Calculated Z =	8.00 + j 44.11	7.10 + j 28.07	5.52 + j 16.72	
	Tested =	8.00 + j 47.00	7.85 + j 28.02	7.10 + j 18.23	

calculated. This gave effective conductor heights up to several thousand feet and a wide variation was found. This assumption always seemed too much in error to the writer because no good method of predicting the impedance of the circuit could be built up from it.

The calculation of the impedance of the fundamental circuit made up of a conductor with ground return requires a study of the distribution of the current in the ground. A solution to this problem has been given by Carson in the *Bell System Technical Journal* for

TABLE B-3—TEST DATA

Pennsylvania Power & Light Co.
Wallenpaupack—Siegfried, 220-Kv. Line
Tests Made Before Ground Wire Installed

Test No.	Total volts	Resistence component	Reactive component	Total amperes	Ground amperes	Ground wire amperes	Line ground wire amperes	Aux. wire volts
	E	R	X	I	I_g	I_o	I_o'	E_t
1	9,747	1,712	9,593	100	100			4,318
2	7,059	1,362	6,918	100	100			4,330
3	6,010	1,107	5,906	100	100			
4	7,680	1,269	7,588	100	55.2	44.8		2,375
5	7,008	1,086	6,948	100	38.1	61.9		
5A	5,052	893	4,970	100	55.7	44.6		
6-7	10,820	1,620	10,700	100				
				Tests made after ground wire installed				
1	8,787	1,466	8,643	100	42.0	58.0		3,442
2	6,143	1,129	6,040	100	42.0	58.0		3,440
3	5,230	993	5,130	100	41.9	59.1		
4	7,453	1,183	7,360	100	25.1	35.2	39.4	2,027
5	6,833	1,067	6,752	100	22.1	36.6	57.7	
5A	4,770	800	4,700	100	31.9	46.1	39.4	
6-7	10,620	1.692	10,480	100				

TABLE B-4—TEST DATA

Pennsylvania Power & Light Co.
Wallenpaupack—Tower 19-5-220 Kv.
Tests Made Before Ground Wire Installed

Test No.	Total volts	Resistance component	Reactive component	Total amperes	Ground amperes	Ground wire amperes	Line ground wire amperes	Aux. wire volts
	E	R	X	I	I_g	I_o	I_t	E
1	8,313	8.055		100	100			7,745
2	7,985	7,928		100	100			7,687
3	7,880	7,825		100	100			
4	3,015	751	2,920	100	18.4	94.1		1,457
5	2.316	468	2,268	100	9.6	95.2		
5A	2,234	651	2,139	100	18.8	94.3		
6-7	3,109	515	3,065	100				
				Tests made after ground wire installed				
1	2,118	474	2,064	100	42	58		
2	1,320	394	1,258	100	42.2	57.8		
3	1,075	358	1,015	100	42.1	57.9		
4	1,933	415	1,888	100	30.9	42.6	26.6	1,967
5	1,878	370	1,841	100	24.3	33.3	42.3	
5A	1,196	300	1,157	100	30.9	42.3	26.8	
6-7	3.102	499	3,064	100				

TABLE B-5—TEST DATA

Appalachian Electric Power Company
Turner—Logan 132-Kv. Line
Tests Made Before Ground Wire Installed

Test No.	Total volts E	Resistance component R	Reactive amperes X	Total amperes I	Ground current I_g	Ground wire I_o	Aux. volts E_t
1	5,955	1,380	5,790	100	100		2,461
2	4,149	851	4,060	100	100		2,473
3	3,608	670	3,545	100	100		
4	4,949	1,414	4,740	100	58.1	41.2	1,435
5	4,530	1,363	4,323	100	40.8	58.2	
5A	3,198	871	3,077	100	58.8	41.3	
6-7	7,001	2,229	6,636	100			
			Tests made after ground wire installed				
1	5.180	1,408	4,985	100	59.6	41.3	1,646
2	3,379	867	3,262	100	60.2	41.8	1,652
3	2,838	715	2,742	100	59.9	41.5	
4	4,662	1,304	4,475	100	40.9	57.9	1,171
5	4,425	1,281	4,235	100	50.8	68.2	
5A	2,908	785	2,802	100	40.9	60.5	
6-7	7,039	2,211	6,678	100			

TABLE B-6—TEST DATA
Alabama Power Co. 110 Kv.
Martin Dam—North Auburn

Test No.	Total volts	Resistance component	Reactive component	Total amperes	Ground amperes	Ground wire amperes	Aux. wire volts
	E	R	X	I	I_g	I_o	E_t
1	3,567	1,353	3,300	100	100		
2	2,415	1,040	2,179	100	100		
3	2,020	923	1,797	100	100		
4	3,163	1,037	2,987	100	65.1	35.5	
5	2,853	833	2,727	100	47.0	49.9	
5A	1,973	755	1,823	100	65.4	35.1	
6-7		1,276	4,532	100			
			North Auburn—State Line				
1		15,493		100	100		
2		15,786		100	100		
3		15,880		100	100		
4	6,590	2,413	6,132	100	21.5	89.2	
5	5,257	1,542	5,024	100	11.2	92.8	
5A	4,867	1,983	4,445	100	23.4	89.8	
6-7		1,825	6,602	100			
			North Auburn—Newnan (with ground wire)				
1	12,130	2,488	11,880	100	100		
2	9,215	1,828	8,980	100	100		
3	8,470	1,541	83,20	100	100		
4	8,936	2,220	8,650	100	47.7	51.6	3,070
5	7,940	1,965	7,700	100	29.1	66.5	
5A	5,975	1,523	5,780	100	47.6	51.4	
6-7	11,530	3,091	11,470	100			

Oct. 1926. With some modification the expression developed by Carson is

$Z = r_w + x_w + Z_g$
Z = total impedance of circuit
r_w = resistance of conductor
x_w = reactance of conductor
Z_g = impedance of ground return.

The reactance of the conductor x_w is calculated as if the return conductor was below the surface of the ground a distance equal to the height of the conductor, in other words it is the one way reactance of the conductor with image return.

TABLE B-7
Consumers Power Co. 140 Kv.
Data Summary
Saginaw—Flint
Tests Made Before Ground Wire Installed

Test	Volts	Watts	Amperes	R	X	Z
1	12,250	629.3	196.3	16.32	60.20	62.40
2	8,237	365.3	191.4	9.92	41.65	42.80
3	7,020	280	192	7.48	35.5	36.27
4	11,020	696	203	16.9	51.6	54.3
5	9,840	637	196	16.51	47.3	50.1
6	14,260	876	183.2	26.1	73.3	77.8
7	11,850	821.3	200.5	20.4	55.5	59.1

Tests made after ground wire installed

Test No.	Volts	Kilowatts	Amperes	R	X	Z
1	11,870	854.4	200.7	21.2	55.3	59.2
2	7,925	539.2	202.4	13.2	37.0	39.2
3	6,580	432.0	199.2	10.9	31.2	33.0
4	10,550	812.8	199.7	20.4	48.8	52.9
5	9,865	777.6	200.3	19.4	45.4	49.3
6	14,425	1,046.4	181.9	31.6	73.6	79.3
7	14,370	1,382.4	241.6	23.7	54.6	59.5

$$X_w = 0.2794 \log \frac{4\,h}{d} + 0.0303 \text{ ohms per mi.—natural logs} \tag{14}$$

The impedance of the earth Z_g, depends upon a qualtity ϕ, which for 60 cycles is defined as

$$\phi = 4196\, h \sqrt{\lambda} \tag{15}$$

$2\,h$ = distance from conductor to its image
d = diameter of conductor
λ = conductivity of the earth.

for any value of ϕ the corresponding value of Z_g for 60 cycles is given by

$$Z_g = 0.243\,(P + j\,Q) \tag{16}$$

and the values of P and Q in ohms per mi. are obtained from Table C-1.

In Table C-1 are given values of the earth resistance "P" in ohms per mi. and the earth reactance "Q" in ohms per mi. as a function of the quantity ϕ and the angle θ. These values were calculated by W. B. Jordan using Carsons formula and cover the range of earth conductivity "λ" as indicated by tests. The values for $\theta = 0$ are used when the self impedance of conductor and earth return is to be calculated. The values for other values of θ are used when it is desired to make calculations involving the mutual impedance in a more rigorous manner than in section A of this paper. For instance, refer to Fig. C-1 and see that

$$\cos\theta = \frac{h + H}{R},\ \sin\theta = \frac{D}{R} \tag{17}$$

The mutual impedance M is then

TABLE B-8—LINE DATA

System	Pennsylvania Pr. & Lt.	Appalachian Elec. Pr. Co.		Alabama Pr. Co.		Consumors Pr. Co.
Line tested	Wallenpaupack-Siegfried	Turner-Logan	Martin Dam-North Auburn	No. Auburn State Line	State Line-Newnan	Saginaw-Flint
Voltage rating	220 kv	132 kv	110 kv	110 kv	110 kv	140 kv.
Length, mi	64.77	41	26.9	40.4	30.1	43.25
Conductor						
Size	ACSR-795,000 cir. mil	ACSR-336,400[3]	ACSR-397,500 cir. mil	ACSR-397,500 cir. mil	ACSR-397,500 cir. mil	Copper 3/0
Arrangement	Horizontal	2 circuit V	Horizontal	Horizontal	Horizontal	2 circuit V
Separation, ft		23-28				10 approx.
Spacing, ft	24.04-H	13-V	17-H	17-H	13[4]-H	12,04-V
Average height, ft	55.1	56.5[1]	43.5	43.1	44.2[4]	38.97[1]
Ground wire						
Number	2[2]		2	2[5]	none	1
Size	ACSR-184,000 cir. mil	ACSR-177,000 cir. mil	ACSR 1/0	ACSR 1/0		Copperweld 3/8
Diameter	0.707					
Spacing, ft	24.04		17	14		
Average height, ft	65.5	94.5	52.7	52.3		74.59

1. To bottom conductor.
2. 19.2 mi. from Wallenpaupack and 4.54 mi. from Siegfried.
3. 5.7 mi. ACSR-292,000 cir. mil.
4. 9.35 mi. at 14 ft. spacing and 43.1 ft. height.
5. Insulated ground wires.

$$M = 0.2794\, L_{ag} \frac{R}{S} + M\, Q \qquad (18)$$

$$M_g = 0.243\, (P + j\, Q) \qquad (19)$$

For the mutual impedance of the earth return M_g, P and Q are obtained for the proper angle θ and a value of ϕ calculated from the expression

$$\phi = 2098\, R \sqrt{\lambda} \qquad (20)$$

In order to apply this method it is necessary to have some idea of the values of the conductivity of the earth since that is the final variable. In Table C-2 the few values of the earth conductivity λ as determined from the tests of this paper are given together with many more values obtained from other sources. The values given apply to specific tests and specific regions but can be used to estimate the value of λ to be used for other places. The striking thing about these data is the wide variation in λ for a relatively small variation in ohms per mile impedance.

TABLE B-9

Appalachian Electric Power Company
Turner-Logan 132-Kv. Line
Comparison of Current in Ground Wire at Middle and End of Line

Test No.	Ground wire current: Middle	Ground wire current: End
1	37.7	41.3
2	40.2	41.8
4	58.5	62.2
5	58.9	56.2
5A	79.9	81.7

Fortunately it is not necessary to know λ with any great degree of accuracy to determine the impedance of a transmission line with earth return within a reasonable degree of accuracy. An error of 1,000 per cent (10-1) in the value of λ will give about 30 per cent error in the earth reactance, which reduces to about 13 per cent error in the reactance of the circuit of conductor with ground return. The relative error dwindles further when the line reactance is in series with transformer and generator reactance.

In Table C-3 is given calculated values of earth reactance and resistance of the earth return path at 60 cycles. These values are shown in the curves in Figs. C-2 and C-3. These curves cover the range of earth conductivity well beyond that found in tests as indicated by Table C-2. To use the curve it is necessary to select the most probable value of earth conductivity for the line in question and then read off the unit values r_g and x_g for the proper value of average height of line.

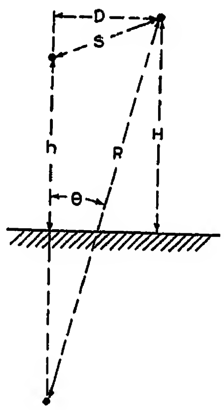

FIG. C-1

Example. As an illustration of the application of the method calculations will be made on the 25-cycle 220-kv. line recently installed in Ontario.[4] This line is 230 miles long, two circuit with 150 ft. separation, horizontally arranged conductors, and has two ground wires. The conductor is 795,000 cir. mils A. C. S. R.

4. *The 220,000-Volt System of the Hydro-Electric Power Commission of Ontario*, E. T. J. Brandon, A. I. E. E. TRANS., Oct. 1930.

and the ground wire is the same as the steel core of this cable. The height of the conductor at the tower is 60 ft. and the sag is 36 ft. The conductor spacing is 25 ft. 3 in. and the ground wires are 13 ft. 4 in. above and spaced 26 ft. 6 in. In the calculations the average height of the conductor will be taken as 38 ft. It appears from the meager information available to the writer that the conductivity of the earth is poor and accordingly it will be taken as 1×10^{-14}.

In Fig. C-4 the line and dimensions are indicated. In some of the calculations the three conductors and the two ground wires of each circuit are grouped together and considered as a single conductor in effect. Fig. C-5 shows the equivalent circuit. When this is done the self impedance of the equivalent conductor is the impedance of three line, or two ground, conductors in parallel; the mutual impedance is unchanged except that it is based upon the logarithmic average of the spacings.

1. Auxiliary Quantities

The ordinary impedance between the line wires, between ground wires, and between line wires and ground wires is needed. By the customary methods there are found to be, for 230 mi. of line

$$\left.\begin{aligned} Z_{ww} &= 54.3 + j\,158.3 \\ Z_{oo} &= 1820 + j\,179.6 \\ Z_{ow} &= 937 + j\,161.2 \end{aligned}\right\} \text{All wires in same circuit}$$

$$\left.\begin{aligned} Z_{ww}' &= 54.3 + j\,194.3 \\ Z_{oo}' &= 1820 + j\,219.8 \\ Z_{ow}' &= 937 + j\,206.6 \end{aligned}\right\} \text{Between circuits}$$

By test

$$Z_{ww} = 63.3 + j\,158.1$$

The difference in resistance between test and calculations indicates that there is some extraneous resistance somewhere. This extra resistance should also appear in the other measurements. However, in the calculations which follow no allowance for this extraneous resistance nor for the resistance to ground at the end points has been made. In actual practise an allowance should be made for these factors as indicated by experience or definite knowledge of the conditions.

2. Impedance:—One Conductor—Ground Return

TABLE C-1—VALUES OF P AND Q FOR CARSON'S FORMULA

	P				Q			
ϕ	$\theta = 0°$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$	$\theta = 0°$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
0.00010	0.39268	0.39268	0.39268	0.39269	4.9132	4.9132	4.9132	4.9131
0.00011	67	68	68	69	4.8655	4.8655	4.8655	4.8655
0.00012	67	68	68	69	4.8220	4.8220	4.8220	4.8219
0.00013	67	67	68	68	4.7820	4.7820	4.7820	4.7820
0.00014	67	67	68	68	4.7449	4.7449	4.7449	4.7449
0.00015	66	67	67	68	4.7104	4.7104	4.7104	4.7104
0.00016	66	67	67	68	4.6782	4.6782	4.6782	4.6782
0.00018	66	66	67	68	4.6193	4.6193	4.6193	4.6193
0.00020	65	66	67	68	4.5666	4.5666	4.5666	4.5666
0.00025	64	65	66	67	4.4551	4.4550	4.4550	4.4550
0.00030	63	64	65	66	4.3639	4.3639	4.3639	4.3639
0.0004	61	62	63	65	4.2201	4.2201	4.2201	4.2200
0.0005	58	60	62	64	4.1085	4.1085	4.1085	4.1085
0.0006	56	58	60	63	4.0174	4.0174	4.0174	4.1073
0.0007	53	56	58	62	3.9404	3.9403	3.9403	3.9403
0.0008	51	54	57	61	3.8736	3.8736	3.8735	3.8734
0.0009	49	52	55	59	3.8147	3.8147	3.8147	3.8146
0.0010	0.39246	0.39250	0.39253	0.39258	3.7621	3.7620	3.7620	3.7620
0.0011	44	48	52	57	3.7145	3.7144	3.7144	3.7143
0.0012	42	46	50	56	3.6710	3.6709	3.6709	3.6707
0.0013	39	43	48	55	3.6310	3.6309	3.6309	3.6308
0.0014	37	41	47	53	3.5939	3.5939	3.5938	3.5938
0.0015	35	39	45	52	3.5595	3.5594	3.5594	3.5593
0.0016	32	37	43	51	3.5272	3.5272	3.5271	3.5270
0.0018	28	33	40	49	3.4684	3.4683	3.4683	3.4682
0.0020	23	29	37	46	3.4157	3.4157	3.4156	3.4155
0.0025	0.39211	19	28	40	3.3043	3.3042	3.3041	3.3040
0.0030	0.39199	0.39209	20	35	3.2132	3.2131	3.2130	3.2129
0.004	77	0.39189	0.39203	23	3.0696	3.0695	3.0694	3.0692
0.005	53	69	0.39187	0.39210	2.9583	2.9581	2.9580	2.9577
0.006	30	49	70	0.39198	2.8674	2.8672	2.8670	2.8667
0.007	0.39107	28	53	86	2.7905	2.7903	2.7901	2.7897
0.008	0.39083	0.39108	37	75	2.7240	2.7238	2.7235	2.7231
0.009	61	0.39088	20	62	2.6653	2.6651	2.6647	2.6643
0.010	0.39038	0.39068	0.39103	0.39151	2.6129	2.6126	2.6122	2.6117

TABLE C-1—VALUES OF P AND Q FOR CARSON'S FORMULA—*Continued*

	P				Q			
ϕ	$\theta = 0°$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$	$\theta = 0°$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
0.010	0.39038	0.39068	0.39103	0.39151	2.6129	2.6126	2.6122	2.6117
0.011	0.39015	48	0.39088	39	2.5655	2.5661	2.5647	2.5642
0.012	0.38992	28	71	27	2.5222	2.5218	2.5214	2.5207
0.013	70	0.39008	54	15	2.4824	2.4820	2.2815	2.4809
0.014	47	0.38989	38	0.39102	2.4456	2.4452	2.4447	2.4440
0.015	24	68	21	0.39090	2.4114	2.4109	2.4103	2.4096
0.016	0.38902	50	0.39004	77	2.3793	2.3788	2.3782	2.3775
0.018	0.38857	0.38910	0.38972	54	2.3209	2.3203	2.3197	2.3188
0.020	0.38812	0.38870	0.38939	0.39029	2.2687	2.2681	2.2673	2.2664
0.025	0.38701	0.38772	0 38856	0.38969	2.1583	2.1575	2.1566	2.1554
0.030	0.3859	0.3868	0.3877	0.3891	2.0683	2.0673	2.0662	2.0648
0.04	37	48	61	79	1.9268	1.9255	1.9241	1.9222
0.05	0.3816	29	45	66	1.8175	1.8160	1.8142	1.8118
0.06	0.3795	0.3910	29	54	1.7286	1.7268	1.7247	1.7218
0.07	74	0.3792	0.3813	41	1.6539	1.6518	1.6493	1.6460
0.08	54	73	0.3797	29	1.5894	1.5870	1.5842	1.5804
0.09	34	55	81	16	1.5327	1.5301	1.5269	1.5228
0.10	0.3714	0.3737	0.3765	0.3803	1.4824	1.4794	1.4759	1.4713
0.11	0.3695	19	49	0.3791	1.4370	1.4338	1.4209	1.4249
0.12	76	0.3702	34	78	1.3957	1.3922	1.3881	1.3826
0.13	57	0.3685	18	65	1.3579	1.3542	1.3497	1.3438
0.14	38	67	0.3703	52	1.3231	1.3191	1.3143	1.3080
0.15	20	50	0.3688	40	1.2908	1.2866	1.2815	1.2747
0.16	0.3602	33	72	27	1.2608	1.2563	1.2509	1.2437
0.18	0.3566	0.3600	42	0.3701	1.2063	1.2013	1.1953	1.1873
0.20	0.3531	0.3567	0.3612	0.3675	1.1580	1.1526	1.1459	1.1371
0.25	0.3448	0.3488	0.3539	0.3612	1.0572	1.0506	1.0426	1.0319
0.30	0.3368	0.3413	0.3468	0.3548	0.9766	0.9690	0.9597	0.9472
0.40	0.3222	0.3270	0.3332	0.3422	0.8534	0.8439	0.8323	0.8163
0.50	0.3089	0.3139	0.3204	0.3300	0.7617	0.7506	0.7369	0.7180
0.60	0.2967	0.3016	0.3082	0.3180	0.6897	0.6773	0.6618	0.6402
0.70	0.2854	0.2903	0.2967	0.3065	0.6312	0.6176	0.6006	0.5766
0.80	0.2750	0.2797	0.2858	0.2953	0.5825	0.5679	0.5494	0.5235
0.90	0.2654	0.2697	0.2755	0.2846	0.5410	0.5255	0.5060	0.4782
1.00	0.2564	0.2604	0.2658	0.2742	0.5052	0.4890	0.4685	0.4392
1.1	0.2480	0.2516	0.2565	0.2644	0.4740	0.4572	0.4358	0.4051
1.2	0.2401	0.2433	0.2478	0.2548	0.4465	0.4292	0.4070	0.3752
1.3	0.2327	0.2355	0.2394	0.2458	0.4200	0.4042	0.3815	0.3487
1.4	0.2257	0.2282	0.2315	0.2370	0.4000	0.3819	0.3587	0.3251
1.00	0.2564	0.2604	0.2658	0.2742	0.5052	0.4890	0.4685	0.4392
1.1	0.2480	0.2516	0.2565	0.2644	0.4740	0.4572	0.4358	0.4051
1.2	0.2401	0.2433	0.2478	0.2548	0.4465	0.4292	0.4070	0.3752
1.3	0.2327	0.2355	0.2394	0.2458	0.4220	0.4042	0.3815	0.3487
1.4	0.2257	0.2282	0.2315	0.2370	0.4000	0.3819	0.3587	0.3251
1.5	0.2192	0.2192	0.2240	0.2287	0.3802	0.3618	0.3382	0.3040
1.6	0.2130	0.2146	0.2169	0.2207	0.3623	0.3437	0.3198	0.2849
1.8	0.2015	0.2024	0.2036	0.2058	0.3309	0.3121	0.2878	0.2522
2.0	0.1913	0.1913	0.1916	0.1922	0.3045	0.2856	0.2611	0.2251
2.5	0.1695	0.1681	0.1661	0.1631	0.2536	0.2348	0.2106	0.1749
3.0	0.1521	0.1494	0.1458	0.1400	0.2169	0.1988	0.1754	0.1411
4.0	0.1260	0.1217	0.1159	0.1066	0.1678	0.1514	0.1303	0.1000
5.0	0.1075	0.1023	0.0959	0.0839	0.1364	0.1218	0.1038	0.0763
6.0	0.0936	0.0878	0.0809	0.0697	0.1148	0.1019	0.0854	0.0623
7.0	0.0828	0.0771	0.0699	0.0587	0.0991	0.0874	0.0728	0.0526
8.0	0.0742	0.0687	0.0615	0.0507	0.0871	0.0765	0.0634	0.0456
9.0	0.0672	0.0619	0.0549	0.0445	0.0776	0.0681	0.0562	0.0403
10.0	0.0614	0.0562	0.0495	0.0397	0.0700	0.0612	0.0505	0.0361

$J = P + jQ$. $Z = J$ D abohms per cm.
To convert to practical ohms per mi., multiply by $5280 \times 12 \times 2.539954 \times 10^{-9} = 1.609315 \times 10^{-4}$.
For a 60-cycle wave, $Z = 0.24268\ (P + j\ Q)$ ohms/mi.

Data calculated from formulas given by John R. Carson, "Wave Propagation in Overhead Lines with Ground Return," reprint B-219 of the Bell Telephone Laboratories, November, 1926.
Data calculated by W. B. Jordan, April, 1930.

TABLE C-2—DETERMINATION OF λ

System or location	Reactance per mile: Line	Reactance per mile: Earth	$\times 10^{-14}$: λ
P P & L	0.976	0.499	1.8
Consumers	1.069	0.321	42.7
Appalachian	0.970	0.434	3.3
Alabama	0.984	0.724	0.065
Cross Keys[1]			42
Glens Falls[1]			175
Massilon[1]			36
Cross Keys[2]			33
U. S.-Canad [3]			200 to 0.1
Vicinity Chicago[4]			9
Eastern Texas			125
West Virginia			55
Penn.—Maryland			0.5
New Jersey			9
New Jersey			30
Ohio			100
Utah			30
Minn			10
Indiana			30
Maryland			8
New York			15
Penn			1
Penn			10
Wisconsin			1
Tenn			1
Ga			0.3
Ill			50
Near Ottawa			5
Oklahoma			100
Alabama			5

1. Gilkeson and Bowen, A. I. E. E. TRANS., Oct. 1930, p. 1375.
2. E. P. Peck, Discussion A. I. E. E. TRANS., Oct. 1930, p. 1379.
3. E. B. King, Discussion, A. I. E. E. TRANS., Oct. 1930, p. 1379. Values given are average of more than 30 tests and the statement is made that 75 per cent of the values of λ are in the range from 100 × 10^{-14}, to 10 × 10^{-14}.
4. This and all the values following given through the courtesy of the joint D & R Committee sponsored by the A. T. & T. and NELA.

Let Z_{w-g} = Impedance single conductor (with image return); generator to fault, one way only. Calculated by the usual method using a spacing of 2 h.

$$Z_{w-g} = 27.1 + j\,89.3$$

For the impedance of the ground return (Z_{g-w}) a little consideration is needed because the line operates at 25 cycles while the curves in Fig. C-2 and C-3 are for 60 cycles. The curves can be entered with an effec-

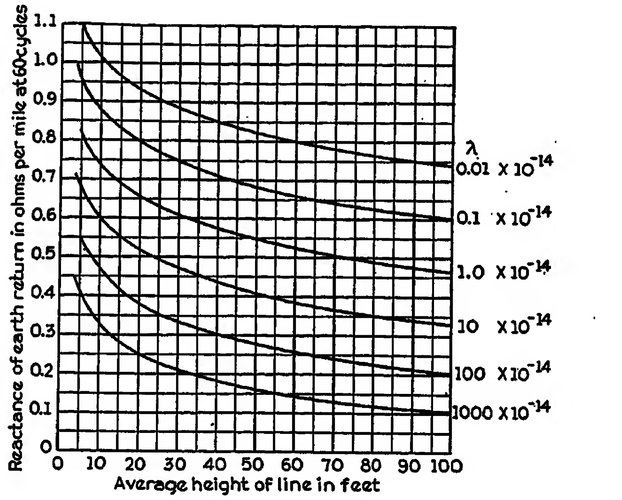

FIG. C-2—THESE CURVES GIVE THE REACTANCE OF THE RETURN PATH THROUGH THE EARTH FOR DIFFERENT VALUES OF THE EARTH CONDUCTIVITY. CALCULATIONS BASED ON CARSONS ANALYSIS.

TABLE C-3

Values of Resistance and Reactance at 60 Cycles of Return Path through the Earth for Various Heights of Line Conductors and Earth Conductivity

λ = 0.01				λ = 0.1				λ = 1.0			
ϕ	h	x	r	ϕ	h	x	r	ϕ	h	x	r
0.0002	4.77	1.10822	0.09529	0.0005	3.77	0.99705	0.09527	0.002	4.77	0.82892	0.09519
0.0004	9.53	1.02413	0.09528	0.001	7.54	0.91299	0.09524	0.004	9.53	0.74493	0.09507
0.0006	14.30	0.97494	0.09527	0.002	15.07	0.02892	0.09519	0.006	14.30	0.69586	0.09496
0.0008	19.07	0.93005	0.09525	0.003	22.61	0.77978	0.09513	0.008	19.07	0.67106	0.09474
0.0012	28.60	0.89088	0.09523	0.004	30.15	0.74493	0.09507	0.012	28.60	0.61209	0.09463
0.0016	38.13	0.85598	0.09521	0.005	37.68	0.71792	0.09502	0.016	38.13	0.57741	0.09441
0.002	47.66	0.82892	0.09519	0.006	45.22	0.69586	0.09496	0.02	47.66	0.55057	0.09419
0.0025	59.58	0.80189	0.09516	0.008	60.29	0.67106	0.09485	0.025	59.58	0.52378	0.09392
0.0030	71.50	0.77978	0.09513	0.010	75.36	0.63410	0.09474	0.030	71.50	0.50194	0.09365
0.0040	95.33	0.74493	0.09507	0.012	90.44	0.61209	0.09463	0.040	95.33	0.46760	0.09312
0.0050	119.16	0.71792	0.09502	0.014	105.51	0.59350	0.09452	0.050	119.16	0.44107	0.09261

$\phi = 4196\, h \sqrt{\lambda}$.

TABLE C-3—*Continued*

λ = 10				λ = 100				λ = 1000			
ϕ	h	x	r	ϕ	h	x	r	ϕ	h	x	r
0.005	3.77	0.71792	0.09502	0.02	4.77	0.55057	0.09419	0.05	3.77	0.44107	0.09261
0.01	7.54	0.63410	0.09474	0.04	9.53	0.46760	0.09312	0.1	7.54	0.35975	0.09013
0.02	15.07	0.55057	0.09419	0.06	14.30	0.41906	0.09210	0.2	15.07	0.28103	0.08569
0.03	22.61	0.50194	0.09365	0.08	19.07	0.38572	0.09110	0.3	22.61	0.23700	0.08173
0.04	30.15	0.46760	0.09312	0.12	28.60	0.33871	0.08921	0.4	30.15	0.20710	0.07819
0.05	37.68	0.44107	0.09261	0.16	38.13	0.30597	0.08741	0.5	37.68	0.18485	0.07496
0.06	45.22	0.41906	0.09210	0.2	47.66	0.28103	0.08569	0.6	45.22	0.16738	0.07200
0.08	60.29	0.38572	0.09110	0.25	59.58	0.25656	0.08368	0.8	60.29	0.14136	0.06674
0.10	75.36	0.35975	0.09013	0.3	71.50	0.23700	0.08173	1.0	75.36	0.12260	0.06222
0.12	90.44	0.33871	0.08921	0.4	95.33	0.20710	0.07819	1.2	90.44	0.10836	0.05827
0.14	105.51	0 32109	0.08829	0.5	119.16	0.18485	0.07496	1.4	105.51	0.09707	0.05477

$\phi = 4196\, h \sqrt{\lambda}$.

tive height $h' = \sqrt{\frac{25}{60}}$. The value read from the curves must also be changed from 60 cycles to 25 cycles by direct proportion.

$$h' = 38 \times 0.646 = 24.5$$

Read the curves for $\lambda = 1 \times 10^{-14}$ and $h = 34.5$ for the unit values of r_g and x_g at 60 cycles, from which the impedance of 230 mi. of line at 25 cycles is found to be

$$Z_{g-w} = 9.1 + j\,60.6$$

The desired impedance is the sum of these two impedances as they are in series,

$$Z_{wg} = 36.2 + j\,149.9$$

When the tests were made the ground wires on both circuits affected the results. With 4 ground wires the expression for the desired impedance is from Table A-1

$$Z_{14} = Z_{wg} - \frac{M_{ow}^2}{Z_{og} - \frac{3}{8} Z_{oo}}$$

and the proper average values of impedances must be inserted into the expression. For the ground wire on the same circuit as the conductor the self impedance of the ground wire should be based on its image return using a spacing of 2 H, but the same ground impedance as for the conductor should be used, so that

$$Z_{og} = 919 + j\,166.2$$

For the other two ground wires the calculations should be made upon the basis of the image distance from the ground wire on one circuit to the conductors of the other circuit. The angle θ, Fig. C-1, is about 60 deg. but the value ϕ is low enough so that the curves can be used, and then

$$Z_{og}' = 919 + j\,162.4$$

For the calculation,

$$\text{Average } Z_{og} = 919 + j\,164.3$$
$$\text{Average } Z_{oo} = 1220 + j\,206.4$$
$$\text{Average } Z_{ow} = 937 + j\,183.9$$

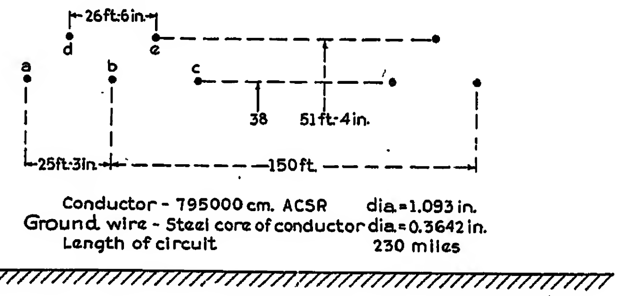

FIG. C-4

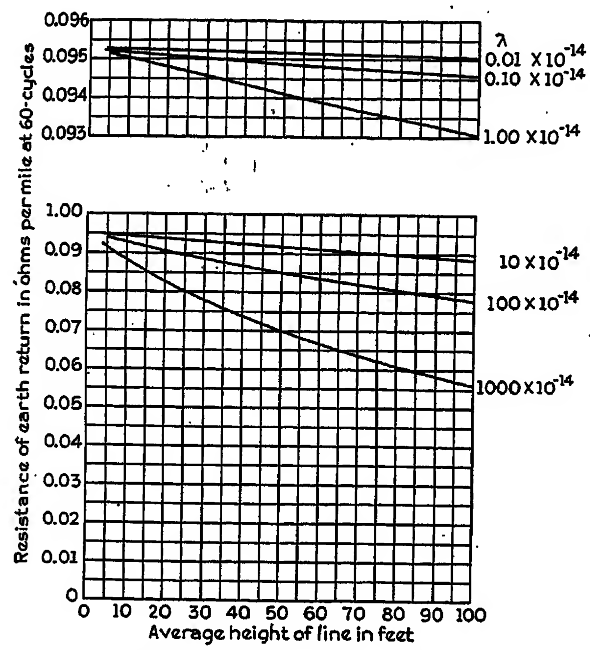

FIG. C-3—THESE CURVES GIVE THE RESISTANCE OF THE RETURN PATH THROUGH THE EARTH FOR DIFFERENT VALUES OF THE EARTH CONDUCTIVITY. CALCULATIONS BASED ON CARSONS ANALYSIS

With these values there results

$$Z_{14} = 36.2 + j\,149.9 - (9.1 + j\,65.7) \times 0.263$$
$$= 33.8 + j\,132.6 \text{ calculated}$$
$$= 51.2 + j\,138.2 \text{ tested}$$

The calculations may also be made by the more rigorous method of the appendix. In which case the impedance is given by the following expression which is obtained from equation (21) of the appendix:

$$Z_{14} = A_a - \frac{I_3}{I_a} B_{a3} - \frac{I_4}{I_a} B_{a4}$$

(See Appendix for usage of A and B)

$A_a = Z_{wg} = 36.2 + j\,149.9$—Self impedance of line wire.

$A_3 = A_4 = 464 + j\,117.7$—Self impedance of two ground wires in parallel.

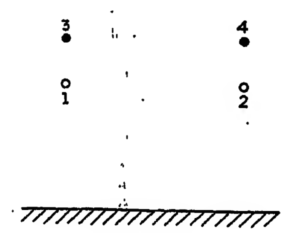

FIG. C-5

$B_{a3} = 9.1 + j\,74.0$ —Mutual impedance as indicated.

$B_{a4} = 9. + j\,52.8$ —Mutual impedance as indicated.

$B_{34} = +. + j\,52.6$ —Mutual impedance as indicated.

From equation (21) the values of currents are found as

$$\frac{I_3}{I_a} = \frac{B_{a3}}{A_3} \, \frac{1 - \frac{B_{a4}}{B_{a3}} \frac{B_{34}}{A_4}}{1 - \frac{B_{34}^2}{A_3 A_y}}$$

$$\frac{I_4}{I_a} = \frac{B_{a4}}{A_4} \frac{1 - \frac{B_{a3}}{B_{a4}} \frac{B_{34}}{A_3}}{1 - \frac{B_{34}^2}{A_3 A_4}}$$

$$\frac{I_3}{I_a} = 0.01549 \qquad \frac{I_4}{I_a} = 0.1102$$

It should be noted that $I_3 = I_a \frac{B_{a3}}{A_3}$ and $I_4 = I_a \frac{B_{a4}}{A_4}$ very closely for the spacings of this line.

$$Z_{14} = 33.8 + j\,132.4$$

This is the same as by the previous calculation and the total ground wire current is 0.265 as compared with 0.263.

3. Impedance:—Three Conductors in Parallel—Ground Return

By the approximate method from Table A-1

$$Z_{14} = 33.8 + j\,132.6$$

$$\frac{1}{3} Z_{ww} = 18.1 + j\,52.8$$

$$\begin{aligned} Z_{34} &= 15.7 + j\,79.8 \quad \text{calculated} \\ &= 34.6 + j\,87.4 \quad \text{tested} \end{aligned}$$

By the rigorous method from equation (**21**) or (**23**)

$$Z_{34} = \frac{A_a + 2B_a}{3} - \frac{I_3}{I} B_{a3} - \frac{I_4}{I} B_{a4}$$

There are two values of B_a that might be used. Since the line conductors are transposed the same current flows in each and the effect of the ground is the same on each of the three conductors, and so

$$B_a = j\,230 \times 0.1165 \text{ Log} \frac{2h}{S} + B_{g \cdot w}$$

$$\begin{aligned} &= j\,10.1 + 9.1 + j\,60.6 \\ &= 9.1 + j\,70.7 \end{aligned}$$

and then

$$Z_{34} = 15.7 + j\,79.6$$

There are those who would prefer to use the image distance in calculating B_{g-w}. The average image distance is 85.45 ft. and from which

$$B_a = 9.1 + j\,70.7$$

which gives the same result.

4. Impedance:—Two Circuits in Parallel.

For this we need to know the impedance of one circuit out and the other return, neglecting earth and ground wires. It is calculated by the usual methods but with three conductors in parallel and transposed.

$$Z_{ww}' = 18.1 + j\,88.8 \text{ (3 in parallel)}$$

and then, by the approximate method using Table A-1

$$Z_{34} = 15.7 + j\,79.8$$

$$\frac{1}{4} Z_{ww}' = 4.5 + j\,22.2$$

$$\begin{aligned} Z_{64} &= 11.2 + j\,57.6 \text{ calculated} \\ &= 28.1 + j\,62.6 \text{ tested} \end{aligned}$$

By the rigorous method of the Appendix, from an adaption of equation (**23**)

$$Z_{64} = \frac{A_a + B_a + B_a'}{6} - \frac{I_3}{I} B_{a3} - \frac{I_4}{I} B_{a4}$$

The mutual factor B_a' is based on the average image distance between the line wires of one circuit and the ground wires of the other.

$$B_a' = 9.0 + j\,52.6$$

and then

$$Z_{64} = 11.2 + j\,59.0$$

5. Impedance:—Out One Circuit—Return Other.

By the rigorous method using equation (**27**) with conductors grouped as in Fig. C-5

$$Z_5 = A_1 - 2B_{12} + A_2 - 2\frac{I_3}{I_1}(B_{13} - B_{14})$$

$$\frac{I_3}{I_1} = \frac{B_{13} - B_{14}}{A_3 - B_{34}}$$

$A_1 = A_2 = \dfrac{A_a + 2B_a}{3} = 18.1 + j\,97.1$ self impedance, 3 line wires in parallel

$A_3 = A_4 = 464 + j\,117.7$ self impedance, 2 ground wires in parallel

$B_{12} = B_a' = 9.0 + j\,52.6$ mutual impedance, as indicated

$B_{13} = B_{a3} = 9.1 + j\,74.0$ mutual impedance, as indicated

$B_{14} = B_{a4} = 9.0 + j\,52.8$ mutual impedance, as indicated

$B_{34} = B_{34} = 9.0 + j\,52.6$ mutual impedance, as indicated

and then

$$\frac{I_3}{I_1} = 0.0461$$

$$Z_5 = 18.2 + j\,89.0 - 0.98$$

$$\begin{aligned} Z_5 &= 18.2 + j\,88.0 \text{ calculated} \\ &= 46.6 + j\,94.0 \text{ tested} \end{aligned}$$

By the approximate method this was calculated as Z_{ww}' under paragraph (4) preceding.

6. Impedance:—Out on One Circuit—Part Way Back on Other.

For this test the fault was located 131.9 mi. from the

generating end. For this condition there are in effect two impedances in series, so that

$$Z_6 = \frac{131.9}{230} Z_{34} + \frac{98.1}{230} Z_5$$

$$Z_6 = 16.8 + j\,83.2 \text{ calculated}$$
$$= 33.0 + j\,91.7 \text{ tested}$$

7. Impedance:—Fault on One Circuit—(Fig. D-1)

For the values of the constants on this line the following equations hold very closely. This is based on an observation made in regard to I_3 and I_4 in the calculations of condition (2)

$$\frac{I_2}{I_1} = 1 + \frac{Z_5}{A_1 - B_{12} - (B_{13} B_{14}) \dfrac{B_{13} - B_{14}}{A_3}}$$

$$\frac{E_1}{I_1} = A_1 + Z_5 - \frac{B_{13}^2 + B_{14}^2}{A_3} + \frac{I_2}{I_1}\left(B_{12} - 2\frac{B_{13} B_{14}}{A_3}\right)$$

When the substitutions are made there results

$$Z_7 = 7.43 + j\,45.4 \text{ calculated}$$
$$= 22.7 + j\,41.3 \text{ tested.}$$

The writer wishes to thank the Pennsylvania Power and Light Company, the Consumers Power Co., the Appalachian Electric Power Co., and the Alabama Power Co., for their assistance in making the tests for the confirmation of the analysis; also the Hydro-Electric Power Commission of Ontario for permission to use its test data for the sample calculation.

D. Appendix

General Solution. For those who desire a more general solution the following is offered. Unfortunately this solution requires one assumption and that is that the earth is homogeneous so that the earth conductivity is everywhere uniform. Consider in Fig. A-1 a system of "*m*" line wires (indicated by number subscript) and "*n*" ground wires (indicated by letter subscript). The circuit voltage of the line wires is E_1, E_2, etc. and of the ground wires is 0. Then

$$\left.\begin{aligned} E_1 &= I_1 A_1 + I_2 B_{12} + I_3 B_{13} + \ldots \\ &\quad + I_a B_{1a} + I_b B_{1b} + \ldots \\ E_2 &= I_1 B_{12} + I_1 A_2 + I_3 B_{23} + \ldots \\ &\quad I_a B_{2a} + I_b B_{2b} + \ldots \\ E_3 &= \text{etc.} \\ \text{etc.} &= \end{aligned}\right\} \quad (21)$$

$$\left.\begin{aligned} 0 &= I_1 B_{a1} + I_2 B_{a2} + I_3 B_{a3} + \ldots \\ &\quad + I_a A_a + I_b B_{ab} + \ldots \\ 0 &= I_1 B_{b1} + I_2 B_{b2} + I_3 B_{bg} + \ldots \\ &\quad I_a B_{ab} + I_b A_b + = \\ 0 &= \text{etc.} \\ \text{etc.} &= \end{aligned}\right\} \quad (22)$$

The solution of this set of equations gives the current distribution among the line and ground wires and the net impedance voltage for any circuit configuration.

The process of solution is as follows. Solve those equations (22) involving the ground wires and which are equated to 0 for the ground wire currents in terms of the line wire currents. Substitute these values of ground wire currents into the equations (21) for the line voltage and then solve for the line wire currents.

In this set of equations

A = self impedance of conductor with ground return.
$A = Z_w + Z_g$
Z_w = self impedance of conductor with image return.
Z_g = Carsonian self impedance of the earth return.
B = mutual impedance between pairs of conductors with ground return.
$B = M_w + M_g$
M_w = mutual impedance of a pair of conductors with image return.
M_g = Carsonian mutual impedance of a pair of conductors with earth return.

The quantities A and B are to be taken for the full

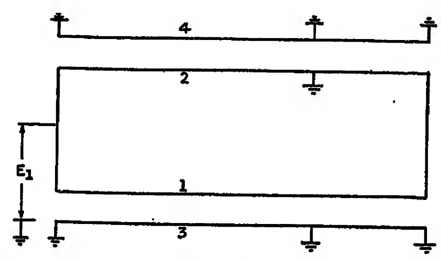

Fig. D-1

length of the line. The application of the equations requires that the average height and spacing of the conductors be known; and that the contact resistance between earth and wire be known, and properly inserted into the expressions.

For any given configuration of line wires and ground wires the results as obtained by this method can be transposed into those obtained in section A by assuming transportation of line conductors and that θ is small enough that Z_g can be used for M_g.

If it is assumed that the line wires are transposed so that they each carry the same current the zero phase sequence reactance is

$$\left.\begin{aligned} Z_o &= A + 2B && \text{no ground wire} \\ &= A + 2B - 3\frac{I_4}{I_1} B_4 && \text{1 ground wire} \\ &= A + 2B - 3\frac{I_4 B_4 + I_5 B_5}{I_1} && \text{2 ground wires} \\ &= A + 2B - 3\frac{I_4 B_4 + I_5 B_5 + I_6 B_6}{I_1} && \text{3 ground wires} \end{aligned}\right\} \quad (23)$$

In these equations I_4, I_5, etc. are the ground wire currents and are obtained from the equations (22) by determinant solution. The quantities B_4, B_5, etc. are the average mutual impedances between each of the ground wire and the line wires. If an average value for mutual impedance between line wires and ground wires were used these equations would, as just mentioned, reduce to those developed under section A.

Parallel Circuits. The solution for parallel circuits will be indicated in a general way only. In Fig. D-1 a fault is indicated on one circuit. In this analysis the line conductors are considered to be transposed so that the quantities A and B involving the line conductors and ground wires may be taken as average values. Note that A and B represent circuit values and not unit values. The method of solution is as follows.

At left of fault

$$\left.\begin{aligned} E_1 &= I_1 A_1 + I_2 B_{12} + I_3 B_{13} + I_4 B_{14} + E \\ E_2 &= I_1 B_{12} + I_2 A_2 + I_3 B_{23} + I_4 B_{24} \end{aligned}\right\} \quad (24)$$

$$\left.\begin{aligned} 0 &= I_1 B_{13} + I_2 B_{23} + I_3 A_3 + I_4 B_{34} \\ 0 &= I_1 B_{14} + I_2 B_{24} + I_3 B_{34} + I_4 A_4 \end{aligned}\right\} \quad (25)$$

At right of fault

$$\left.\begin{aligned} E_1' &= I_1' A_1' + I_2' B_{12}' + I_3' B_{13}' + I_4' B_{14}' \\ E_2' &= I_1' B_{12}' + I_2' A_2' + I_3' B_{23}' + I_4' B_{24}' \end{aligned}\right\} \quad (26)$$

$$\left.\begin{aligned} 0 &= I_1' B_{13}' + I_2' B_{23}' + I_3' A_3' + I_4' B_{34}' \\ 0 &= I_1' B_{14}' + I_2' B_{24}' + I_3' B_{24}' + I_4' A_4' \end{aligned}\right\} \quad (27)$$

$$E = E_1' - E_2'$$

Solve equations (27) for the ground wire currents I_3' and I_4'. Substitute these values in equations (26)—and note that $I_1' = I_1$, and $I_2' = -I_1$. The value of E is now found.

Now solve equations (25) for the ground wire currents I_3 and I_4. Substitute these together with the value of E in equations (24) and then solve for I_1 and I_2. The zero phase sequence impedance is $\dfrac{E_1}{I_1 + I_2}$ not forgetting that $E_2 = E_1$.

Discussion

P. A. Jeanne: The method of developing the formulas for zero sequence impedance given by the author in the main body of the paper, involves initially an arbitrary separation of the impedance of a circuit formed by an overhead wire with ground return into two parts, one assigned to the overhead portion and the other to the underground portion of the circuit. In the end it is then necessary to rearrange or combine the terms to form impedances of complete ground return or metallic circuits so that the terms in the formulas will be measurable quantities. It therefore seems to me that the method outlined in the appendix, which only involves the use of complete ground return circuit impedances, is the preferable and more direct. The Joint Subcommittee on Development and Research of the N. E. L. A. and Bell System has been using formulas developed by this latter method.

With regard to the influence of earth resistivity, I should like to point out that although large variations in this quantity produce relatively small variations in the zero sequence impedance and even less in total fault current it does affect ground wire currents quite appreciably; for example, taking a 220-kv. line with two high conductivity (186,000-cir. mil ACSR) ground wires, a change in earth resistivity from 1,000 to 100,000 ohms per cm. cube (conductivity of 10^{-12} to 10^{-14} abmhos per cm. cube) changes the zero sequence impedance about nine per cent, while it changes the ground wire current about 26 per cent on the assumption of the same fault current in the two cases.

F. J. Grueter: One of the simplifying assumptions made by Mr. Clem in developing his method of calculating zero sequence impedance is that of uniform current along the length of the ground wire, and he cites some tests on the Turner-Logan line which agree with this assumption. This condition holds only when the ground wire is terminated in impedances to ground which are small compared to the resistance to ground of the tower footings. Such an assumption is valid when the ground wires are connected to station ground at both ends and the fault is outside of the section of line for which the zero sequence impedance is being determined.

Recently the Joint Subcommittee on Development and Research of the N. E. L. A.-Bell System made a series of tests to determine the variation of ground wire current along the length of the line. Fig. 1 gives some of the results of these tests which were made on a 220-kv. transmission line, equipped with two 200,000-cir. mil ACSR ground wires in a region where the earth resistivity was about 20,000 ohms per cm. cube, (earth conductivity of 5×10^{-14} abmhos per cm. cube) and where the average tower footing resistance is about 6.8 ohms. At one end of the line, where the ground wires were terminated, the three-phase conductors were multiplied and energized to ground, while at a point about 23 miles distance, the conductors were short-circuited together and to ground through a tower. The ground wires extended about three miles beyond this point of grounding.

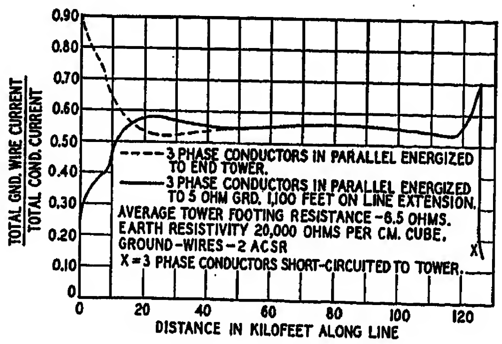

Fig. 1—Variation of Ground-Wire Current Along a Transmission Line when Energized with Ground-Return Current

The ground wire current observed was a stepped function, due to the lumped leakage of the ground wires at the towers. The smooth curves on the plot were drawn giving averaged results and the general shape of these curves, including the overshoot, has been checked from theoretical considerations. The solid line presents the ground wire currents observed when the conductors were energized to a 5-ohm ground about 1,100 ft. beyond the end of the line. The dotted line gives the values when the line is energized directly to the end tower footing. At the energized end, the increase or decrease of ground wire currents, over the value at the middle of the line, persists for a considerable distance. Due to the continuation of the ground wires beyond the point of fault, the change in ground wire current at the fault end was observed over considerably shorter distances.

It should be pointed out that the distribution of the currents in the ground wire at the energized end is not representative of the condition which exists when a ground wire is connected to the

station ground of low resistance, relative to the tower footing resistance. Under the latter arrangement the current in the ground wire entering the station would be equal to that in the middle of the line. Under these conditions, the ground wire current, with the exception of the case of a fault very close to the substation, will be of substantially constant value practically throughout the length of the line. In the case where the ground wire terminates on a tower not connected to the station structure, the solid curve of the slide would be typical of the resulting conditions.

The results of tests which are referred to here are a part of a study which has been conducted by the Joint Subcommittee on the general subject of the magnitude and distribution of currents induced in grounded conductors due to their presence in a magnetic field produced by current in a ground return circuit. The effects of these currents in grounded conductors on the zero sequence impedance of power circuits and of their effects on the voltages induced in nearby communication circuits have also been studied and it is expected that this information will be made generally available to the industry in a short time.

George Waschecks: In determining the zero phase sequence impedance of transmission lines Mr. Clem treats the current in the ground wires as a constant quantity throughout the length of the line. Where ground wires are connected to a station ground of low resistance compared to tower grounds, and the fault is distant from the station it is quite accurate for the calculations of the zero phase sequence impedance to assume the current in the ground wire to be constant along the line. However, with a fault within a few miles of the station the constant value of ground wire current may not exist for any appreciable length. A theoretical investigation has been made, in conjunction with work of the Joint Subcommittee on Development and Research of the N. E. L. A. and Bell System, of the possible effect of this condition on the zero phase sequence impedance.

For the solution of the impedance of such a system, in which the ground wires and towers assume the form of a ladder network (tower distances and ground resistances assumed constant) an evaluation is required of the average current in the ground wires. By means of finite difference equations the current in the nth section may be derived and the summation of the currents in all the sections becomes a geometrical series which may readily be evaluated. This, divided by the number of sections, becomes the average or equivalent constant current to be inserted in the circuit equations.

The resulting formula for the zero phase sequence impedance has been put into the following form, consisting of three terms:

$$Z_O = Z_P - n\frac{Z_{PG}^2}{Z_G} + n\,Z_G\left[1 - \frac{Z_{PG}}{Z_G}\right]^2\left[\frac{m_1^{2K} - 1}{K\; m_1^{2K-1}(m_1^2 - 1)}\right]$$

where:

Z_O = zero phase sequence impedance.

Z_P = grounded impedance of one phase wire with the other phase wires carrying equal currents, and in the absence of ground wires.

Z_G = series impedance of ground wires with earth return. (Equal and constant currents in all ground wires, no current in phase wires).

Z_{PG} = mutual impedance between ground wires and phase wires.

R = resistance of tower to ground.

s = length of section between towers in miles.

K = number of sections.

n = number of phase wires.

$$m_1 = 1 + \frac{s\,Z_G}{2R} + \frac{1}{2}\sqrt{\frac{4\,s\,Z_G}{R} + \left(\frac{s\,Z_G}{R}\right)^2}$$

The formula is based on the assumption of solidly grounded ground wires at Station C and a fault to a tower with the line extending indefinitely beyond, although the formula for any combination of the end groundings is determinable.

It is seen that the first term is the zero phase sequence impedance in the absence of ground wires, the second, the additional term that would be obtained if the ground wires were solidly grounded at each end of the circuit involved, and the third, the contribution of the varying ground wire currents near the point of grounding.

An application of this formula was made to a three-phase transmission line of flat construction of 28.5-ft. spacing with two ACSR ground wires of the same spacing placed 13.9 ft. symmetrically above the phase wires. The three factors in the above equation were evaluated for assumed tower ground resistances of 5 and of 50 ohms each. The results may be seen in Fig. 2. A is the zero phase sequence impedance per mile with no ground wire, B the term to be subtracted vectorially with constant current assumed in the ground wire (the ends solidly grounded), and C

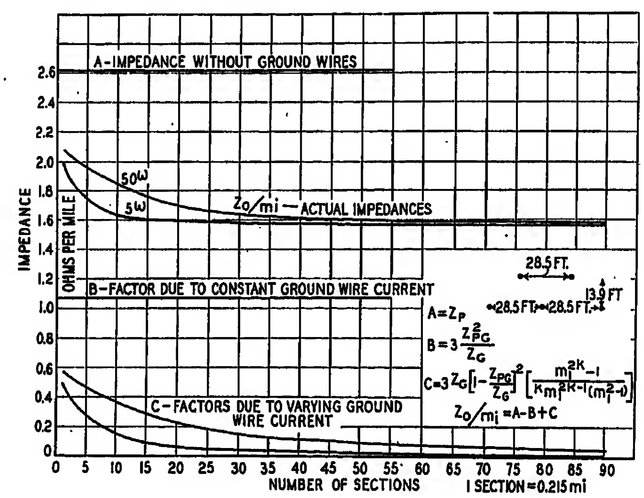

FIG. 2—ZERO-PHASE SEQUENCE IMPEDANCES

(Variation of zero-phase sequence impedance per mile and its components *versus* circuit length for two values of tower ground resistances. Ground wire solidly grounded at station O. Fault taken to any tower with line indefinitely long beyond used transmission line configuration shown below with phase wires 70 ft. above ground. Frequency 60 cycles per sec.)

the correction due to the varying currents in the end sections near the fault. Parts A and B are constant but C is variable with length and with tower ground resistance. The curve marked $Z_O/Mi.$ is the vector sum of all three components and is the actual zero phase sequence impedance per mile. Thus, with a fault five sections from the station (about one mile) the zero phase sequence impedance per mile with 5-ohm grounds is 1.74 ohms per mile; with 50-ohm grounds 1.96 ohms per mile. This is in contrast to 1.56 ohms per mile should the station ground wire current have been assumed. The following table shows a comparison between that of the net zero phase sequence impedance per mile against that assumed with the constant current of solidly grounded ground wires.

It is to be observed that the error is quite small for a fault distant from the station and that for the two cases illustrated the effect of increased current in the ground wire near the fault is to increase the zero phase sequence impedance. It appears that the additional impedance of each tower to ground over that were the fault tower solidly grounded is sufficient to offset the reduction in impedance to be expected by the larger ground wire current.

No. of sections	Length miles	Zo per mi. with constant ground wire current	Zo per mi. allowing for varying current in ground wire		Per cent increase in magnitude	
		5 ohms or 50 ohms	5 ohms	50 ohms	5 ohms	50 ohms
2	1/2	$1.56\underline{/75.6}$	$1.92\underline{/67.4}$	$2.05\underline{/67.8}$	23.0	31.5
5	1	"	$1.74\underline{/68.4}$	$1.96\underline{/67.4}$	11.5	25.5
9	2	"	$1.65\underline{/71}$	$1.86\underline{/67.6}$	5.8	19.2
14	3	"	$1.61\underline{/72}$	$1.77\underline{/68.2}$	3.2	13.5
23	5	"	$1.59\underline{/74}$	$1.67\underline{/69.5}$	1.9	7.0
47	10	"	$1.58\underline{/74.5}$	$1.61\underline{/72.2}$	1.3	3.2

H. M. Trueblood: Carson's expression for the self-impedance of a ground-return circuit is

$$Z_{iw} + j\,2\,\omega \log \frac{\rho''}{a} + 4\,\omega \int_o^\infty (\sqrt{\mu^2 + j} - \mu)\, e^{-2h'\mu}\, d\,\mu$$

Mr. Clem speaks of the second term in this formula as representing the reactance of the wire with image return. More fully expressed, it is the reactance, external to the wire, of the complete ground-return circuit *on the hypothesis that the earth is of infinite conductivity.* On this hypothesis, the third term of the formula would vanish, and there would be neither flux nor current in the body of the earth. If the wire also were of infinite conductivity, the second term would represent the entire impedance, since Z_{iw}, which is the internal impedance of the wire, would in that case likewise be zero. All flux would thread between the surface of the wire and that of the earth, there being none within the material of either, and, of course, no resistance anywhere.

Should we then regard the first and third terms of the formula as "correction" terms, introduced to take care of departures from the ideal condition in which both the wire and the earth have infinite conductivity? This is a permissible point of view, but the third term does not strictly represent an "impedance of the earth," in the sense in which Z_{iw} represents an impedance of the wire, *viz.*, its *internal* impedance. The third term, as Carson points out, formulates the effect of the finite conductivity of the earth. This effect does not consist solely in causing flux and current to appear within the body of the earth; it consists also, in part, in changing the amount and distribution of the *external* flux, *i. e.*, the flux between the surfaces of the wire and the earth. The third term, therefore, is not directly or simply identified with an impedance associated with the earth.

However, it is entirely possible to assign a definite meaning to the expression "internal impedance of the earth" and the conception is of some value in the physical visualization of a circuit consisting of a straight long horizontal wire with ground return. In such a circuit, the internal impedance of the earth, per unit length of the circuit, may be taken as the electric force, parallel to the wire, at the earth's surface directly below the wire. This is identical with the voltage induced per unit length in a second grounded conductor lying on the earth's surface directly below the first wire, when unit current flows in the ground-return circuit of which the first wire is one side.

With this understanding, the *total* impedance of a ground-return circuit can be written

$$Z_{iw} + j\,\omega\,\Phi_o + Z_{ie}$$

in which Z_{iw} is the internal impedance of the wire, identical with the first term of Carson's expression, Z_{ie} is the internal impedance of the earth and $j\ \omega\ \Phi_o$ is the voltage drop due to the flux threading between the surface of the wire and the surface of the earth. The last two terms are not the same respectively as the last two terms of Carson's formula, although, of course, the *sum* of the last two terms is the same in both expressions.

This expression is of exactly the same form as appears in writing the total self-impedance of a circuit consisting of two *wires*. It is true, of course, that Z_{ie}, the internal impedance of the earth, depends on the height of the wire above the earth, whereas, with wires, we commonly think of the internal impedance as a property of the wire alone, at a given frequency. This difference is apparent rather than real, however, as is obvious if we think for a moment of such problems as that of the proximity effect in two closely adjacent wires at high frequencies, where the current distribution, and hence the internal impedance, of either wire, is affected by its separation from the other. The ground-return circuit problem, in fact, is really nothing but a case of proximity effect, as can be seen by first imagining two wires forming a "metallic" circuit, and then allowing the radius of one to increase indefinitely while the distance between the surfaces remains constant. The wire with indefinitely expanding radius ultimately represents the earth, and the "proximity effect" consists essentially in the resulting non-uniform current distribution in this conductor, and associated phenomena. Cases can be easily imagined in which a noticeable proximity effect would be maintained in the wire whose radius remains constant, thus making the analogy between the two-wire circuit and the wire-earth circuit complete. In other cases, also, the usual simple conception of internal impedance becomes impossible and must be generalized; for example, in circuits of which railroad rails form a part.

C. F. Wagner: Mr. Clem's paper is particularly valuable because it collects together a large mass of data corroborating Carson's formulas for the calculation of ground return circuits. His Table C-2 enumerates the available data for ground conductivity which forms a basis for the calculation of the ground return impedances.

I have been using Carson's formulas, also, for this purpose but the form is somewhat different from that which Mr. Clem presents. This is given in the April, 1931 issue of *The Electric Journal*, Vol. 28, No. 4, pages 241-244.

S. Whitehead and P. D. Morgan: The present paper is a valuable contribution to the important subject of the impedance of a line when one or more phases is connected to earth as may occur under fault conditions. The author has performed a real service to engineering in publishing Table C1 by means of which Carson's formula may be readily computed for any special case. The analysis employed by the author in treating the effect of earth wires in parallel with the earth differs from ours in that the partial impedance coefficients the L's and M's of the first part of the A's and B's of the Appendix are eliminated by expressing them in terms of complete coefficients relating to go and return circuits, *i. e.* the Z's. We, on the other hand (*J. I. E. E.*, March 1930, page 367 *et seq*) have used the partial coefficients as determinable quantities by relating them to the plane of the earth as assumed neutral plane. The present author's method is superior to ours in that the application of formulas such as Carson's or Pollaczek's is simpler.

In the case of Rudenberg's formula for the impedance of line and earth return we may put—

$$\text{Impedance} = \left[\pi^2 f + \omega j \left[1 + 2 \log h \left(\frac{2}{y\,K\,h}\right)\right] + R + 2\,\omega\,j \log h \frac{h}{r}\right] 10^{-9} \text{ ohms/cm.}$$

Where

f = frequency

$\omega = 2\,\pi\,f$

$K = 2\pi \sqrt{f/u}$

u = specific resistance of the earth in abohms

h = height of conductor in cm.

r = over-all radius of conductor in cm.

y = 1.781

R = resistance of conductor in abohms per cm.

The first two terms may be taken as referring to the earth and the second to the conductor with the earth as a neutral plane. In such a case our method and that of the author lead to identical results. Our method is, however, easier to apply to a multiple earthed earth wire or an underground cable where the current in the earth wire or cable sheaths varies along the line. Both the methods suffer from the disadvantage that they may be rigorously applied only to concentrated currents such as in a conductor since otherwise the distributive and commutative laws of the coefficients no longer hold. Fortunately, however, the aerial field of the earth current is usually negligible while the earth field of the aerial conductors is rapidly attenuated and may also be neglected. We used Rudenberg's formula on account of simplicity but for most lines it agrees fairly closely with those of Carson and Pollaczek.

The theory of Rudenberg is a simplification of the more rigorous methods of the other two authors and its errors appear when the induced voltage at a distance from the power line is required, the Rudenberg theory giving too concentrated a current distribution in the earth. For the case of the English "grid" lines examined by us (*ibid.* pp. 406-7) it has been verified that the present author's methods give the same results as ours and this will be the case for nearly all power lines, divergence is only to be anticipated for lines at a great height from the ground or for frequencies considerably above those used for normal power transmission. It would appear, therefore, in view of the extensive tests made by the author in America and ourselves in England that the calculations of the earth or zero-phase impedance may be approached with confidence by engineers.

We wish, however, to draw attention to the fact that the earth wire current may vary considerably along the line with a high-conductivity earth wire such as is used in England unless the fault resistance to the earth wire is the same as the "earth plate" resistance, at the fault. In one of our tests there was only an additional 1¼ ohms fault resistance to earth as compared with the resistance to the earth wire but theory and experiment showed that the earth wire current fell from 70 per cent to about 30 per cent from the end to the middle of the line. The effect on the impedance is however, usually negligible as shown in our paper (*ibid.*). The same effect occurs with cables and may produce large voltage rises at the bonds adjacent to the fault.

In our paper the actual fault impedance but not the zero-phase impedance for a three-phase line was given. This omission was rectified in the discussion (*J. I. E. E.*, June 1930, page 779) where it is also mentioned that short-circuit calculations may be much simplified in practise by treating all the impedances as pure reactances, that is, neglecting the phase angles. The errors introduced by this on fairly short lines would probably not exceed about 10 per cent so that this approximation may often be useful.

In our tests the value adopted for the resistivity of the earth was based partly on measurements of the voltage induced in neighboring telephone lines which are very sensitive to the magnitude of the effective resistivity. A certain amount of agreement was observed with surface resistivity measurements and in considering similar tests made elsewhere in Europe it appears possible that some correlation between the a-c. resistance and the known electrical properties of various geological strata might be obtained. Has the author been in a position to attempt such a correlation and, if so, has he come to any conclusion as to the feasibility of this?

Finally we should like to mention, as in our paper, that a second earth wire placed below the conductors appears to have considerable advantages notably from the standpoint of reducing inductive interference from short-circuit currents.

Simultaneous Faults on Three-Phase Systems

BY EDITH CLARKE*

Associate, A. I. E. E.

Synopsis.—The method of symmetrical components now so extensively used to determine short-circuit currents and stability limits during transient conditions for three-phase transmission systems when a fault involving one or more of the three conductors occurs at any one point of the system, has been extended to apply to three-phase systems during simultaneous faults at two or more points of the system.

A general equivalent circuit is developed to replace, in the positive phase diagram, two simultaneous faults involving any combination of the six conductors. An approximate equivalent circuit to be used with the d-c. calculating table when resistance is neglected is also given.

Special equivalent circuits are employed to replace two simultaneous faults and the lines upon which they occur, when the lines are unloaded feeders radiating from a common point or lines of equal impedances bussed at both ends.

The methods and formulas given in this paper were developed in answer to such questions as the following:

1. Which is a more severe shock to a system, a double line-to-ground fault on one circuit or two single line-to-ground faults occurring simultaneously on two separate circuits?

2. Do simultaneous double line-to-ground faults which involve the same phases, a and b, on two circuits, result in more or less ground current than faults which involve phases a and b on one circuit and phases b and c on the other?

* * * * *

When double circuit towers carry two three-phase circuits, disturbances may involve one or more conductors of each circuit. From published records[1] of the number of flashovers on double circuit towers which have tripped out both circuits, and from opinions expressed by operating engineers of various power companies who have been consulted, it seems reasonable to conclude that in the neighborhood of 20 per cent of the faults on double circuit towers involve conductors of both circuits. In addition there are instances where faults in substations have involved conductors of circuits not on the same towers.

It seems worth while therefore, to have in convenient form, methods for calculating short-circuit currents, and of determining the stability limit of a system when faults occur simultaneously at two separate and distinct points of the system. The general case will cover simultaneous faults at any two points of the system, involving one, two or three conductors at each point, while short circuits on parallel circuits on the same tower will be a special case in which the two points of fault are symmetrical with respect to the system, although they will not be symmetrical with respect to ground unless the faults are on the same phase or phases in both circuits.

Mr. C. L. Fortescue has shown[2] that any system of three vectors may be replaced by three sets of balanced components. The fundamental equations expressing actual currents and voltages in terms of their symmetrical components, and expressing the symmetrical components of current and voltage in terms of the actual currents and voltages respectively are given in Appendix A.

Single Fault

When one or more of the conductors of a three-phase circuit becomes grounded, the voltages to ground on the three conductors and the currents in the three lines are no longer balanced. If the three unbalanced voltages to ground, V_a, V_b and V_c, at the point of fault, and three line currents flowing into the fault, I_a, I_b and I_c, are resolved into their symmetrical components, it will be found that there are enough relations existing between these six components to allow the positive component of voltage at the fault, V_{a1}, to be expressed in terms of the positive component of current in the fault, I_{a1}, and the zero and negative phase impedances, Z_0 and Z_2 respectively, viewed from the fault. To do this requires five equations.

Since there is no generated zero nor negative phase sequence voltage, and the positive direction for all component currents is taken towards the fault,

$$0 - V_{a0} = I_{a0} Z_0 \tag{1}$$

$$0 - V_{a2} = I_{a2} Z_2 \tag{2}$$

At a point of fault there are certain relations between the positive, negative, and zero components of current which flow into the fault, and also between the positive, negative, and zero components of voltage at the fault. These relations between the components of current and voltage, depending upon the type of fault, provide additional equations connecting the unknowns. For all types of fault there will be three independent equations connecting the components of current and voltage at the point of fault. These equations are tabulated in Table I, and the manner of their derivation shown in Appendix B.

From five equations with six unknowns, the four unknowns V_{a0}, V_{a2}, I_{a0} and I_{a2} may be eliminated, and V_{a1} expressed in terms of I_{a1} thus:

$$V_{a1} = K\, I_{a1} \tag{3}$$

where K is a function of Z_0 and Z_2 depending upon the type of fault. For line-to-ground faults $K = Z_0 + Z_2$, for line-to-line faults $K = Z_2$, and for double- line-to-ground faults $K = \dfrac{Z_0 Z_2}{Z_0 + Z_2}$.

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*Central Station Engg. Dept. General Electric Company, Schenectady, N. Y.

1. For references see Bibliography.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

Equation (3) will be satisfied, and the positive phase current and voltage distribution may be determined if the fault in the positive phase diagram is replaced[3] by the shunt impedance K, the value of K being determined by Z_0, Z_2 and the type of fault.

Two Simultaneous Faults

The method of procedure outlined above for determining the positive phase current and voltage distribution when a fault involves one or more of the three conductors at a single point, may be followed for simultaneous faults at two distinct points.

If the two points of fault are C and D and the conductors at C are a, b and c and those at D are α, β and γ (a and α being conductors of the same phase, as are b and β, and c and γ), the six components of voltage and current at C will be V_{a0}, V_{a1}, V_{a2}, I_{a0}, I_{a1} and I_{a2}, and those at D will be $V_{\alpha 0}$, $V_{\alpha 1}$, $V_{\alpha 2}$, $I_{\alpha 0}$, $I_{\alpha 1}$ and $I_{\alpha 2}$. It will be shown that there are ten independent equations connecting these twelve unknowns. It is proposed to eliminate the eight unknowns V_{a0}, $V_{\alpha 0}$, V_{a2}, $V_{\alpha 2}$, I_{a0}, $I_{\alpha 0}$, I_{a2} and $I_{\alpha 2}$ and to reduce the number of equations to two, expressing the positive components of voltage, V_{a1} and $V_{\alpha 1}$, in terms of the positive components of current, I_{a1} and $I_{\alpha 1}$, and the known zero and negative impedances.

Zero Phase System. With a single fault, the zero phase current in the fault varies directly as the zero phase voltage to ground at the fault. When there are two simultaneous faults, the zero phase current in either fault depends upon the zero phase voltages at both points of fault. In order to readily express the two zero phase voltages in terms of the two zero phase currents, the zero phase impedance diagram[4] will be simplified, remembering that all points of zero potential for zero phase voltage may be considered bussed at the same point, S. Equivalent circuits to replace two parallel transmission lines with mutual impedance between them in the zero phase system are given in Appendix C. When there are more than two parallel lines it may not be possible to accurately represent them by a simple equivalent circuit in the zero phase system but an approximate equivalent circuit may usually be obtained. When the zero phase system can be represented by an equivalent impedance diagram, it is always possible to reduce it to an equivalent Y, connecting the two points of fault, C and D, and the zero potential point, S. In some systems this may be done by $\Delta - Y$ or $Y - \Delta$ transformations,[5] but in complicated systems it may be necessary to use an a-c. or d-c. calculating table to determine the branches of the equivalent Y or Δ connecting the points C, D, and S.

Determination of Equivalent Y for the Zero Phase Impedance Network by Means of the Calculating Table. For an exact determination, the use of an a-c. calculating table such as the M. I. T. Network Analyzer[6] is necessary, in which case the currents and voltages measured as described below would have to be in vector form, requiring the use of a watt-meter as well as an ammeter and voltmeter. For many purposes the resistance in the networks can be neglected and the impedances considered to consist of reactance alone. In this latter case the d-c. calculating table may be used. The general procedure in either case is as follows:

(a) Set up the zero phase impedance network on the calculating table. Apply a voltage to ground, V_c, at point C with point D ungrounded, but all zero potential points, S, grounded. Measure the total ground current, I_s. Current I_c at C is the same as the total ground current I_s at S.

(b) Apply a voltage to ground, V_c, at C with point D grounded but all zero potential points, S, ungrounded. Measure the current I_d at D.

(c) Apply a voltage to ground, V_d, at point D with point C ungrounded, but all zero potential points, S, grounded and measure the current I_s.

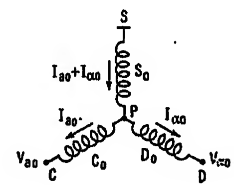


Fig. 1—Equivalent Y for Zero Phase Sequence Network

If the branches of the equivalent Y are C_0, D_0, and S_0, then

$$\frac{V_c}{I_s} = C_0 + S_0 \qquad \text{(a)}$$

$$\frac{V_c}{I_d} = C_0 + D_0 \qquad \text{(b)}$$

$$\frac{V_d}{I_s} = D_0 + S_0 \qquad \text{(c)}$$

From equations (a), (b) and (c) the branches of the Y are determined.

Fig. 1 represents the equivalent Y for the zero phase network connecting C, D and S with its branch impedances, C_0, D_0 and S_0. Let the positive direction for zero phase currents be taken towards the faults C and D. Then since there is no generated zero phase voltage, by superposing the voltages due to the two component currents, the following equations are obtained:

$$0 - V_{a0} = -V_{a0} = I_{a0}(C_0 + S_0) + I_{\alpha 0} S_0 \qquad (5)$$

$$0 - V_{\alpha 0} = -V_{\alpha 0} = I_{a0} S_0 + I_{\alpha 0}(D_0 + S_0) \qquad (6)$$

Equations (5) and (6) express the zero components of voltage at the two points of fault in terms of the two zero phase currents flowing into the faults and the branch impedances of the equivalent Y.

Rewriting equations (5) and (6) to express the cur-

rents I_{a0} and $I_{\alpha 0}$ in terms of V_{a0} and $V_{\alpha 0}$, or from Fig. 1 directly:

$$I_{a0} = - V_{a0} \frac{D_0 + S_0}{Z_{00}} + V_{\alpha 0} \frac{S_0}{Z_{00}} \qquad (7)$$

$$I_{\alpha 0} = V_{a0} \frac{S_0}{Z_{00}} - V_{\alpha 0} \frac{C_0 + S_0}{Z_{00}} \qquad (8)$$

where $\quad Z_{00} = C_0 D_0 + C_0 S_0 + D_0 S_0 \qquad (9)$

Equations (7) and (8) are not independent of equations (5) and (6). There are four unknowns, V_{a0}, $V_{\alpha 0}$, I_{a0} and $I_{\alpha 0}$ in the zero phase system and two independent equations connecting them.

Negative Phase System. In the negative phase systen, just as in the zero phase system, there is no generated voltage, and the positive direction for negative phase currents is taken from the neutrals of the machines towards the faults. The neutrals of generators and loads are points of zero potential for the negative phase system. The negative phase network[4] of a system may be reduced to an equivalent Y or Δ connecting the points of fault C and D and the points of zero potential, which may be considered bussed at a common point, S. It is important to note that S, the point of

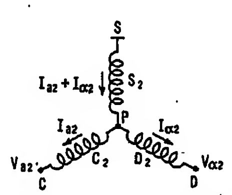


FIG. 2—EQUIVALENT Y FOR NEGATIVE PHASE SEQUENCE NETWORK

zero potential for zero phase voltage is not necessarily the same point of the actual system as S, the point of zero potential for negative phase voltage. Neither are the two P points identical. The same letters are used, however, to obtain symmetry in form for the zero and negative phase current and voltage equations.

From Fig. 2 which represents the equivalent Y for the negative phase network connecting C, D and S, the two negative phase voltages may be expressed in terms of the two negative phase currents and the impedances of the Y, thus:

$$- V_{a2} = I_{a2} (C_2 + S_2) + I_{\alpha 2} S_2 \qquad (10)$$

$$- V_{\alpha 2} = I_{a2} S_2 + I_{\alpha 2} (D_2 + S_2) \qquad (11)$$

Rewriting equations (10) and (11) to express the currents I_{a2} and $I_{\alpha 2}$ in terms of V_{a2} and $V_{\alpha 2}$, or from Fig. 2 directly:

$$I_{a2} = - V_{a2} \frac{D_2 + S_2}{Z_{22}} + V_{\alpha 2} \frac{S_2}{Z_{22}} \qquad (12)$$

$$I_{\alpha 2} = V_{a2} \frac{S_2}{Z_{22}} - V_{\alpha 2} \frac{C_2 + S_2}{Z_{22}} \qquad (13)$$

where $\quad Z_{22} = C_2 D_2 + C_2 S_2 + D_2 S_2 \qquad (14)$

Equations (12) and (13) are not independent of equations (10) and (11). There are four unknowns V_{a2}, $V_{\alpha 2}$, I_{a2} and $I_{\alpha 2}$ in the negative phase system, and two independent equations connecting them.

Relations between Components of Current and of Voltage at Two Points of Fault. At each point of fault, the relations between the components of current flowing into the fault, and between the components of voltage to ground at the fault are independent of the rest of the system. Table I gives three equations connecting the components of current or of voltage at a point of fault for various types of fault. When there are two points of fault, there will be three equations connecting components of current or voltage at each point. By replacing a, b and c in Table I by α, β and γ respectively, the three equations connecting components of current or voltage at the second point of fault, D, are obtained.

Simultaneous Equations. Three equations expressing relations between the components of current or voltage at each of the two points of fault, two equations connecting zero phase currents and voltages and two connecting negative phase currents and voltages give the ten equations needed to eliminate the eight unknowns V_{a0}, $V_{\alpha 0}$, I_{a0}, $I_{\alpha 0}$, V_{a2}, $V_{\alpha 2}$, I_{a2} and $I_{\alpha 2}$ so that the positive components of voltage V_{a1} and $V_{\alpha 1}$ may be expressed in terms of the positive components of current, I_{a1} and $I_{\alpha 1}$, and the known zero and negative phase impedances of the system. Since the ten equations are linear the two resulting equations can be put in the form:

$$V_{a1} = k\, I_{a1} + m\, I_{\alpha 1} \qquad (15)$$

$$V_{\alpha 1} = n\, I_{a1} + l\, I_{\alpha 1} \qquad (16)$$

where k, l, m and n depend upon the branch impedances of the equivalent Y's replacing the negative and zero phase networks, and the particular combination of conductors involved in the simultaneous faults. It should be noted that k, l, m and n do not involve positive phase impedances.

Equations giving k, l, m and n for faults which involve various combinations of the six conductors are derived in Appendix D and tabulated in Table II.

Positive Phase System. The positive phase system differs from the negative and zero phase systems because positive phase sequence voltages are generated at various points of the positive phase system. In general there will be as many separate positive phase generated voltages as there are separate generators or synchronous motors on the system. When the operating conditions just previous to the occurrence of the faults are known, these generated voltages are determined in magnitude and in phase. For steady state calculations, the excitation voltage or voltage behind synchronous reactance is required, while for transient calculations, the voltage behind transient reactance should be known. However, in either case the internal generated voltages are determined from the given operating conditions.

TABLE I

Relations between the symmetrical components of current in the fault and between the components of voltage to ground at the fault for various types of fault.

The operators a and a^2 have been defined in equations (**13a**) and (**14a**).

Case	Type of fault	Phases involved	Current relations	Voltage relations
A	(a) Line to ground	a	$I_{a0} = I_{a1},\ I_{a2} = I_{a1}$	$V_{a1} = -(V_{a0} + V_{a2})$
	(b) Line to ground	b	$\begin{cases} I_{a0} = a^2 I_{a1};\ I_{a1} = a\ I_{a0} \\ I_{a2} = a\ I_{a1};\ I_{a1} = a^2 I_{a2} \end{cases}$	$V_a = -(a\ V_{a0} + a^2\ V_{a2})$
	(c) Line to ground	c	$\begin{cases} I_{a0} = a\ I_{a1};\ I_{a1} = a^2 I_{a0} \\ I_{a2} = a^2 I_{a1};\ I_{a1} = a\ I_{a2} \end{cases}$	$V_{a1} = -(a^2\ V_{a0} + a\ V_{a2})$
B	(a) Line to line	b, c	$I_{a0} = 0,\ I_{a2} = -I_{a1}$	$V_{a2} = V_{a1}$
	(b) Line to line	a, c	$\begin{cases} I_{a0} = 0 \\ I_{a2} = -a\ I_{a1};\ I_{a1} = -a^2 I_{a2} \end{cases}$	$V_{a2} = a\ V_{a1};\ V_{a1} = a^2\ V_{a2}$
	(c) Line to line	a, b	$\begin{cases} I_{a0} = 0 \\ I_{a2} = -a^2 I_{a1};\ I_{a1} = -a\ I_{a2} \end{cases}$	$V_{a2} = a^2\ V_{a1};\ V_{a1} = a\ V_{a2}$
C	(a) Double line to ground	b, c	$I_{a1} = -(I_{a0} + I_{a2})$	$V_{a0} = V_{a1},\ V_{a2} = V_{a1}$
	(b) Double line to ground	a, c	$I_{a1} = -(a\ I_{a0} + a^2 I_{a2})$	$\begin{cases} V_{a0} = a^2\ V_{a1};\ V_{a1} = a\ V_{a0} \\ V_{a2} = a\ V_{a1}\ V_{a1} = a^2\ V_{a2} \end{cases}$
	(c) Double line to ground	a, b	$I_{a1} = -(a^2 I_{a0} + a\ I_{a2})$	$\begin{cases} V_{a0} = a\ V_{a1};\ V_{a1} = a^2\ V_{a0} \\ V_{a2} = a^2\ V_{a1};\ V_{a1} = a\ V_{a2} \end{cases}$
D	(a) Three-phase	a, b, c	$I_{a0} = 0$	$V_{a1} = 0,\ V_{a2} = 0$
	(b) Three-phase to ground	a, b, c		$V_{a1} = 0,\ V_{a0} = 0\ \ V_{a2} = 0.$

At point C there is a positive phase voltage, V_{a1}, to neutral and a positive phase current, I_{a1}, flowing into the fault. At point D there is a positive phase voltage, $V_{\alpha 1}$, to neutral and a positive current, $I_{\alpha 1}$, flowing into the fault, the relations between voltages and currents being given in equations (**15**) and (**16**).

EQUIVALENT CIRCUITS REPLACING TWO SIMULTANEOUS FAULTS IN POSITIVE PHASE SYSTEM

Equations (**15**) and (**16**) may be written

$$V_{a1} = (k - n)\, I_{a1} + \frac{m + n}{2}\,(I_{a1} + I_{\alpha 1}) + \frac{m - n}{2}\,(I_{\alpha 1} - I_{a1}) \qquad (17)$$

$$V_{\alpha 1} = (l - m)\, I_{\alpha 1} + \frac{m + n}{2}\,(I_{a1} + I_{\alpha 1}) + \frac{m - n}{2}\,(I_{\alpha 1} - I_{a1}) \qquad (18)$$

The last term of (**17**) and of (**18**) may also be written

$$\frac{n - m}{2}\,(I_{a1} - I_{\alpha 1})$$

Case I. m and n equal.

When m and n are equal (**17**) and (**18**) become

$$V_{a1} = (k - m)\, I_{a1} + m\,(I_{a1} + I_{\alpha 1}) \qquad (19)$$
$$V_{\alpha 1} = (l - m)\, I_{\alpha 1} + m\,(I_{a1} + I_{\alpha 1}) \qquad (20)$$

The relations expressed in (**19**) and (**20**) are satisfied by substituting for the faults a Y network having the branch impedances $(k - m)$, $(l - m)$ and m connecting the points C, D and ground as shown in Fig. 3. In this case the positive phase voltages and currents of the system may be determined as in any other balanced load distribution problem: that is, by calculation, or by means of a calculating table, the equivalent Y replacing the two faults in the positive phase network.

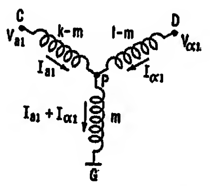


FIG. 3—EQUIVALENT Y REPLACING TWO FAULTS IN POSITIVE PHASE SEQUENCE NETWORK FOR SPECIAL CASE OF $m = n$

Case II. m and n unequal.

When m and n are not equal, the positive distribution of currents and voltage due to the faults can not be obtained by inserting an equivalent Y or Δ between the points, C, D and ground.

A. General Equivalent Circuit. Fig. 4 shows an equivalent circuit which may replace the two faults in the positive phase system. The equivalent circuit consists of a Y having branch impedances $k - n$, $l - m$ and $\frac{m + n}{2}$ connected between points C, D and F,

TABLE II

Values of k, l, m and n to be substituted in equations (15) and (16) for simultaneous faults at two points of the system. The operators a and a^2 have been defined in equations (13a) and (14a), and Z_{00} and Z_{22} in equations (9) and (14). For the sake of brevity let,

$$Z_{cs} = C_0 + S_0 + C_2 + S_2$$
$$Z_{ds} = D_0 + S_0 + D_2 + S_2$$
$$Z_{cds} = (C_0 + C_2)(D_0 + D_2) + (C_0 + C_2)(S_0 + S_2) + (D_0 + D_2)(S_0 + S_2)$$

Case E. Single Line-to-Ground Faults at Two Points.

(a) Phases a and α.

$k = Z_{cs}$ $\qquad m = S_0 + S_2$

$n = S_0 + S_2$ $\qquad l = Z_{ds}$

(b) Phases b and α.

$k = Z_{cs}$ $\qquad m = a\,S_0 + a^2\,S_2$

$n = a^2\,S_0 + a\,S_2$ $\qquad l = Z_{ds}$

(c) Phases c and α.

$k = Z_{cs}$ $\qquad m = a^2\,S_0 + a\,S_2$

$n = a\,S_0 + a^2\,S_2$ $\qquad l = Z_{ds}$

Case F. Line-to-Line Faults at Two Points.

(a) Phases b, c and β, γ.*

$k = C_2 + S_2$ $\qquad m = S_2$

$n = S_2$ $\qquad l = D_2 + S_2$

(b) Phases a, c and β, γ.

$k = C_2 + S_2$ $\qquad m = a^2\,S_2$

$n = a\,S_2$ $\qquad l = D_2 + S_2$

(c) Phases a, b and β, γ.

$k = C_2 + S_2$ $\qquad m = a\,S_2$

$n = a^2\,S_2$ $\qquad l = D_2 + S_2$

Case G. Double Line-to-Ground Faults at Two Points.

(a) Phases b, c and β, γ.*

$$k = \frac{Z_{22}(C_0 + S_0) + Z_{00}(C_2 + S_2)}{Z_{cds}} \qquad m = \frac{Z_{22}\,S_0 + Z_{00}\,S_2}{Z_{cds}}$$

$$n = \frac{Z_{22}\,S_0 + Z_{00}\,S_2}{Z_{cds}} \qquad l = \frac{Z_{22}(D_0 + S_0) + Z_{00}(D_2 + S_2)}{Z_{cds}}$$

(b) Phases a, c and β, γ.

$$k = \frac{Z_{22}(C_0 + S_0) + Z_{00}(C_2 + S_2)}{Z_{cds} + 3\,S_0\,S_2} \qquad m = \frac{a\,S_0\,Z_{22} + a^2\,S_2\,Z_{00}}{Z_{cds} + 3\,S_0\,S_2}$$

$$n = \frac{a^2\,S_0\,Z_{22} + a\,S_2\,Z_{00}}{Z_{cds} + 3\,S_0\,S_2} \qquad l = \frac{Z_{22}(D_0 + S_0) + Z_{00}(D_2 + S_2)}{Z_{cds} + 3\,S_0\,S_2}$$

(c) Phases a, b and β, γ.

$$k = \frac{Z_{22}(C_0 + S_0) + Z_{00}(C_2 + S_2)}{Z_{cds} + 3\,S_0\,S_2} \qquad m = \frac{a^2\,S_0\,Z_{22} + a\,S_2\,Z_{00}}{Z_{cds} + 3\,S_0\,S_2}$$

$$n = \frac{a\,S_0\,Z_{22} + a^2\,S_0\,Z_{00}}{Z_{cds} + 3\,S_0\,S_2} \qquad l = \frac{Z_{22}(D_0 + S_0) + Z_{00}(D_2 + S_2)}{Z_{cds} + 3\,S_0\,S_2}$$

Case H. Three-Phase Faults at Two Points.

$k = 0$ $\qquad m = 0$

$n = 0$ $\qquad l = 0$

Case I. Line-to-Line Fault at C and Single Line-to-Ground Fault at D.

(a) Phases b, c and α.

$k = C_2 + S_2$ $\qquad m = -\,S_2$

$n = -\,S_2$ $\qquad l = Z_{ds}$

(b) Phases a, c and α.

$k = C_2 + S_2$ $\qquad m = -\,a^2\,S_2$

$n = -\,a\,S_2$ $\qquad l = Z_{ds}$

(c) Phases a, b and α.

$k = C_2 + S_2$ $\qquad m = -\,a\,S_2$

$n = -\,a^2\,S_2$ $\qquad l = Z_{ds}$

*See "Equivalent Y vs. Equivalent Δ" page 927.

TABLE II—*Continued*

Case J. Double Line-to-Ground Fault at C and Single Line-to Ground Faults at D.

(a) Phases b, c and α.

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}} \qquad m = -\frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{Z_{cs}}$$

$$n = -\frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{Z_{cs}} \qquad l = Z_{ds} - \frac{(S_0 - S_2)^2}{Z_{cs}}$$

(b) Phases a, c and α.

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}} \qquad m = -\frac{a\,S_0(C_2 + S_2) + a^2 S_2(C_0 + S_0)}{Z_{cs}}$$

$$n = -\frac{a^2 S_0(C_2 + S_2) + a\,S_2(C_0 + S_0)}{Z_{cs}} \qquad l = Z_{ds} - \frac{S_0^2 + S_0 S_2 + S_2^2}{Z_{cs}}$$

(c) Phases a, b and α.

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}} \qquad m = -\frac{a^2 S_0(C_2 + S_2) + a\,S_2(C_0 + S_0)}{Z_{cs}}$$

$$n = -\frac{a\,S_0(C_2 + S_2) + a^2 S_2(C_0 + S_0)}{Z_{cs}} \qquad l = Z_{ds} - \frac{S_0^2 + S_0 S_2 + S_2^2}{Z_{cs}}$$

Class K. Three-phase Fault at C and Single Line-to-Ground Fault at D.

(a) Three-phase fault not involving ground.

$$k = 0 \qquad m = 0$$

$$n = 0 \qquad l = D_0 + S_0 + D_2 + \frac{C_2 S_2}{C_2 + S_2}$$

(b) Three-phase fault involving ground.

$$k = 0 \qquad m = 0$$

$$n = 0 \qquad l = D_0 + \frac{C_0 S_0}{C_0 + S_0} + D_2 + \frac{C_2 S_2}{C_2 + S_2}$$

---

and between F and ground an impedance, $\frac{m - n}{2}$ or $\frac{n - m}{2}$, paralleled by an adjustable voltage, V_s. Equations (17) and (18) will be satisfied if current $(I_{\alpha 1} - I_{a1})$

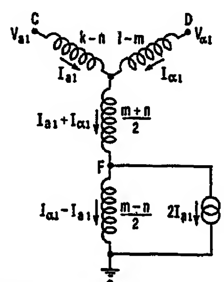


FIG. 4—EXACT GENERAL EQUIVALENT CIRCUIT REPLACING TWO FAULTS IN POSITIVE PHASE SEQUENCE NETWORK

is made to flow through the impedance $\frac{m - n}{2}$, or if current $(I_{a1} - I_{\alpha 1})$ flows through the impedance $\frac{n - m}{2}$. If the voltage, V_s, is adjusted in phase and magnitude until the current through it to ground is double I_{a1}, the current entering the fault at C, then the current $(I_{\alpha 1} - I_{a1})$ will flow in the impedance $\frac{m - n}{2}$. If the impedance $\frac{n - m}{2}$ is used, V_s must be adjusted until the current through it is double $I_{\alpha 1}$, the current entering the fault at D.

B. Approximate Equivalent Circuit for Use on the D-C. Calculating Table. In short-circuit studies it is customary to neglect capacitance and resistance, and to consider all generators operating with no load excitations. The system can then be represented on a d-c. calculating table.

A study of Table II shows that with resistance and capacitance neglected, k and l have no real components, but are positive reactive impedances larger in magnitude than m and n. When m and n are equal, they also have no real components and are positive reactive impedances; but when m and n are unequal they have real components which are equal and opposite, while their reactive components are equal in magnitude and of the same sign.

The error made by neglecting the real components of

m and n will ordinarily be less than the error made by neglecting line resistances. When the real components of m and n are neglected, $\frac{m - n}{2} = 0$, and the general equivalent circuit in Fig. 4 becomes a Y connecting C, D, and G, with branch impedances $(k - n)$, $(l - m)$ and $\frac{m + n}{2}$, see Fig. 5. The branches $k - n$, and

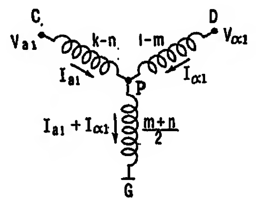


FIG. 5—APPROXIMATE EQUIVALENT CIRCUIT REPLACING TWO FAULTS IN POSITIVE PHASE SEQUENCE NETWORK

$(l - m)$ will be positive reactive impedances and therefore can be represented on the d-c. calculating table, while $\frac{m + n}{2}$ may be either positive or negative. If positive it can also be represented on the d-c. table.

It will be remembered that the branch impedance $\frac{m + n}{2}$ is connected to ground as are the neutrals of the generators in the positive phase diagram. This impedance is therefore in series with the generator reactance when there is a single generating source and may be combined with it. When $\frac{m + n}{2}$ is negative and it is not possible to combine it with any other reactance, this branch of the Y between P and G may be set to zero and the distribution of currents obtained, these currents to be increased by the ratio $\dfrac{X_p}{X_p + \dfrac{m + n}{2}}$, where X_p is the equivalent impedance between generator neutrals and P, and is found by dividing generator voltage by total ground current when P is grounded.

C. Special Equivalent Circuits. Lines upon which the faults occur, together with the faults may be represented by a particular equivalent circuit when the faults occur; (1) on unloaded feeders radiating from a common point and (2) on lines of equal impedances bussed at both ends.

When the equivalent circuit represents the lines upon which the faults occur as well as the faults, the sum of the currents in the faulted lines, the total ground current, and the positive phase current and voltage distribution for the rest of the system may readily be obtained on the calculating table. Then the division of positive phase currents between the faulted lines, and the positive phase voltages at the points of fault may be determined by calculation.

(1) Equivalent circuit for simultaneous faults on unloaded feeders radiating from a common point

Let the fault points C and D be on unloaded feeders radiating from a common point H, then referring to Fig. 6A, let

V_h = positive phase voltage at H.
V_{a1} = positive phase voltage at C.
$V_{\alpha 1}$ = positive phase voltage at D.
Z_{ch} = positive phase impedance between H and C.
Z_{dh} = positive phase impedance between H and D.
I_h = positive phase current at H.
I_{a1} = positive phase current at C.
$I_{\alpha 1}$ = positive phase current at D.

Then

$$I_h = I_{a1} + I_{\alpha 1} \tag{21}$$
$$V_h - V_{a1} = I_{a1} Z_{ch} \tag{22}$$
$$V_h - V_{\alpha 1} = I_{\alpha 1} Z_{dh} \tag{23}$$

subtracting (23) from (22)

$$- V_{a1} + V_{\alpha 1} = I_{a1} Z_{ch} - I_{\alpha 1} Z_{dh} \tag{24}$$

subtracting (16) from (15)

$$V_{a1} - V_{\alpha 1} = (k - n) I_{a1} - (l - m) I_{\alpha 1} \tag{25}$$

adding (15) to (22) and (24) to (25)

$$V_h = (Z_{ch} + k) I_{a1} + m I_{\alpha 1} \tag{26}$$
$$0 = (Z_{ch} + k - n) I_{a1} - (Z_{dh} + l - m) I_{\alpha 1} \tag{27}$$

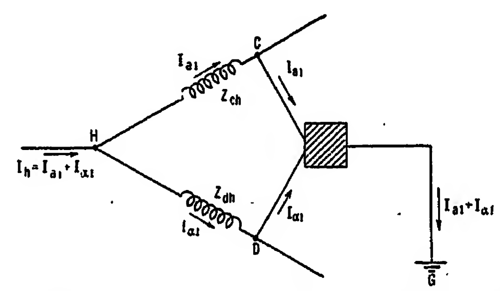


FIG. 6A—UNLOADED FEEDERS WITH FAULTS AT C AND D

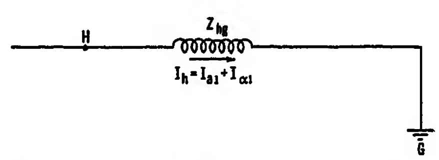


FIG. 6B—EQUIVALENT IMPEDANCE REPLACING THE TWO FAULTS AND FEEDERS OF FIG. 6A

Solving equations (26) and (27) for I_{a1} and $I_{\alpha 1}$:

$$I_{a1} = V_h \frac{Z_{dh} + l - m}{(Z_{ch} + k)(Z_{dh} + l) - m n} \tag{28}$$

$$I_{\alpha 1} = V_h \frac{Z_{ch} + k - n}{(Z_{ch} + k)(Z_{dh} + l) - m n} \tag{29}$$

adding (28) and (29) and replacing $I_{a1} + I_{\alpha 1}$ by I_h

$$I_h = V_h \frac{Z_{ch} + k + Z_{dh} + l - (m + n)}{(Z_{ch} + k)(Z_{dh} + l) - m\,n} \tag{30}$$

Substituting V_h from (30) in (28) and (29)

$$I_{a1} = I_h \frac{Z_{dh} + l - m}{Z_{ch} + k + Z_{dh} + l - (m + n)} \tag{31}$$

$$I_{\alpha 1} = I_h \frac{Z_{ch} + k - n}{Z_{ch} + k + Z_{dh} + l - (m + n)} \tag{32}$$

From equation (30)

$$\frac{V_h}{I_h} = \frac{(Z_{ch} + k)(Z_{dh} + l) - m\,n}{Z_{ch} + k + Z_{dh} + l - (m + n)} \tag{33}$$

but $\dfrac{V_h}{I_h} = Z_{hg} =$ impedance between point H and ground, which is the equivalent circuit sought.

Therefore, two unloaded feeders radiating from H with two simultaneous faults may be replaced by a single lumped impedance, Z_{hg}, whose value is given by equation (33). The total ground current and the positive phase current and voltage distribution for the system exclusive of the feeders themselves may then be obtained. Knowing I_h and V_h, the positive phase currents in the feeders, I_{a1} and $I_{\alpha 1}$, may be obtained from equations (31) and (32), and the voltages at the points of fault, V_{a1} and $V_{\alpha 1}$, from equations (22) and (23).

(2) Equivalent circuit for two lines which have two points in common or are bussed at both ends and two simultaneous faults

This case will include simultaneous faults on any two lines which, on the positive phase diagram, have their ends terminating at common points. Two feeders radiating from a common point with impedance loads replaced by shunt impedances to ground, have the ground as a common terminal point. Even when the lines upon which the faults occur do not have two points in common by appropriate $\Delta - Y$ or $Y - \Delta$ transformations it is often possible to represent them by two equivalent lines which do have two points in common.

Fig. 7A represents two lines bussed at R and T with faults at C and D, in which Z_{cr}, Z_{ct}, Z_{dr} and Z_{dt} represent positive phase impedances between C and R, C and T, D and R, and D and T respectively, and V_r, V_t, V_{a1} and $V_{\alpha 1}$ the voltage at R, T, C and D. I_r and I_t represent the currents at R and T respectively, positive direction being towards the faults and I_{a1} and $I_{\alpha 1}$ the currents flowing into the faults from C and D.

From Fig. 7A

$I_r + I_t = I_{a1} + I_{\alpha 1} = I_g =$ total ground current

Let I_r divide into I_x and $I_r - I_x$, and I_t divide into I_y and $I_t - I_y$.

Let $I_{a1} = I_x + I_y$ and $I_{\alpha 1} = I_r + I_t - I_x - I_y$.

Let $Z_{cr} = a$, $Z_{ct} = b$, $Z_{dr} = c$, $Z_{dt} = d$ and $a + b + c + d = S$.

Then

$$V_{a1} = V_r - a\,I_x \tag{34}$$
$$V_{a1} = V_t - b\,I_y \tag{35}$$
$$V_{\alpha 1} = V_r - c\,(I_r - I_x) \tag{36}$$
$$V_{\alpha 1} = V_t - d\,(I_t - I_y) \tag{37}$$

Substituting the above values for I_{a1} and $I_{\alpha 1}$ in (15) and (16)

$$V_{a1} = (k - m)(I_x + I_y) + m\,(I_r + I_t) \tag{38}$$
$$V_{\alpha 1} = (n - l)(I_x + I_y) + l\,(I_r + I_t) \tag{39}$$

From the six simultaneous equations (34)—(39) the four unknowns V_{a1}, $V_{\alpha 1}$, I_x and I_y will be eliminated and V_r and V_t expressed in terms of I_r and I_t.

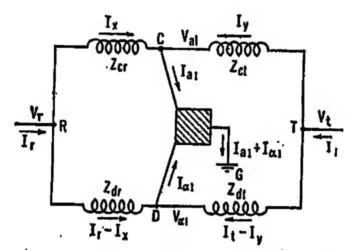


Fig. 7A—Two Lines Bussed at Both Ends with Two Simultaneous Faults

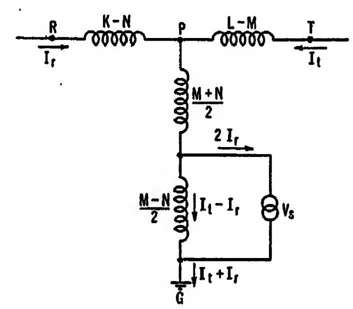


Fig. 7B—Exact Equivalent Circuit Replacing the Two Faulted Lines of Fig. 7A

Substituting (38) and (39) in (34) and (37) respectively, and transposing:

$$V_r = m\,I_r + m\,I_t + (a + k - m)\,I_x + (k - m)\,I_y \tag{40}$$
$$V_t = l\,I_r + (d + l)\,I_t + (n - l)\,I_x + (n - l - d)\,I_y \tag{41}$$

Subtracting the sum of equations (35) and (36) from the sum of (34) and (37); and the sum of (34) and (39) from the sum of (36) and (38):

$$0 = c\,I_r - d\,I_t - (a + c)\,I_x + (b + d)\,I_y \tag{42}$$

$$0 = (m - c - l)\,I_r + (m - l)\,I_t + (a+c+k+l-m-n)\,I_x + (k+l-m-n)\,I_y \tag{43}$$

Expressing I_x and I_y from (42) and (43) in terms of I_r and I_t, and replacing $a + b + c + d$ by S.

$$I_x = \frac{\left[\begin{array}{c}[(b + c + d)(l - m) + c\,(b + d + k - n)]\,I_r \\ + [b\,(l - m) - d\,(k - n)]\,I_t\end{array}\right]}{S\,(k + l - m - n) + (a + c)(b + d)} \tag{44}$$

$$I_y = \frac{\left[\begin{array}{c}[a\,(l-m) - c\,(k-n)]\,I_r \\ +[(a+c+d)(l-m)+d(a+c+k-n)]I_t\end{array}\right]}{S\,(k+l-m-n) + (a+c)\,(b+d)} \quad (45)$$

Substituting (44) and (45) in (40) and (41)

$$V_r = \frac{\left[\begin{array}{c}S\,(k\,l - m\,n) - a\,c\,(m+n) + c\,(a+b+d)\,k \\ + a\,(b+c+d)\,l + a\,c\,(b+d)\end{array}\right]}{S\,(k+l-m-n) + (a+c)\,(b+d)}\, I_r$$

$$+ \frac{S\,(k\,l - m\,n) + b\,c\,m + a\,d\,n + c\,d\,k + a\,b\,l}{S\,(k+l-m-n) + (a+c)\,(b+d)}\, I_t \quad (46)$$

$$V_t = \frac{S\,(k\,l - m\,n) + a\,d\,m + b\,c\,n + c\,d\,k + a\,b\,l}{S\,(k+l-m-n) + (a+c)\,(b+d)}\, I_r$$

$$+ \frac{\left[\begin{array}{c}S\,(k\,l - m\,n) - b\,d\,(m+n) + d\,(a+b+c)\,k \\ + b\,(a+c+d)\,l + b\,d\,(a+c)\end{array}\right]}{S\,(k+l-m-n) + (a+c)\,(b+d)}\, I_t \quad (47)$$

(*a*) *General Case.* A comparison of the coefficient of I_t in (46) with that of I_r in (47) shows that they are not identical, and therefore a simple impedance Y or Δ can not replace the lines with the two simultaneous faults in the general case where m and n are unequal and there is no fixed relation between the impedances Z_{cr}, Z_{cd}, Z_{ct} and Z_{dt}.

The general equivalent circuit or the approximate equivalent circuit may be used to replace the two lines with the two simultaneous faults, rather than just to replace the two faults, if it is found advantageous to do so. To obtain this circuit, equations (46) and (47) may be written:

$$V_r = K\,I_r + M\,I_t \quad (48)$$

$$V_t = N\,I_r + L\,I_t \quad (49)$$

If k, l, m and n in Figs. 4 and 5 are replaced by K, L, M and N as defined in equations (46)—(49) and and I_{a1} and $I_{\alpha 1}$ by I_r and I_t the desired circuits are obtained. The general equivalent circuit is shown in Fig. 7B.

(*b*) *Special Case.* The coefficients of I_t and I_r in equations (46) and (47) respectively will be identical if $b\,c\,m + a\,d\,n = a\,d\,m + b\,c\,n$, that is if $m = n$ or $a\,d = b\,c$. If either of these conditions is satisfied the two lines with the two simultaneous faults may be replaced by an equivalent Y having the branch impedances $K - M$, $L - M$ and M.

Equivalent circuit for two transmission lines of equal impedances bussed at both ends and two simultaneous faults

Since $Z_{dr} = Z_{cr} = a$ and $Z_{dt} = Z_{ct} = b$, c and d in equations (46) and (47) may be replaced by a and b respectively and the equations rewritten thus

$$V_r = \left[\frac{a}{2}\, I_r + \frac{a\,b\,(k+l+m+n) + 2\,(a+b)\,(k\,l - m\,n)}{4\,a\,b + 2\,(a+b)\,(k+l-m-n)}\,(I_r + I_t)\right] \quad (50)$$

$$V_t = \frac{b}{2}\, I_t + \frac{a\,b\,(k+l+m+n) + 2\,(a+b)\,(k\,l - m\,n)}{4\,a\,b + 2\,(a+b)\,(k+l-m-n)}\,(I_r + I_t) \quad (51)$$

Equations (50) and (51) will be satisfied if the two lines and the two simultaneous faults are replaced by a Y between the points R, T and G, Fig. 8B, the branch impedances of the Y being $\frac{a}{2}$, $\frac{b}{2}$ and

$$\frac{a\,b\,(k+l+m+n) + 2\,(a+b)\,(k\,l - m\,n)}{4\,a\,b + 2\,(a+b)\,(k+l-m-n)}.$$

In problems where the division of currents between the two lines and the voltages at the points of fault are required they may be determined by calculations from the voltages and currents at the ends of the lines. If the positive phase current and voltage distribution has been determined with the equivalent Y replacing the

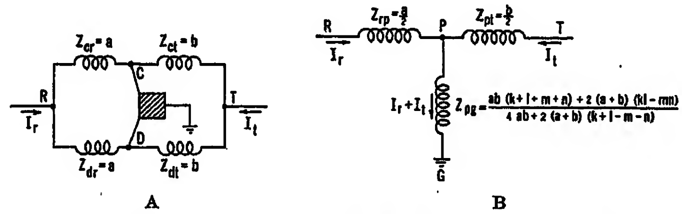


FIG. 8A—SPECIAL CASE OF FIG. 7A WHEN LINES HAVE EQUAL IMPEDANCE AND FAULT POINTS ARE EQUIDISTANT FROM ONE END

FIG. 8B—EXACT EQUIVALENT CIRCUIT FOR FIG. 8A

lines and the two simultaneous faults, then V_r, V_t, I_r and I_t will be known. I_x and I_y may be obtained by substituting the values of I_r and I_t in equations (44) and (45); V_{a1} may be obtained from equations (34) or (35); and $V_{\alpha 1}$ from (36) or (37).

Equivalent Y vs. Equivalent Δ. In the zero and negative phase systems, equivalent Y's were used to replace the network connected between the two fault points and the points of zero potential, assumed bussed at S. Equivalent Δ's might have been used through this paper. The choice of equivalent Y's and impedances has given, in the majority of cases, equations of simpler form for the constants k, l, m and n than would have been obtained with equivalent Δ's and admittances. In the case of double line-to-ground faults, however, the solution with equivalent Δ's and admittances is a simple one and the form of the resulting equations of special interest when the faults are on the same phases. Let the admittances of the Δ replacing the equivalent Y for the zero phase system be Y_{0cd}, Y_{0cs} and Y_{0ds} and those for the negative system be

Y_{2cd}, Y_{2cs} and Y_{2ds}, the first subscript indicating the sequence and the second and third the terminal points.

Replacing[5] the impedances of the Y's by the admittances of the Δ's in equations (**13d**) and (**14d**) of Appendix D:

$$I_{a1} = V_{a1}(Y_{0cs} + Y_{2cs} + Y_{0cd} + Y_{2cd}) - V_{\alpha 1}(Y_{0cd} + Y_{2cd})$$

$$I_{\alpha 1} = - V_{a1}(Y_{0cd} + Y_{2cd}) + V_{\alpha 1}(Y_{0ds} + Y_{2ds} + Y_{0cd} + Y_{2cd})$$

These equations are satisfied if the zero and negative phase networks are connected in parallel between the points C, D and G of the positive phase system, points C in the three systems being connected, also points D, and the points of zero potential of all three systems. With these connections on the calculating table, the zero and negative phase currents for phase a as well as the positive, may be read directly for all parts of the system.

Analytical Solution of Positive Phase Network

When a calculating table is not available, or when greater accuracy is required than can be secured from either the d-c. table or the a-c. network analyzer, the distribution of positive phase currents and voltages may be determined by calculation.

Equations may be written by Kirchoff's law expressing the internal voltages of the various machines and the positive phase voltages, V_{a1} and $V_{\alpha 1}$, at points C and D respectively, in terms of the positive phase impedances of the systems and the positive phase currents. If there are n machines, there will be a minimum of $(n + 2)$ unknown currents, *i. e.*, the n machines currents and the two fault currents; and a minimum of $(n + 2)$ voltage equations, *i. e.*, the n machine internal voltage equations and the two fault voltage equations. Each loop in the system introduces an additional unknown current, but also an additional equation, the voltage drop around the loop being zero. In these $(n + 2)$ equations the n machines internal voltages are known, the $(n + 2)$ currents and the two fault voltages are unknown. With equations (15) and (16), (which are independent of the positive phase sequence impedances and the internal voltages of the machine, but depend upon the types of fault and the zero and negative phase impedances) there will be $(n + 4)$ equations and $(n + 4)$ unknowns. It is possible, therefore, to solve for the positive currents in terms of the internal generated voltages and the positive, negative, and zero phase impedances of the system. Knowing the positive currents, the positive voltages for the system may be obtained.

Solution at Positive Phase Network by Superposition

To avoid the additional equations and unknowns due to loops in the positive phase system, the following equations may be used:

Positive Phase Sequence Equations Expressing Positive Voltages in Terms of Positive Currents and Impedances*

$$E_1 = I_1 Q_{11} + I_2 Q_{12} + \ldots\ldots + I_n Q_{1n} + I_{a1} Q_{1c} + I_{\alpha 1} Q_{1d}$$
$$E_2 = I_1 Q_{21} + I_2 Q_{22} + \ldots\ldots + I_n Q_{2n} + I_{a1} Q_{2c} + I_{\alpha 1} Q_{2d}$$

$$E_n = I_1 Q_{n1} + I_2 Q_{n2} + \ldots\ldots + I_n Q_{nn} + I_{a1} Q_{nc} + I_{\alpha 1} Q_{nd}$$
$$V_{a1} = I_1 Q_{c1} + I_2 Q_{c2} + \ldots\ldots + I_n Q_{cn} + I_{a1} Q_{cc} + I_{\alpha 1} Q_{cd}$$
$$V_{\alpha 1} = I_1 Q_{d1} + I_2 Q_{d2} + \ldots\ldots + I_n Q_{dn} + I_{a1} Q_{dc} + I_{\alpha 1} Q_{dd}$$

where E_1, $E_2 \ldots\ldots E_n$ are known internal voltages on machines 1, 2 and n, and I_1, I_2 and In are the corresponding currents. V_{a1} and $V_{\alpha 1}$ are the positive phase voltages at the points of fault C and D respectively, and I_{a1} and $I_{\alpha 1}$ are the corresponding currents flowing into the faults. Positive direction for machine currents is taken from the generator neutrals. Positive direction for faults current, I_{a1} and $I_{\alpha 1}$, is taken from the system into the faults.

Also

$Q_{11} = \dfrac{V_1}{I_1}$ where V_1 is the internal voltage of machine I, when no fault currents and no machine currents are flowing except I_1.

$Q_{1n} = \dfrac{V_1}{I_n}$ when no fault nor machine currents are flowing except I_n.

$Q_{cd} = \dfrac{V_{a1}}{I_{\alpha 1}}$ when no machine currents are flowing and no fault current except $I_{\alpha 1}$.

$Q_{nn} = \dfrac{V_n}{I_n}$ when V_n is the internal voltage of machine, or when no fault currents and no machine currents are flowing except I_n.

The Q's associated with I_{a1} and $I_{\alpha 1}$ in the above equations will be negative, due to the arbitrary assumptions for positive direction of current flow.

When there is a point of zero potential on the positive phase system at which no voltage is generated, such as the neutral of a shunt impedance load, there will be a return path for the currents when one current only is flowing, and the Q's may readily be obtained on the calculating table. When there is no such point it will be necessary to ground one machine, No. 2, to provide a return path for the currents when the only current flowing is the one under consideration. The voltage equation $E_2 = I_1 Q_{a1} + \ldots\ldots + I_{\alpha 1} Q_{2d}$ will then become indeterminate but may be replaced by the current equation $I_1 + I_2 \ldots\ldots + I_n = I_{a1} + I_{\alpha 1}$. The current distribution will first be determined with machine No. 2 grounded ($E_2 = 0$), and then with all machines except No. 2 grounded. By superposing the two sets of currents the distribution of positive phase currents will be determined.

*These equations were suggested by Mr. R. H. Park.

Solution of Negative and Zero Phase Networks

When the positive phase voltages at the points of fault and the positive phase currents flowing into the faults have been determined, four of the eight unknowns V_{a0}, $V_{\alpha 0}$, V_{a2}, $V_{\alpha 2}$, I_{a0}, $I_{\alpha 0}$, I_{a2}, and $I_{\alpha 2}$ may readily be obtained from the relations given in Table I. The four remaining unknowns, two of which will be in the zero phase and two in the negative phase system, may be obtained from equations (5), (6), (10) and (11), or from (7), (8), (12) and (13).

In general, the voltages at the two points of fault of either system will not be in phase with each other nor with the reference voltage of the positive phase system. The currents in the network of either system may be determined by superposition, *i. e.*, by adding vectorially the currents due to the fault voltage at C with point D grounded, and the currents due to the fault voltage at D with C grounded.

When an a-c. calculating table is available the current and voltage distribution in the zero phase system may be obtained by applying voltages to ground, V_{a0} and $V_{\alpha 0}$, at points C and D respectively of the zero phase network; and in a similar manner the negative phase current and voltage distribution will be determined by applying voltages V_{a2} and $V_{\alpha 2}$ at point C and D of the negative phase system.

System Currents and Voltages

By use of the fundamental symmetrical component equations given in Appendix A, the positive, negative, and zero components of currents and voltages may be combined to give actual currents and voltages over the entire system.

Three Simultaneous Faults

The method used for determining currents and voltages when simultaneous faults occur at two separate and distinct points of the system may be extended to apply to three or more simultaneous faults.

With three simultaneous faults on the system, three independent equations may be written expressing the three zero phase voltages at the points of fault in terms of the three zero phase currents flowing into the faults. The equations given above for use in determining the positive phase currents may be used to advantage in this connection; or if three equivalent Y's for the zero phase system are drawn, each preserving the identity of the points of zero potential considered bussed at a common point, S, and two of the three fault points, the three equations each expressing one zero phase fault voltage in terms of the three zero phase currents flowing into the faults may be written by inspection.

In like manner the negative phase system will provide three independent equations connecting negative phase fault voltages and currents.

At each of the three points of fault there will be three independent equations connecting components of current or of voltage.

Nine equations expressing relations between the components of current or voltage at the three fault points, three equations connecting zero phase currents and voltages, and three connecting negative phase currents and voltages, provide fifteen equations. The unknown zero and negative phase currents and voltages may be eliminated from these equations, the number being reduced to three, and the three positive phase voltages to ground at the fault points expressed in terms of the three positive phase currents flowing into the faults and the negative and zero phase impedances of the system.

In the positive phase system there will be one more unknown voltage and one more unknown current than with two simultaneous faults, but there will be two additional voltage equations, one involving positive phase currents flowing into the faults and negative and zero phase impedances, and the other involving positive phase line currents and positive phase impedances. A complete determination of the positive phase currents, therefore, is possible, and from these, the currents and voltages over the entire system may be obtained.

Generalizing from the above, it may be seen that n simultaneous faults would be solved in a similar manner.

Single-Pole Switching

In the past there has been some discussion as to the advisability of switching out one phase and retaining the two good phases when a ground occurs on one phase only of a three-phase system. When grounds occur at two separate and distinct points of the system but each involves one conductor only, it may be desirable to investigate the possibility of simultaneous single-pole switching.

When a conductor is disconnected from the system by opening the breakers at its terminals, the current in it and in the other two conductors and the three voltages to ground at the terminals of the line are the same as they would be if the conductor were open at one point only, provided there is no ground on the conductor and capacitance is negligible. For convenience, when a conductor is open at both ends it will be considered open at one point only, the point having any convenient location along the line. This will not change the currents in the conductors nor the voltages at the ends of the section under consideration. The voltage to ground of the open conductor, however, will depend upon the location of the opening.

When one conductor of a three-phase transmission line is open the voltages to ground on the two sides of the opening in general will be different, and a voltage will exist across the opening. Fig. 9 represents three conductors a, b, and c with conductor a open at point C. Let e_a, e_b, e_c be the differences in voltage on the two sides

of the opening for conductors a, b, and c respectively, and i_a, i_b and i_c the line currents at point C.

(a) conductor a open.

Fault conditions: $e_b = e_c = 0$, and $i_a = 0$

From (1a), (10a)—(12a)

$$i_{a1} = -(i_{a0} + i_{a2}) \tag{52}$$

$$e_{a1} = e_{a0} = e_{a2} \tag{53}$$

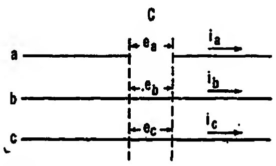


FIG. 9—THREE-PHASE LINE WITH ONE CONDUCTOR OPEN SHOWING SERIES VOLTAGES USED IN ANALYSIS

(b) conductor b open.

Fault conditions: $e_a = e_c = 0$, and $i_b = 0$

From (2a) and (10a)—(12a)

$$i_{a1} = -(a\, i_{a0} + a^2\, i_{a2}) \tag{54}$$

$$e_{a1} = a\, e_{a0} = a^2\, e_{a2} \tag{55}$$

(c) conductor c open.

By analogy from (b)

$$i_{a1} = -(a^2\, i_{a0} + a\, i_{a2}) \tag{56}$$

$$e_{a1} = a^2\, e_{a0} = a\, e_{a2} \tag{57}$$

It is of interest to note that equations (52)—(57) are identical in form with the equations given in Table I, C for double line-to-ground faults. In a similar manner it may be shown that when two conductors are opened the resulting equations between components of series voltages and between components of line currents will be identical in form with those given in Table I, A for single line-to-ground faults.

Series Voltages and Series Impedances. It should be noted that e_a, e_{a0}, e_{a1} and e_{a2} are series voltages. Due to the opening, a series voltage, e_{ac}, appears in the zero phase network at point C and a line current, i_{a0}, which meets a series impedance z_0. Likewise the series voltage, e_{a2}, appears in the negative phase network at C and a line current, i_{a2}, which meets the series impedance z_2. Expressed in equations:

$$-i_{a0} = \frac{e_{a0}}{z_0} \tag{58}$$

$$-i_{a2} = \frac{e_{a2}}{z_2} \tag{59}$$

where z_0 and z_2 are the series impedances for the zero and negative phase networks respectively, viewed from the point C.

Single-Pole Switching on One Circuit. It has been shown[8] that the effect on the positive phase currents and voltages of the system of opening a single conductor may be represented by inserting the zero and negative phase series impedance circuits viewed from the opening in parallel across the opening in the positive phase impedance diagram; and if two conductors are opened, by inserting the zero and negative phase series impedance circuits in series. This is seen by adding (58) and (59) and substituting (52) and (53) in the sum, then transposing:

$$e_{a1} = \frac{z_0\, z_2}{z_0 + z_2}\, i_{a1} \tag{60}$$

and when two conductors are open, by analogy from a single line-to-ground fault, it follows that

$$e_{a1} = (z_0 + z_2)\, i_{a1} \tag{61}$$

Single-Pole Switching on Two Circuits Simultaneously. The relations between the components of line current and the components of series voltages at a point where a conductor is open are independent of the rest of the system. These relations are given in equations (52)—(57) for conductors a, b, and c. By substituting α, β and γ for a, b, and c respectively, the relations existing at a second point of opening are obtained.

In both the zero and negative phase networks there will be two series voltages introduced by the opening. If the zero and negative phase networks are reduced to their simplest form consisting of three branches each, as shown in Figs. 10A and 10B, the identity of the lines in which the conductors are open being preserved, equations connecting components of current and voltage may be written thus

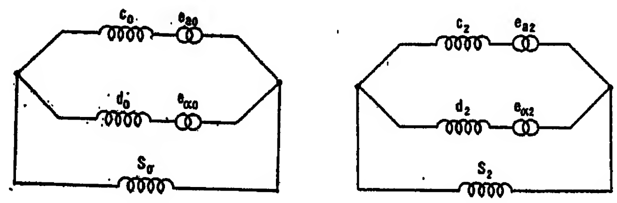
$$-e_{a0} = i_{a0}(c_0 + s_0) + i_{\alpha 0}\, s_0 \tag{62}$$

$$-e_{\alpha 0} = i_{a0}\, s_0 + i_{\alpha 0}(d_0 + s_0) \tag{63}$$

$$-e_{a2} = i_{a2}(c_2 + s_2) + i_{\alpha 2}\, e_2 \tag{64}$$

$$-e_{\alpha 2} = i_{a2}\, s_2 + i_{\alpha 0}(d_2 + s_2) \tag{65}$$

It is seen that equations (62)—(65) have the same form as (5), (6), (10), and (11), and since equations (52)—(57) have the same form as the equations of Table I,



A B

FIG. 10A—ZERO PHASE SEQUENCE SERIES EQUIVALENT CIRCUIT FOR TWO LINES WITH CONDUCTORS OPEN AT TWO POINTS

FIG. 10B—NEGATIVE PHASE SEQUENCE SERIES EQUIVALENT CIRCUIT FOR TWO LINES WITH CONDUCTORS OPEN AT TWO POINTS

C for double line-to-ground faults, it follows by analogy from the work on simultaneous faults that the positive series voltages e_{a1} and $e_{\alpha 1}$ at points C and D respectively, when one conductor is open at each point may be expressed in terms of the positive phase line currents i_{a1} and $i_{\alpha 1}$ thus:

$$e_{a1} = k'\, i_{a1} + m'\, i_{\alpha 1} \tag{66}$$

$$e_{\alpha 1} = n'\, i_{a1} + l'\, i_{\alpha 1} \tag{67}$$

where k', l', m' and n' are functions of c_0, d_0, s_0, c_2, d_2 and s_2, the branches of the simplified circuits representing the zero and negative phase networks, and are given in Table II, G if k', l', m' and n' are substituted for k, l, m, n and c_0, d_0, s_0, c_2, d_2 and s_2 for C_0, D_0, S_0, C_2, D_2 and S_2 respectively.

Equivalent Circuits Replacing Openings in Two Transmission Lines in Positive Phase System. The effect of an opening in a conductor in the positive phase system is to introduce a series voltage opposing the flow of line current. When there are two openings, two series voltages will be introduced, one at each opening, the relations between these series voltages and the line currents being given by equations (**66**) and (**67**). Equivalent circuits may replace the two openings just as they may replace the two simultaneous faults, but since these equivalent circuits are to be inserted in series with the lines in which the conductors are open they will depend upon the end connections of the lines.

When the lines are bussed at one end, and m' and n' are equal, an equivalent Y having the branch impedances $k' - m'$, $l' - m'$ and m' may be inserted by opening the lines at the common point and connecting the branch $k' - m'$ to the line in which the opening occurs at C, the branch $l' - m'$ to the line with the opening at D and the branch m' to the common point. When m' and n' are not equal the general equivalent circuit or the approximate equivalent circuit may be inserted in the same manner. These circuits may be obtained from Figs. 4 and 5 if k, l, m, n, I_{a1} and $I_{\alpha 1}$ are replaced by k', l', m', n', i_{a1} and $i_{\alpha 1}$, respectively.

When the lines are not bussed at either end the positive currents may be determined analytically as under two simultaneous faults, remembering that the voltages at C and D are series voltages and the currents are line currents. In the special case where m' and n' are equal the openings in the two conductors may be replaced by a four terminal mesh network similar to that shown in Fig. 17B, Appendix C. If terminals 1 and 2 are connected across the opening at point C and 3 and 4 across the opening at point D, the impedances of the six branches may be determined from equations (**1c**) Z_a, Z_b and Z_{ab} being replaced by k', l', and m' respectively.

When the lines are bussed at both ends a single impedance may replace the lines with the openings in the positive phase system, Fig. 11.

Equivalent circuit replacing in the positive phase system two lines bussed at both ends each having one conductor open. Let the lines with the openings at C and D, having positive phase impedances between terminals Z_c and Z_d, respectively, be bussed at points R and T. Let the sum of the currents entering the lines at R and leaving them at T be I_r, and let V_r and V_t be the voltages at R and T, respectively, Fig. 11A.

Then

$$I_r = i_{a1} + i_{\alpha 1} \tag{68}$$

$$Z_{rt} = \frac{V_r - V_t}{i_{a1} + i_{\alpha 1}} = \text{equivalent impedances} \tag{69}$$

$$V_r - V_t = Z_c i_{a1} + e_{a1} \tag{70}$$

$$V_r - V_t = Z_d i_{\alpha 1} + e_{\alpha 1} \tag{71}$$

Substituting (**66**) and (**67**) in (**70**) and (**71**), respectively,

$$V_r - V_t = (Z_c + k')\, i_{a1} + m'\, i_{\alpha 1} \tag{72}$$

$$V_r - V_t = n'\, i_{a1} + (Z_d + l')\, i_{\alpha 1} \tag{73}$$

Solving (**72**) and (**73**) for i_{a1} and $i_{\alpha 1}$

$$i_{a1} = \frac{Z_d + l' - m'}{(Z_c + k')\,(Z_d + l') - m'\, n'}\,(V_r - V_t) \tag{74}$$

$$i_{\alpha 1} = \frac{Z_c + k' - n'}{(Z_c + k')\,(Z_d + l') - m'\, n'}\,(V_r - V_t) \tag{75}$$

Adding (**74**) and (**75**) and substituting in (**69**)

$$Z_{rt} = \frac{(Z_c + k')\,(Z_d + l') - m'\, n'}{Z_c + Z_d + k' + l' - m' - n'} \tag{76}$$

Equation (**76**) gives the equivalent impedance, Z_{rt}, which replaces the two lines bussed at both ends each having one conductor open. Fig. 11B.

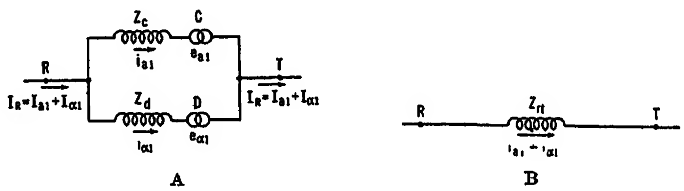


FIG. 11A AND FIG. 11B—POSITIVE PHASE SEQUENCE EQUIVALENT CIRCUITS REPLACING TWO LINES BUSSED AT BOTH ENDS, EACH HAVING ONE CONDUCTOR OPEN

SHORT-CIRCUIT AND STABILITY PROBLEMS

Methods have been given for determining the currents and voltages in all parts of the system during simultaneous faults when there are any number of connected machines. For practical problems it is not ordinarily required to know the exact currents and voltages in all parts of the system.

For short-circuit studies when a high degree of precision is not required, resistance and capacitance are neglected, and all generated voltages are assumed equal and in phase.

In stability studies one equivalent machine is often used to replace several machines or groups of machines, so that the number of machines involved in the calculations is small. It is necessary to know the generated voltages on all machines or equivalent machines in order to calculate the power which each sends out or receives, but it is not required to know the currents if power is calculated from voltages, angular displacements, and impedances. In order to determine the stability[7,8] of a system during transient conditions the transfer[8] impedance between the various machines before, during, and after the disturbance must be known,

and also the driving point[8] impedance of each machine when resistance is taken into account.

The way in which the analytical methods developed in the preceding part of the paper may be applied to actual problems is shown in the two examples given below.

Problem I. Short-Circuit Study. Simultaneous double line-to-ground faults occur on two circuits; phases a and c are involved at point C, and phases b and c (β and γ) at point D. Find the currents in the six conductors in "per unit"[9] neglecting resistance and assuming no-load excitations on all synchronous machines.

Fig. 12A gives the simplified positive phase diagram of the system. Since the currents in the faulted lines are the only ones required, the rest of the system has been reduced to an equivalent generator viewed from H, with transient reactance $x_d' = 0.34$ and excitation $E_{a1} = 1.00$. Negative phase impedances are taken equal to positive phase impedances, Fig. 12C. The zero phase impedance between C and the zero potential point, S, is 0.904 and between D and S is 1.80, and there is no mutual between the two circuits, Fig. 12B.

Since the zero and negative phase diagrams are already in simplest form the impedances of the branches of the Y's may be tabulated:

$$S_0 = 0 \qquad C_0 = j\,0.904 \qquad D_0 = j\,1.80$$
$$S_2 = j\,0.34 \qquad C_2 = j\,1.06 \qquad D_2 = j\,0.75$$

Substituting these branch impedances in Table II, G (*b*), k, l, m and n will be calculated.

$$Z_{22} = j\,1.06 \times j\,0.34 + j\,1.06 \times j\,0.75 + j\,0.34 \times j\,0.75 = -\,1.410$$

$$Z_{00} = j\,0.904 \times j\,1.80 = -\,1.627$$

$$Z_{cds} = j\,1.964 \times j\,2.55 + j\,1.964 \times j\,0.34 + j\,2.55 \times j\,0.34 = -\,6.54$$

$$3\,S_0\,S_2 = 0$$

$$k = \frac{-\,1.410 \times j\,0.904 - 1.627 \times j\,1.40}{-\,6.54} = j\ 0.543$$

$$l = \frac{-\,1.410 \times j\,1.80 - 1.627 \times j\,1.09}{-\,6.54} = j\ 0.660$$

$$m = \frac{a^2 \times j\,0.34 \times (-\,1.627)}{-\,6.54} = 0.0732 - j\,0.0423$$

$$n = \frac{a \times j\,0.34 \times (-\,1.627)}{-\,6.54} = -\,0.0732 - j\,0.0423$$

In the positive phase diagram the equivalent circuit for two unloaded feeders radiating from the same point H, may be used and Z_{hg} calculated from equation (33).

$$Z_{hg} = \frac{(j\,1.06 + j\,0.543)\,(j\,0.75 + j\,0.660) + 0.007}{j\,1.06 + j\,0.543 + j\,0.75 + j\,0.660 + j\,0.846}$$

$$= \frac{-\,2.253}{j\,3.098} = j\,0.727$$

$$I_h = I_{a1} + I_{\alpha 1} = \frac{E_{a1}}{j\,0.34 + j\,0.727} = \frac{1.00}{j\,1.067} = -j\ 0.937$$

$$V_h = E_{a1} - (j\,0.34) \times (-j\,0.937) = 1. - 0.3186 = 0.681$$

From (31) and (32)

$$I_{a1} = -\,j\,0.937\,\frac{j\,0.75 + j\,0.66 - (0.0732 - j\,0.0423)}{j\,3.098}$$
$$= 0.022 - j\,0.439$$

$$I_{\alpha 1} = -\,j\,0.937\,\frac{j\,1.06 + j\,0.543 - (-\,0.0732 - j\,0.0423)}{j\,3.098}$$
$$= -\,0.022 - j\,0.498$$

From equations (22) and (23)

$$V_{a1} = 0.681 - (0.0221 - j\,0.439)\,(j\,1.06)$$
$$= 0.216 - j\,0.023$$

$$V_{\alpha 1} = 0.681 - (-\,0.0221 - j\,0.498)\,(j\,0.75)$$
$$= 0.308 + j\,0.016$$

From Table I, C(b) and C(a)

$$V_{a0} = a^2\,V_{a1} = -\,0.128 - j\,0.1755$$
$$V_{a2} = a\,V_{a1} = -\,0.088 + j\,0.1985$$
$$V_{\alpha 0} = V_{\alpha 2} = V_{\alpha 1} = 0.308 + j\,.016$$

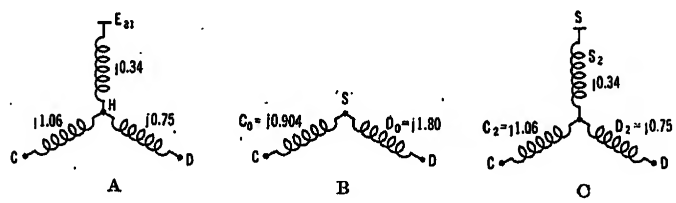


FIG. 12

A—POSITIVE PHASE SEQUENCE DIAGRAM FOR PROBLEM I

B—EQUIVALENT Y FOR ZERO PHASE SEQUENCE DIAGRAM

C—EQUIVALENT Y FOR NEGATIVE PHASE SEQUENCE DIAGRAM

From Fig. 12B and equations (5) and (6)

$$I_{a0} = -\,\frac{V_{a0}}{j\,0.904} = 0.194 - j\,0.142$$

$$I_{\alpha 0} = -\,\frac{V_{\alpha 0}}{j\,1.80} = -\,0.009 + j\,0.171$$

From Fig. 12C and equations (12) and (13)

$$I_{a2} = -\,(-\,0.088 + j\,0.1985)\,\frac{j\,1.09}{-\,1.410} + (0.308 + j\,0.016)\,\frac{j\,0.34}{-\,1.410} = -\,0.150 - j\,0.142$$

$$I_{\alpha 2} = -\,(-\,0.088 + j\,0.1985)\,\frac{j\,0.34}{-\,1.410} - (0.308 + j\,0.016)\,\frac{j\,1.40}{-\,1.410} = 0.032 + j\,0.327$$

Substituting the component currents in equations (1a)—(3a), the line currents are determined.

$$I_a = 0.066 - j\,0.723$$
$$I_b = 0$$
$$I_c = 0.515 + j\,0.298$$
$$I_\alpha = 0$$
$$I\beta = -\,0.729 + j\,0.303$$
$$I\gamma = 0.701 + j\,0.210$$

If the approximate equivalent circuit, Fig. 5, is used in the positive phase diagram, Fig. 13 would result.

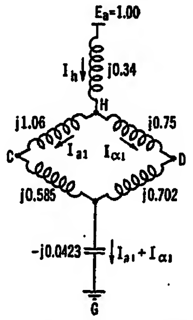


FIG. 13—APPROXIMATE POSITIVE PHASE SEQUENCE DIAGRAM FOR PROBLEM I

From Fig. 13, $I_h = -\,j\,0.935$, $I_{\alpha 1} = -\,j\,0.438$, $I_{\alpha 1} = -$ j 0.497. These values check those obtained by the exact method very closely, and the line currents calculated by using the approximate equivalent circuit would differ very little from those given above.

Problem II. Transfer Impedances (for Stability Study). Find the transfer impedance between the generator *G*. and the equivalent motor, *M*, in Fig.14A with simultaneous line-to-ground faults at points *C* and *D*, conductor *b* being grounded at *C* and *α* at *D*. Find also the transfer impedance when conductors *b* and *α* are open. Reactances are given in per unit on the generator base. Transformers are Δ-*Y*, solidly grounded on the high sides. The four transmission lines are on double-circuit towers, which are not on the same right-of-way, and have light steel ground wires. Power is delivered at two points, *B* and *E*, to a system which has been replaced by an equivalent motor, *M*, and two impedances.

The negative phase reactances of the system have been indicated on Fig. 14A when they differ from the positive phase values. The zero phase diagram has been constructed, Fig. 14B, using the average zero phase line and mutual reactances given in Appendix C.

(a) *Transfer impedance between G and M with fault on.* The negative and zero diagrams reduced to equivalent *Y*'s are shown in Figs. 14C and 14D, respectively.

Tabulating the branch impedances of these equivalent *Y*'s:

$C_0 = j\,0.175 \qquad D_0 = j\,0.175 \qquad S_0 = j\,0.0066$

$C_2 = j\,0.16 \qquad D_2 = j\,0.16 \qquad S_2 = j\,0.13$

Substituting these values in Table II, *E*(b).

$$k = j\,0.4716$$
$$l = j\,0.4716$$
$$m = -\,0.1067 - j\,0.0683$$
$$n = 0.1067 - j\,0.0683$$

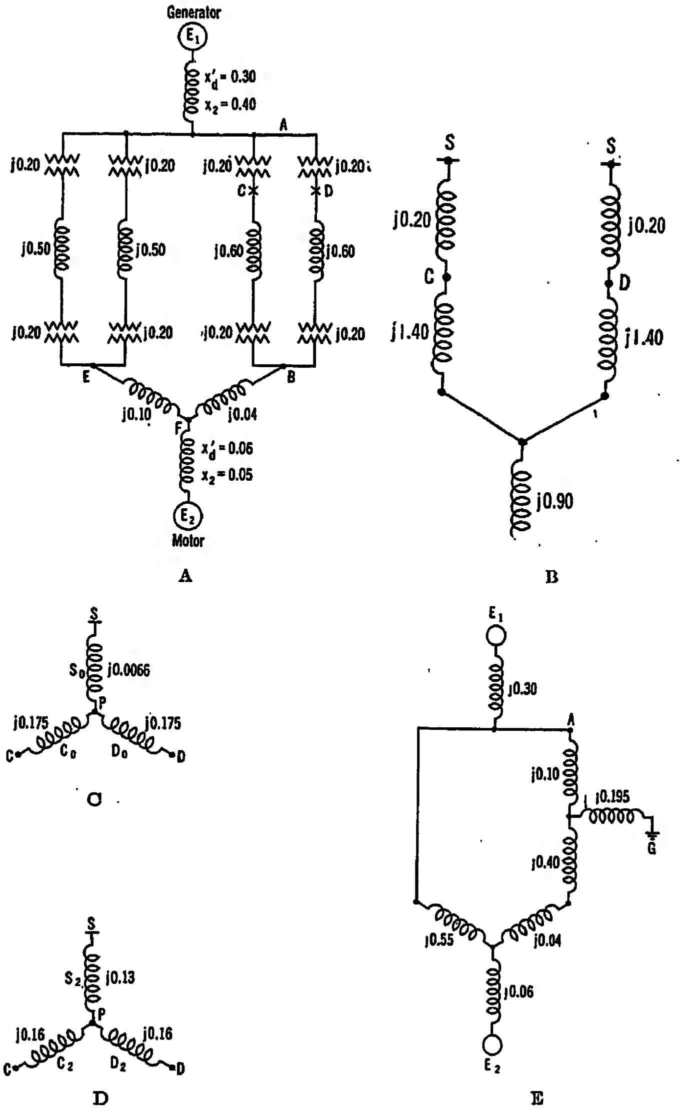


FIG. 14

A—SIMPLIFIED SYSTEM DIAGRAM FOR PROBLEM IIA

B—ZERO PHASE SEQUENCE DIAGRAM

C—EQUIVALENT *Y* FOR ZERO PHASE SEQUENCE DIAGRAM

D—EQUIVALENT *Y* FOR NEGATIVE PHASE SEQUENCE DIAGRAM

E—EQUIVALENT POSITIVE PHASE SEQUENCE DIAGRAM WITH LINES AND FAULTS REPLACED BY EQUIVALENT *Y*

The two faults with the lines upon which they occur may be replaced by an equivalent *Y*, Fig. 8B, having the branch impedances

$$Z_{rp} = \frac{a}{2} = j\,0.10$$

$$Z_{tp} = \frac{b}{2} = j\,0.40$$

$$Z_{pg} = \frac{-\ .16 \times j.8066 + 2 \times j1.00 \times (-\,0.2063)}{-\ 0.64 + 2 \times j\,1.00 \times j\,1.080} = j0.195$$

Inserting this equivalent Y in the positive phase diagram, Fig. 14E is obtained, from which the transfer impedance, Z_{12}, between E_1 and E_2 is readily determined

$$Z_{12} = j\,1.052$$

(b) *Transfer impedance berween G and M with two conductors open.* Figs. 15A and B show the zero and negative phase networks respectively reduced to their simplest series impedance diagrams, the identity of the lines with the open conductors being preserved.

If C_0, D_0, S_0, C_2, D_2 and S_2 are replaced by c_0, d_0, s_0, c_2, d_2 and s_2 respectively, k', l', m' and n' may be calculated from Table II, G(b).

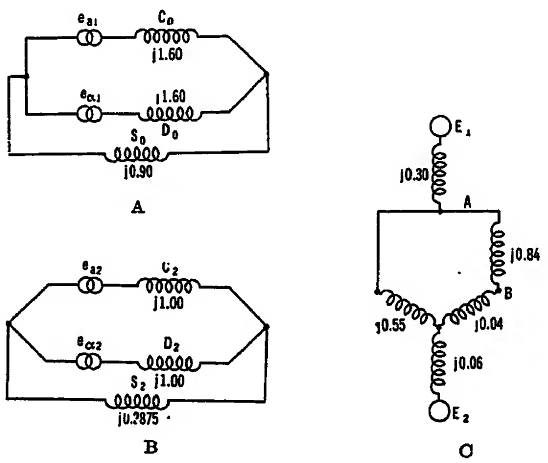


FIG. 15

A—ZERO PHASE SEQUENCE SERIES DIAGRAM FOR PROBLEM IIb

B—NEGATIVE PHASE SEQUENCE SERIES DIAGRAM FOR PROBLEM IIb

C—POSITIVE PHASE SEQUENCE DIAGRAM WITH LINES CONTAINING OPEN CONDUCTORS REPLACED BY SINGLE IMPEDANCE

$$c_0 = j\,1.60 \qquad d_0 = j\,1.60 \qquad s_0 = j\ 0.90$$
$$c_2 = j\,1.00 \qquad d_2 = j\,1.00 \qquad s_2 = j\ 0.2875$$

$$Z_{00} = c_0\,d_0 + c_0\,s_0 + d_0\,s_0 = -\ 5.44$$
$$Z_{22} = c_2\,d_2 + c_2\,s_2 + d_2\,s_2 = -\ 1.575$$
$$Z_{cds} = j\,2.60 \times j\,2.60 + j\,2.60 \times j\,1.1875 + j\,2.60 \times j\,1.1875 = -\ 12.935$$
$$3\,s_0\,s_2 = 3 \times j\,0.90 \times j\,0.2875 = -\ 0.776$$
$$Z_{cds} + 3\,S_0\,S_2 = -\ 13.71$$
$$k' = j\,0.799$$
$$l' = j\,0.799$$
$$m' = 0.0093 - j\,0.1086$$
$$n' = -\ 0.0093 - j\,0.1086$$

Substituting these values of k', l', m', n' in equation (76) the value of the impedance, Z_{rt}, to replace the two lines with the two open conductors may be obtained. Z_c and Z_d are the positive phase reactances of the two lines.

$$Z_c = Z_d = j\,1.00$$

$$Z_{rt} = \frac{j\,1.799 \times j\,1.799 + 0.0119}{j\,3.598 + j\,0.217} = j\,0.840$$

If the impedance Z_{rt} is inserted in the positive phase diagram to replace the two lines with two conductors open, Fig. 15C is obtained, from which the transfer impedance, Z_{12}, between E_1 and E_2 may be calculated.

$$Z_{12} = 0.798$$

ACKNOWLEDGMENT

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Appendix A

Fundamental Symmetrical Component Equations:[2]

$$I_a = I_{a0} + I_{a1} + I_{a2} \qquad (1a)$$
$$I_b = I_{a0} + a^2\,I_{a1} + a\,I_2 \qquad (2a)$$
$$I_c = I_{a0} + a\,I_{a1} + a^2\,I_{a2} \qquad (3a)$$
$$V_a = V_{a0} + V_{a1} + V_{a2} \qquad (4a)$$
$$V_b = V_{a0} + a^2\,V_{a1} + a\,V_{a2} \qquad (5a)$$
$$V_c = V_{a0} + a\,V_{a1} + a^2\,V_{a2} \qquad (6a)$$
$$I_{a0} = \frac{1}{3}\,(I_a + I_b + I_c) \qquad (7a)$$
$$I_{a1} = \frac{1}{3}\,(I_a + a\,I_b + a^2\,I_c) \qquad (8a)$$
$$I_{a2} = \frac{1}{3}\,(I_a + a^2\,I_b + a\,I_c) \qquad (9a)$$
$$V_{a0} = \frac{1}{3}\,(V_a + V_b + V_c) \qquad (10a)$$
$$V_{a1} = \frac{1}{3}\,(V_a + a\,V_b + a^2\,V_c) \qquad (11a)$$
$$V_{a2} = \frac{1}{3}\,(V_a + a^2\,V_b + a\,V_c) \qquad (12a)$$

where

$$a = -\,\frac{1}{2} + j\,\frac{\sqrt{3}}{2} = 1.00\ \underline{/120^\circ} \qquad (13a)$$

$$a^2 = -\,\frac{1}{2} - j\,\frac{\sqrt{3}}{2} = 1.00\ \underline{/240^\circ} = 1.00\ \underline{/-\,120^\circ} \qquad (14a)$$

Notation

I_a, I_b and I_c are the three line currents at any point of the system.

V_a, V_b and V_c are the three voltages to ground at any point of the system.

I_{a1}, I_{b1} and I_{c1} are the positive phase sequence currents in the three conductors. By definition these currents are equal in magnitude and I_{a1} leads I_{b1} by 120 deg. and I_{c1} by 240 deg.

I_{a2}, I_{b2} and I_{c2} are the negative phase sequence currents in the three conductors. By definition these currents are equal in magnitude and I_{a2} leads I_{c2} by 120 deg. and I_{b2} by 240 deg.

I_{a0}, I_{b0} and I_{c0} are the zero phase sequence currents in the three conductors. By definition these currents are equal in magnitude and in phase.

Notation for components of voltage corresponds to that for components of currents.

Appendix B

Relations Between the Components of Current and of Voltage at a Single Fault for Various Types of Fault

Let I_a, I_b and I_c be the currents flowing *into* the fault from lines a, b and c respectively, and V_a, V_b and V_c the voltages to ground of phases a, b and c respectively, all at point of fault, C.

When a ground occurs on a three-phase system the conductors which become grounded have zero voltage to ground and the conductors which are not involved have zero current flowing into the fault. When the fault is between conductors and not to ground, voltages to ground on the faulted conductors are equal and the sum of the currents flowing into the fault is zero. Thus three equations may be written in terms of actual currents or voltages, which when used with the fundamental equations (1a)—(12a) of Appendix A permit zero and negative phase currents and voltages to be expressed in terms of positive phase currents and voltage, respectively.

In these fundamental equations phase a is taken as reference phase, and the currents and voltages of phases b and c are expressed in terms of the symmetrical components of phase a.

Negative and zero phase voltages and currents will be expressed in terms of positive phase voltages and currents respectively for typical fault conditions.

A. Line-to-ground Fault.

Fault on phase b, Fig. 16A

Fault conditions: $I_a = I_c = 0$ and $V_b = 0$.

From (5a): $V_{a1} = -(a\, V_{a0} + a^2\, V_{a2})$

From (7a), (8a) and (9a):

$$I_{a0} = \frac{I_b}{3},\ I_{a1} = \frac{a\, I_b}{3},\ I_{a2} = \frac{a^2\, I_b}{3}$$

$$\therefore\ I_{a0} = a^2\, I_{a1} \text{ or } a\, I_{a0} = I_{a1}$$
$$\text{and } I_{a2} = a\, I_{a1} \text{ or } a^2\, I_{a2} = I_{a1}$$

B. Line-to-Line Faults.

Fault on phases a and b, Fig. 16B

Fault conditions: $I_c = 0$, $I_b = -I_a$ and $V_a = V_b$.

From (7a): $I_{a0} = 0$

$$\text{From (8a): } I_{a1} = \frac{I_a\,(1-a)}{3}$$

$$\text{From (9a): } I_{a2} = \frac{I_a\,(1-a^2)}{3}$$

$$\therefore\ I_{a2} = \frac{1-a^2}{1-a} I_{a1} = -a^2\, I_{a1} \text{ or } -a\, I_{a2} = I_{a1}$$

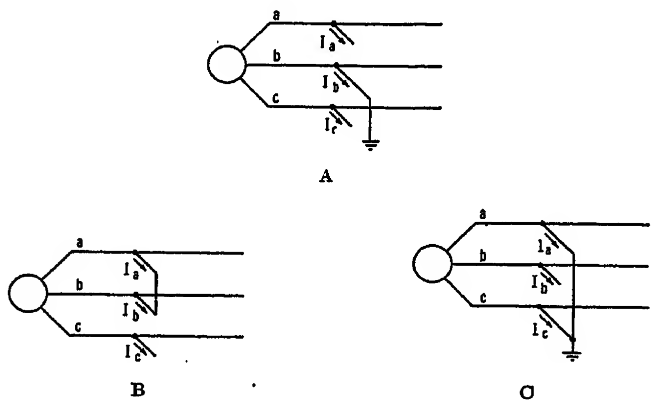

Fig. 16

A—Currents Flowing Into Single Line-to-Ground Fault

B—Currents Flowing Into Line-to-Line Fault

C—Currents Flowing Into Double Line-to- Ground Fault

From (4a) and (5a):

$$V_a - V_b = (1-a^2)\, V_{a1} + (1-a)\, V_{a2} = 0$$

$$\therefore\ V_{a2} = -\frac{1-a^2}{1-a} V_{a1} = a^2\, V_{a1} \text{ or } V_{a1} = a\, V_{a2}$$

C. Double Line-to-Ground Fault.

Fault on phases a and c, Fig. 16C

Fault conditions: $V_a = V_c = 0$ and $I_b = 0$.

From (2a) $I_b = I_{a0} + a^2 I_{a1} + a\, I_{a2} = 0$

$$\therefore\ I_{a1} = -(a\, I_{a0} + a^2\, I_{a2})$$

From (10a), (11a), (12a):

$$V_{a0} = \frac{V_b}{3},\ V_{a1} = \frac{a\, V_b}{3},\ V_{a2} = \frac{a^2\, V_b}{3}$$

$$\therefore\ V_{a0} = a^2\, V_{a1} \text{ or } V_{a1} = a\, V_{a0}$$
$$\text{and } V_{a2} = a\, V_{a1} \text{ or } V_{a1} = a^2\, V_{a2}$$

D. Three-Phase Fault.

(a) Conductors not grounded.

Fault conditions: $V_a = V_b = V_c$ and $I_a + I_b + I_c = 0$
From (**7a**): $I_{a0} = 0$
From (**11a**): $V_{a1} = 0$
From (**12a**): $V_{a2} = 0$

(b) Conductors grounded.

Fault conditions: $V_a = V_b = V_c = 0$
From (**10a**): $V_{a0} = 0$
From (**11a**): $V_{a1} = 0$
From (**12a**): $V_{a2} = 0$

Appendix C

Zero Phase Equivalent Circuits for Two Parallel Transmission Lines

It will be assumed that the conductors of all circuits are completely transposed, so that the same impedance is offered to zero phase currents by each of the three conductors of any circuit.

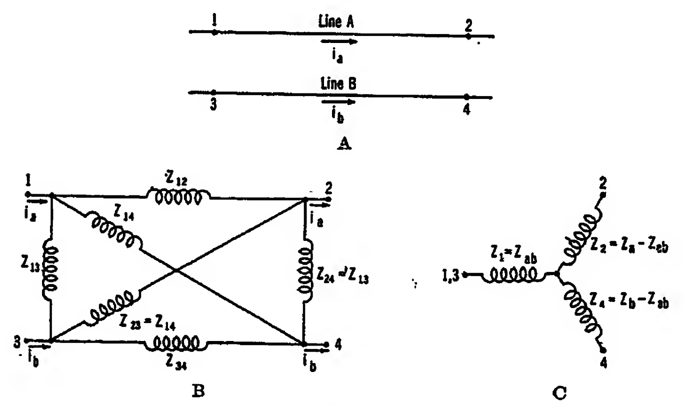

Fig. 17

A—Parallel Transmission Lines Having Impedance Z_a and Z_b and Mutual Z_{ab}

B—Equivalent Circuit to Replace the Two Parallel Lines Between Terminals 1, 2, 3 and 4

C—Equivalent Circuit to Replace Two Parallel Lines Bussed at One End but Not at the Other

1. Two Parallel Transmission Lines.

(*a*) *General Case.* Any two parallel transmission lines or portions of parallel transmission lines having impedances Z_a and Z_b and mutual reactance, Z_{ab}, between them may be replaced at their terminals by an equivalent circuit consisting of a six-branch network directly connecting the four terminals.[10] If 1 and 2 are the terminals of line A, and 3 and 4 the terminals of line B, see Fig. 17, then the six impedances of the equivalent circuit are:

$$Z_{12} = \frac{Z_a Z_b - Z_{ab}^2}{Z_b} = \text{impedance between points 1 and 2.}$$

$$Z_{34} = \frac{Z_a Z_b - Z_{ab}^2}{Z_a} = \text{impedance between points 3 and 4.}$$

$$Z_{13} = Z_{24} = \frac{Z_a Z_b - Z_{ab}^2}{Z_{ab}} = \text{impedance between points 1 and 3 and between points 2 and 4.}$$

$$Z_{14} = Z_{23} = -\frac{Z_a Z_b - Z_{ab}^2}{Z_{ab}} = \text{impedance between points 1 and 4 and between points 2 and 3.}$$

(**1c**)

Equations, (**1c**), were derived by analogy from similar equations for the equivalent circuit of a two winding transformer given by Mr. George A. Campbell in "Cisoidal Oscillations," p. 890, Proc. A. I. E. E., 1911.

(*b*) *Two Lines bussed at both ends.* If two parallel lines are bussed at both ends the equivalent circuit of Fig. 17B reduces to a single impedance branch Z_e where

$$Z_e = \frac{1}{\dfrac{1}{Z_{12}} + \dfrac{1}{Z_{14}} + \dfrac{1}{Z_{23}} + \dfrac{1}{Z_{34}}} = \frac{Z_a Z_b - Z_{ab}^2}{Z_a + Z_b - 2 Z_{ab}} \tag{2c}$$

If $Z_a = Z_b$

$$Z_e = \frac{Z_a + Z_{ab}}{2} \tag{3c}$$

Neglecting Resistance

$$X_e = \frac{X_a + X_{ab}}{2} \tag{4c}$$

(*c*) *Two Lines bussed at one end but not at the other.* If the lines are bussed so that points 1 and 3 are together Fig. 17B becomes a Δ connecting points 1, 2 and 4, which may be converted into a Y, Fig. 17C having branch impedances Z_1, Z_2 and Z_4 where

$$\left.\begin{aligned} Z_1 &= Z_{ab} \\ Z_2 &= Z_a - Z_{ab} \\ Z_4 &= Z_b - Z_{ab} \end{aligned}\right\} \tag{5c}$$

This case may be extended to include parallel lines which are bussed through unequal impedances. On the zero phase diagram of an actual system which includes two parallel lines it is usually possible to find a point where the parallel lines are bussed at one end or the other through impedances. Such a point may be the ground or any branch point of the system, or it may be obtained by Δ-Y or Y-Δ transformations.

If the impedance from the branch point 5 to the terminals 1 and 3 of the lines A and B are Z_α and Z_β respectively, an equivalent Y may be used between points 5, 2 and 4, having the branch impedances Z_5, Z_2 and Z_4, where

$$Z_5 = Z_{ab}$$
$$Z_2 = Z_a + Z_\alpha - Z_{ab}$$
$$Z_4 = Z_b + Z_\beta - Z_{ab}$$

These above relations follow directly from (**17c**) if Z_a and Z_b are the impedances from the junction point

and therefore include the external impedances Z_α and Z_β.

Mutual Reactance Between Two Parallel Transmission Lines

From (4c)

$$X_{ab} = 2\,X_e - X_a \tag{6c}$$

Equation (6c) gives the mutual reactance X_{ab} in terms of X_a, the reactance of one line alone, and X_e the reactance of the two lines in parallel. X_{ab} may be expressed in terms of X_1, the positive phase reactance of one line, by substituting for X_a and X_e their values in terms of X_1.

For lines without ground wires or with light steel ground wires.[7]

If $X_a = 3.5\,X_1$ and $X_e = 2.5\,X_1$ then

$$X_{ab} = 1.5\,X_1$$

For heavy copper ground wires.

If $X_a = 2\,X_1$ and $X_e = 1.5\,X_1$ then

$$X_{ab} = X_1$$

These approximate values for mutual reactance may be used in determining zero phase equivalent circuits for two parallel transmission lines when extreme accuracy is not required.

Appendix D

Relations Between Positive Phase Components of Voltage at the Faults and Positive Phase Currents Flowing Into the Faults for Two Simultaneous Faults

Table I expresses negative and zero phase voltages at the point of fault, and currents flowing into the fault in terms of the positive phase voltage at the fault and the positive phase current into the fault when grounds occur on one or more of the three conductors of a three-phase system, taking phase a as reference phase. Similar relations for a second point of fault are obtained by substituting α, β, γ for a, b, and c in Table I. These two sets of relations from Table I substituted in combinations of equations (5)—(14) allow V_{a1} and $V_{\alpha 1}$ to be expressed in terms of I_{a1} and $I_{\alpha 1}$.

Case E. Single Line-to-Ground Faults at Two Points.

(a) corresponding phases a and α.

Adding equations (5) and (10) and (6) and (11) and substituting from Table I, A (a):

$$V_{a1} = (C_0 + S_0 + C_2 + S_2)\,I_{a1} + (S_0 + S_2)\,I_{\alpha 1} \tag{1d}$$

$$V_{\alpha 1} = (S_0 + S_2)\,I_{a1} + (D_0 + S_0 + D_2 + S_2)\,I_{\alpha 1} \tag{2d}$$

(b) Phases b and α.

Multiplying equations (5) by a and equation (10) by a^2 and adding: adding equations (6) and (11); substituting from Table I, A (b) and A (a):

$$V_{a1} = (C_0 + S_0 + C_2 + S_2)\,I_{a1} + (a\,S_0 + a^2\,S_2)\,I_{\alpha 1} \tag{3d}$$

$$V_{\alpha 1} = (a^2\,S_0 + a\,S_2)\,I_{a1} + (D_0 + S_0 + D_2 + S_2)\,I_{\alpha 1} \tag{4d}$$

(c) Phases c and α.

Multiplying equation (5) by a^2 and equation (10) by a and adding; adding equations (6) and (11); substituting from Table I, A (c) and A (a):

$$V_{a1} = (C_0 + S_0 + C_2 + S_2)\,I_{a1} + (a^2\,S_0 + a\,S_2)\,I_{\alpha 1} \tag{5d}$$

$$V_{\alpha 1} = (a\,S_0 + a^2\,S_2)\,I_{a1} + (D_0 + S_0 + D_2 + S_2)\,I_{\alpha 1} \tag{6d}$$

Case F. Line-to-Line Faults at Two Points.

(a) Corresponding phases b, c and β, γ.

Replacing negative phase currents and voltages in equations (10) and (11) by their values in terms of positive phase currents and voltages from Table I, B(a):

$$V_{a1} = (C_2 + S_2)\,I_{a1} + S_2\,I_{\alpha 1} \tag{7d}$$

$$V_{\alpha 1} = S_2\,I_{a1} + (D_2 + S_2)\,I_{\alpha 1} \tag{8d}$$

(b) Phases a, c and β, γ.

Substituting from Table I, B (b) and B (a) in equation (10) multiplied by a^2, and in equation (11):

$$V_{a1} = (C_2 + S_2)\,I_{a1} + a^2\,S_2\,I_{\alpha 1} \tag{9d}$$

$$V_{\alpha 1} = a\,S_2\,I_{a1} + (D_2 + S_2)\,I_{\alpha 1} \tag{10d}$$

(c) Phases a, b and β, γ.

Substituting for Table I, B (c) and B (a) in equation (10) multiplied by a, and in equation (11):

$$V_{a1} = (C_2 + S_2)\,I_{a1} + a\,S_2\,I_{\alpha 1} \tag{11d}$$

$$V_{\alpha 1} = a^2\,S_2\,I_{a1} + (D_2 + S_2)\,I_{\alpha 1} \tag{12d}$$

Case G. Double Line-to-Ground Faults at Two Points.

(a) Corresponding phases b, c and β, γ.

Adding equations (7) and (12), and (8) and (13), and substituting from Table I, C (a).

$$I_{a1} = V_{a1}\left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}}\right) - V_{\alpha 1}\left(\frac{S_0}{Z_{00}} + \frac{S_2}{Z_{22}}\right) \tag{13d}$$

$$I_{\alpha 1} = -V_{a1}\left(\frac{S_0}{Z_{00}} + \frac{S_2}{Z_{22}}\right) + V_{\alpha 1}\left(\frac{C_0 + S_0}{Z_{00}} + \frac{C_2 + S_2}{Z_{22}}\right) \tag{14d}$$

Expressing V_{a1} and $V_{\alpha 1}$ in terms of I_{a1} and $I_{\alpha 1}$ by solving equations (13d) and (14d) and making use of equations (9) and (14):

$$V_{a1} = \frac{[Z_{22}\,(C_0 + S_0) + Z_{00}\,(C_2 + S_2)]\,I_{a1} + [Z_{22}\,S_0 + Z_{00}\,S_2]\,I_{\alpha 1}}{(C_0 + C_2)\,(D_0 + D_2) + (C_0 + C_2)\,(S_0 + S_2) + (D_0 + D_2)\,(S_0 + S_2)} \tag{15d}$$

$$V_{\alpha 1} = \frac{[Z_{22}\,S_0 + Z_{00}\,S_2]\,I_{a1} + [Z_{22}\,(D_0 + S_0) + Z_{00}\,(D_2 + S_2)]\,I_{\alpha 1}}{(C_0 + C_2)\,(D_0 + D_2) + (C_0 + C_2)\,(S_0 + S_2) + (D_0 + D_2)\,(S_0 + S_2)} \tag{16d}$$

(b) Phases a, c and β, γ.

Multiplying (7) by a and (12) by a^2 and adding: adding (8) and (13): substituting from Table I, C (b) and C(a):

$$I_{a1} = V_{a1}\left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}}\right) - V_{\alpha 1}\left(\frac{a\ S_0}{Z_{00}} + \frac{a^2 S_2}{Z_{22}}\right) \tag{17d}$$

$$I_{\alpha 1} = -V_{a1}\left(\frac{a^2 S_0}{Z_{00}} + \frac{a\ S_2}{Z_{22}}\right) + V_{\alpha 1}\left(\frac{C_0 + S_0}{Z_{00}} + \frac{C_2 + S_2}{Z_{22}}\right) \tag{18d}$$

Solving (17d) and (18d) for V_{a1} and $V_{\alpha 1}$, remembering that $a \times a^2 = 1$ and $a + a^2 = -1$:

$$V_{a1} = \frac{[Z_{22}(C_0 + S_0) + Z_{00}(C_2 + S_2)]\, I_{a1} + (a\, S_0 Z_{22} + a^2 S_2 Z_{00})\, I_{\alpha 1}}{(C_0 + C_2)(D_0 + D_2) + (C_0 + C_2)(S_0 + S_2) + (D_0 + D_2)(S_0 + S_2) + 3\, S_0 S_2} \tag{19d}$$

$$V_{\alpha 1} = \frac{(a^2 S_0 Z_{22} + a\, S_2 Z_{00})\, I_{a1} + [Z_{22}(D_0 + S_0) + Z_{00}(D_2 + S_2)]\, I_{\alpha 1}}{(C_0 + C_2)(D_0 + D_2) + (C_0 + C_2)(S_0 + S_2) + (D_0 + D_2)(S_0 + S_2) + 3\, S_0 S_2} \tag{20d}$$

(c) Phases a, b, and β, γ.

This case may be solved by analogy from (b), a replacing a^2, and a^2 replacing a in equations (19d) and (20d).

Case H. Three-Phase Faults at Two Points.

From Table I, D.

$$V_{a1} = 0 \tag{21d}$$

$$V_{\alpha 1} = 0 \tag{22d}$$

Case I. Line-to-Line Fault at C and Single Line-to-Ground Fault at D.

(a) Phases b, c and α.

Substituting from Table I, B (a) and A (a) in equations (10) and in (6) plus (11)

$$V_{a1} = I_{a1}(C_2 + S_2) - I_{\alpha 1} S_2 \tag{23d}$$

$$V_{\alpha 1} = -I_{a1} S_2 + I_{\alpha 1}(D_0 + S_0 + D_2 + S_2) \tag{24d}$$

(b) Phases a, c and α.

Substituting from Table I, B (b) and A (a) in equation (10) multiplied by a^2, and in equation (6) plus (11):

$$V_{a1} = I_{a1}(C_2 + S_2) - I_{\alpha 1} a^2 S_2 \tag{25d}$$

$$V_{\alpha 1} = -a\, S_2 I_{a1} + I_{\alpha 1}(D_0 + S_0 + D_2 + S_2) \tag{26d}$$

(c) Phases a, b, and α.

By analogy from (b) replacing a by a^2, and a^2 by a.

Case J. Double-Line-to-Ground Fault at C, Single Line-to-Ground Fault at D.

(a) Phases b, c and α.

Adding equations (7) and (12) and substituting from Table I, C (a):

$$I_{a1} = V_{a1}\left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}}\right) - V_{\alpha 0}\frac{S_0}{Z_{00}} - V_{\alpha 2}\frac{S_2}{Z_{22}} \tag{27d}$$

Transposing and simplifying equations (8) and (13) to express $V_{\alpha 0}$ in terms of V_{a0} and $I_{\alpha 0}$ and $V_{\alpha 2}$ in terms of V_{a2} and $I_{\alpha 2}$: then replacing V_{a0} and V_{a2} by V_{a1} from Table I, C (a) and $I_{\alpha 0}$ and $I_{\alpha 2}$ by $I_{\alpha 1}$ from A (a):

$$V_{\alpha 0} = V_{a1}\frac{S_0}{C_0 + S_0} - I_{\alpha 1}\frac{Z_{00}}{C_0 + S_0} \tag{28d}$$

$$V_{\alpha 2} = V_{a1}\frac{S_2}{C_2 + S_2} - I_{\alpha 1}\frac{Z_{22}}{C_2 + S_2} \tag{29d}$$

Substituting (28d) and (29d) in (27d), and replacing Z_{00} and Z_{22} by their values given in (9) and (14):

$$I_{a1} = V_{a1}\left(\frac{1}{C_0 + S_0} + \frac{1}{C_2 + S_2}\right) + I_{\alpha 1}\left(\frac{S_0}{C_0 + S_0} + \frac{S_2}{C_2 + S_2}\right) \tag{30d}$$

Transposing (30d)

$$V_{a1} = \frac{(C_0 + S_0)(C_2 + S_2)}{C_0 + S_0 + C_2 + S_2} I_{a1} - \frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{C_0 + S_0 + C_2 + S_2} I_{\alpha 1} \tag{31d}$$

Adding equations (28d) and (29d), and replacing $(V_{\alpha 0} + V_{\alpha 2})$ by $-V_{\alpha 1}$ from Table I, A (a) and V_{a1} by its value from (31d)

$$V_{\alpha 1} = -\frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{C_0 + S_0 + C_2 + S_2} I_{a1} + \left(D_0 + D_2 + S_0 + S_2 - \frac{(S_0 - S_2)^2}{C_0 + S_0 + C_2 + S_2}\right) I_{\alpha 1} \tag{32d}$$

(b) Phases a, c and α.

Multiplying equation (7) by a, equation (12) by a^2; adding and substituting from Table I, C (b):

$$I_{a1} = V_{a1}\left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}}\right) - a\, V_{\alpha 0}\frac{S_0}{Z_{00}} - a^2 V_{\alpha 2}\frac{S_2}{Z_{22}} \tag{33d}$$

From equations (8) and (13) and Table I, C (b) and A (a)

$$V_{\alpha 0} = a^2 V_{\alpha 1} \frac{S_0}{C_0 + S_0} - I_{\alpha 1} \frac{Z_{00}}{C_0 + S_0} \quad (34d)$$

$$V_{\alpha 2} = a V_{\alpha 1} \frac{S_2}{C_2 + S_2} - I_{\alpha 1} \frac{Z_{22}}{C_2 + S_2} \quad (35d)$$

Substituting (34d) and (35d) in (33d) and transposing:

$$V_{\alpha 1} = \frac{(C_0 + S_0)(C_2 + S_2)}{C_0 + S_0 + C_2 + S_2} I_{\alpha 1} - \frac{a S_0 (C_2 + S_2) + a^2 S_2 (C_0 + S_0)}{C_0 + S_0 + C_2 + S_2} I_{\alpha 1} \quad (36d)$$

Adding (34d) and (35d), replacing $(V_{\alpha 0} + V_{\alpha 2})$ by $- V_{\alpha 1}$ from Table I, A (a) and $V_{\alpha 1}$ by its value from (36d):

$$V_{\alpha 1} = - \frac{a^2 S_0 (C_2 + S_2) + a S_2 (C_0 + S_0)}{C_0 + S_0 + C_2 + S_2} I_{\alpha 1} + \left(D_0 + D_2 + S_0 + S_2 - \frac{S_0^2 + S_0 S_2 + S_2^2}{C_0 + S_0 + C_2 + S_2} \right) I_{\alpha 1} \quad (37d)$$

(c) Phases a, b and α.

By analogy from (36d) and (37d) replacing a by a^2, and a^2 by a.

Case K. Three-Phase Fault at C and Single Line-to-Ground Fault at D, .

(a) Three-phase fault not involving ground.

Substituting $I_{a0} = 0$ from Table I, D (a) in equations (5) and (6), and $V_{\alpha 2} = 0$ in equation (10):

$$- V_{\alpha 0} = I_{\alpha 0} S_0 \quad (38d)$$

$$- V_{\alpha 0} = I_{\alpha 0} (D_0 + S_0) \quad (39d)$$

$$I_{a2} = - I_{\alpha 2} \frac{S_2}{C_2 + S_2} \quad (40d)$$

Substituting (40d) in (11) and simplifying

$$- V_{\alpha 2} = I_{a2} \left(D_2 + \frac{C_2 S_2}{C_2 + S_2} \right) \quad (41d)$$

adding (39d) and (41d) and substituting from Table I, A (a):

$$V_{\alpha 1} = I_{\alpha 1} \left(D_0 + S_0 + D_2 + \frac{C_2 S_2}{C_2 + S_2} \right) \quad (42d)$$

From Table I, D.

$$V_{a1} = 0 \quad (43d)$$

(b) Three-phase fault involving ground.

Substituting $V_{a0} = 0$, and $V_{a2} = 0$ from Table I, D (b) in (5) and (10) and expressing I_{a0} and I_{a2} in terms of $I_{\alpha 0}$ and $I_{\alpha 2}$.

$$I_{a0} = - I_{\alpha 0} \frac{S_0}{C_0 + S_0} \quad (44d)$$

$$I_{a2} = - I_{\alpha 2} \frac{S_2}{C_2 + S_2} \quad (45d)$$

Adding (6) and (11), replacing I_{a0} and I_{a2} by their values given in (44d) and (45d), and substituting from Table I, A (a) and simplifying:

$$V_{\alpha 1} = \left(D_0 + \frac{C_0 S_0}{C_0 + S_0} + D_2 + \frac{C_2 S_2}{C_2 + S_2} \right) I_{\alpha 1} \quad (46d)$$

$$V_{a0} = 0 \quad (47d)$$

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Discussion

M. MacFerran: Miss Clarke's paper is of great practical value because, as is pointed out in the introduction, simultaneous faults do occur quite frequently especially when multicircuit tower lines are in use. For example, on the system of the Southern California Edison Co., Ltd., there are many tower lines carrying from two to twelve circuits, and there have been several cases in which trouble on such multicircuit structures has involved two lines going to entirely different parts of the system. Another very interesting case of simultaneous faulting occurred at a substation which was being operated with buses sectionalized, one section being on one major station, the other on another. Trouble originating on one bus spread to the other, thus producing a condition quite similar to that covered by the numerical example in the paper.

From the standpoint of one who is faced with the task of calculating simultaneous faults, the outstanding features of Miss Clarke's paper are first, the derivation of the values of k, l, m, and n for all conceivable combinations of faults; and second, the development of the approximate equivalent fault wye which can be used with a high degree of accuracy even when m is not equal to n. These two features make it possible to work out the complete story of any type of simultaneous fault in a quite reasonable length of time, and therefore enable one to give due attention to such faults in setting relays or analyzing their performance. This in turn represents an important step forward in protection engineering.

H. W. Bibber: While Miss Clarke's paper deals very exhaustively with problems involving simultaneous faults as they are

met in cases of greatest practical importance,—*i. e.*, where one fault is on one circuit and another is on a different circuit of the same *grounded* system—it seems possible that there may be cases where a solution is desired for simultaneous faults on an *ungrounded* system.

Perhaps because ungrounded systems are more common in Europe, the special case of simultaneous faults on a single-circuit line of an isolated neutral system has been treated to a limited extent in the European technical press,* but Miss Clarke has presented a much more general method which can be extended to cover such a special case.

Employing methods similar in nature to those which Miss Clarke uses for simultaneous faults on grounded systems, to solve for currents and voltages in isolated neutral systems gives interesting results.

On an isolated system, it is obvious that both of two simultaneous faults must involve ground or there will be no zero phase currents.

The equivalent circuit for the zero phase system if both faults involve ground will be a simple loop which will usually be easily obtainable by calculation. Should complication of the network make the use of a calculating table advisable, only one reading is necessary, that to determine $(C_o + D_o)$, following Miss Clarke's nomenclature.

The significant fact regarding the zero phase sequence currents at the fault points in an isolated system, is that one is the negative of the other. Since the zero phase circuit is a simple loop there will be a single independent voltage equation that can be written. The two relations to be noted are therefore, (cf. equations (7) and (8).

$$I_{ao} = - I_{\alpha o}$$

$$I_{ao} = \frac{V_{ao} - V_{\alpha o}}{C_o + D_o}$$

As far as the negative phase system is concerned, the grounding of the system is immaterial, so that an equivalent Y can be obtained for the isolated system as described by Miss Clarke. Equations (10) to (14) inclusive of her paper hold for this case.

As she has shown, there will be six equations connecting the components of current and the components of voltage to ground at the fault points, and in addition to the two equations connecting negative phase currents and voltages, such as (12) and (13), there are the two equations connecting the zero phase currents and voltages I have previously given. This provides ten simultaneous equations so that eight unknowns can be eliminated and the positive components of current, I_{a1} and $I_{\alpha 1}$ may be expressed in terms of each other, the positive components of voltage, V_{a1} and $V_{\alpha 1}$, and the known zero and negative phase impedances. Here as in Miss Clarke's paper, the ten equations are linear and we may write:

$$I_{a1} = A\, V_{a1} + B\, V_{\alpha 1}$$

$$I_{\alpha 1} = F\, I_{a1}$$

where A, B, and F depend on the branch impedances of the equivalent Y replacing the negative phase network $(C_o + D_o)$, and the type of faults. Values of A, B and F, can be derived for faults which involve various combinations of conductors, and tabulated in a form similar to Table II of Miss Clarke's paper.

In the time that has been available I have not been able to find any simple general equivalent circuit which will replace the two simultaneous faults in the positive phase system. For the special cases of two unloaded feeders radiating from a single point and two lines bused at both ends I have worked out equivalent impedances to replace the faults.

*"Double Ground Faults on a Single-Circuit Line Fed from Two Points in the Light of Symmetrical Components," by G. Oberdorfer, *Wissenschaftliche Veroffentlichen aus dem Siemens*—Konzern IX Band 1930.

For other than the special cases mentioned, it is evident that the method of analytical solution described by Miss Clarke may be used for isolated systems, until such time as simple equivalent circuits may be brought forth. The two zero phase current and voltage relations I stated earlier as holding for the isolated neutral case, are substituted for equations (15) and (16). There will then be $(n + 4)$ equations with $(n + 4)$ unknowns when there are n machines in the system, and a complete solution can be obtained.

Although the case of two simultaneous faults on a single-circuit line of a *grounded* system has not been specifically mentioned by Miss Clarke, it is evident that her general methods hold for such a case, as C and D are any points at all on a system.

Viewed from the many different angles presented by different types of systems and fault conditions, it seems that Miss Clarke has pointed the way to the solution of almost any problem involving simultaneous faults that might arise.

W. C. Hahn: The material in this paper will be needed for the general case of simultaneous faults or open circuits on three-phase systems. There are three special cases, however, where the procedure can be simplified in making use of either the a-c. or d-c. calculating table. These are the cases of simultaneous three-phase, line-to-line, or double line-to-ground faults occurring on the same phases.

For simultaneous three-phase faults, of course, all that is necessary is to apply the several faults simultaneously on the usual set up.

For the case of simultaneous line-to-line faults on the same phase, such as discussed in Case F, Appendix D, sub-head "a," it is necessary to set up the positive and negative phase sequence diagrams simultaneously. Connections between the two networks are made to join the corresponding fault positions. The short-circuit plug is then applied at the neutral point of the negative phase sequence set up. A little consideration will show that this satisfies all the conditions given in Table I, providing all the faults are between the same two phases of the system.

The case of simultaneous double line-to-ground faults such as is given in Case G, Appendix D, sub-head "a," can also be represented by setting up the positive, negative and zero phase sequence diagrams on the calculating table. In this case connections between the networks are made between corresponding fault positions in all three networks.

It will be noted that these simplifications cannot be used for simultaneous line-to-ground faults, because in this case, it is necessary to insert, in the connection between the positive and zero sequence setup a negative phase sequence voltage corresponding to that fault. It may be possible to find an equivalent circuit for the negative phase sequence network, which will satisfy this condition, but it appears that such a procedure may be more complicated than following that given in the paper.

It is also not feasible to use these simplifications for simultaneous faults on different phases when using the d-c. calculating table, because from Table I, it will appear that in at least one of the connections between the networks, a phase shift of 120 deg. must be introduced both in current and in voltage. While it is possible to produce the equivalent of this shift on the d-c. calculating table, by various reconnections of the networks, it is believed that the method outlined in the paper is simpler and occasions less confusion.

H. L. Hazen: Until rather recently unsymmetrical short circuits on power system networks have been unmanageable in any but the simplest cases. Each of the single unsymmetrical faults, as Miss Clarke has pointed out, has previously been reduced to an equivalent circuit, representing the fault, which can be applied to the ordinary positive phase sequence diagram, thus reducing the problem to the relatively simple study of a

balanced polyphase system. This brings it within the scope of calculating-table attack, rendering solutions practicable even with large and complicated networks.

Miss Clarke in the present paper has extended this treatment to cover the much more complicated case of two simultaneous unsymmetrical faults, treating not only each of the various kinds of unsymmetrical faults but most of the combinations in which these can occur; and, which is very important from the system engineer's point of view, she has put it in a form well suited to calculating-table treatment. This she has done in a very clear and complete manner.

As she has pointed out, either the d-c. calculating table or a-c. network analyzer can be used for the calculations; the former where the relatively rough approximations involved in its use are satisfactory; the latter where the desire for more accurate results dictates the use of the a-c. device. The a-c. network analyzer of course enables one to deal with all qualities in complex form, to use actual values of individual generator excitations, and thus to obtain results which are a good approximation to the truth.

In making calculating-table solutions of various sorts involving equivalent circuits one not infrequently encounters the serious difficulty that the real portions of certain impedances are negative. These impedances therefore are not physically realizable as such by any simple means. For practical purposes this limitation can be overcome with relative ease in the following way: insert in place of the unrealizable impedance element an adjustable voltage which, in the alternating-current case, is variable in both phase and magnitude. With voltmeter, ammeter and wattmeter measure the voltage, current, and power of this element. By a cut-and-try method adjust this voltage until the desired vector relation $V = I\,Z$ exists as shown by the instrument readings. Provided the network as a whole is dissipative—which must be the case where the equivalent circuits are a representation of actual physical circuits—there is only one vector value of the inserted voltage which will satisfy this condition. Practically this process is not difficult to carry out, as has been demonstrated by its use on the M. I. T. Network Analyzer, and it removes what otherwise might be an embarrassing restriction upon certain calculating-table solutions.

Edith Clarke: It is gratifying to learn from Miss Macferran's discussion that an operating engineer has found the paper of practical value.

Mr. H. W. Bibber by extending the methods of the paper to the case of simultaneous faults on *ungrounded* systems, has materially enlarged its scope.

Mr. W. C. Hahn in his discussion emphasizes the important fact that simultaneous line-to-line or double-line-to-ground faults on the same phases may be treated most simply as special cases. I am glad Mr. Hahn has brought out this point, for although the case of simultaneous double line-to-ground faults on the same phases has been covered as a special case in the paper, no reference to this was made in Table II where the constants k, l, m and n are tabulated.

Under the heading "Equivalent Y vs. Equivalent Δ" (page 927 of the paper), it is shown that positive sequence currents and voltages at the fault points are satisfied if the zero and negative sequence networks are connected in parallel between the fault points of the positive sequence system, corresponding fault points in the three networks being connected, and also the points of zero potential for the three systems.

In a similar manner from equations **(7d)** and **(8d)**, by expressing I_{a1} and $I_{\alpha 1}$ in terms of V_{a1} and $V_{\alpha 1}$ and replacing the impedances of the equivalent Y by the admittances of the equivalent Δ, a proof may be obtained of Mr. Hahn's statement that simultaneous line-to-line faults on the same phases may be solved on the a-c. or d-c. calculating table by setting up the positive and negative sequence networks and joining corresponding fault points and points of zero potential in the two systems.

It is interesting to note from Mr. Hazen's discussion that circuits in which the real portions of certain impedances are negative (negative resistances) may now be represented on the M. I. T. Network Analyzer with the same degree of precision as the familiar positive resistances.

Power Equipment at New KDKA Station

BY R. L. DAVIS[1]
Associate, A. I. E. E.

THE new KDKA Station at Saxonburg, Pa., has been designed both for broadcasting and experimental work dealing with the development of large high-power transmitters. On this account the equipment is considerably larger than is found in any ordinary broadcasting station, the main rectifier being the largest high-voltage unit ever constructed for such use. The control and transfer features of the various power units are also considerably different than would be required for a conventional broadcasting transmitter, and designed not only with a view to safety and convenience for normal operation, but to facilitate experimental work as well.

The equipment and power apparatus has been laid out to provide the necessary special power supplies to two complete and independent high-power broadcasting transmitters, one being operated normally at 980 kilocycles, using the familiar signature of KDKA, while the other is planned for higher frequency work. Both of these equipments are designed to operate separately but for experimental purposes arrangements are made to operate either transmitter from the various power supplies of the other, thus providing the greatest variation possible in the range of power circuits as well as providing virtually duplicate equipment throughout for the main broadcasting transmitter. All of the power equipment, except transformers and reactors which are mounted outdoors, is arranged on the ground floor of the main station building, thus leaving the second floor entirely free for the radio apparatus and control boards. A view of the station building is shown in Fig. 1.

To meet possible future requirements of high power transmitters which quite likely may employ tubes much larger than any now known, it was deemed necessary to provide a plate voltage supply of 900 kw. at 30 kv. of high-voltage direct-current energy for the standard broadcast transmitter. Similarly, the filament heating requirements, were set at 3,000 amperes, 40 volts, and the grid voltage supply at 3,000 volts direct current. In addition to the foregoing, a moderate amount of power at 10 to 15 kv. and still smaller amounts at 3 kv. and 400 volts are needed for the various intermediate amplifiers and the master oscillator.

Because of the probable limitation of the high-frequency tubes it was not considered necessary to provide as large a main plate power supply system for the short-wave transmitter and therefore the second rectifier unit was designed for only 450 kw. at 22 kv. However, since either plate supply system may be used with either transmitter, the maximum plate power can also be used for any high-frequency experimental work that may require it. Approximately the same power supplies are needed for the intermediate amplifiers and master oscillator of both the high-frequency and the standard broadcast transmitter and therefore these were designed in duplicate to provide the greatest interchangeability of equipment. Filament heating requirements for the high-frequency transmitter were set at 2,000 amperes.

Since all high quality broadcasting equipment required pure direct current for grid plate and filament supplies, and because all commercial power circuits are alternating current, it can readily be recognized that the problem of converting the normal three-phase 60-cycle supply to meet the special needs of broadcasting is one of greatest importance in radio station design.

The two main rectifiers are of the polyphase type and operate directly from the 2,300-volt three-phase station

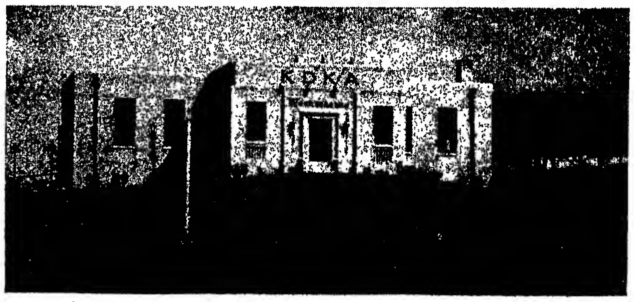

FIG. 1—STATION BUILDING

supply. The 900-kw. rectifier employs six 190-kva. single-phase transformers to provide the proper voltage and phase relations for rectification into the desired direct-current energy. Each of these six rectifier plate transformers has double high-voltage windings and these are connected to form four stars, the branches of the stars being displaced in phase by special connections so as to produce twelve-phase rectification. Each of the stars is connected to multi-anode mercury cathode rectifier tubes and the direct current output of the four tubes is connected in series so that while the rectification takes place in three-phase units the result is the same as if a single high-voltage multi-anode tube of twelve phases were employed. Connecting four stars in series in this manner also requires only a moderate inverse peak voltage per tube in spite of the unusually high d-c. voltage developed and by keeping the inverse voltage well within the conservative upper limit for mercury vapor tubes, conditions most suitable for a long trouble-free life are obtained. So far no noticeable troubles due

1. Westinghouse Electric and Mfg. Co., Chicopee Falls, Mass.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

to flashbacks and similar disturbances have been experienced with the tubes now in service at the new station.

The smaller, or 450-kw. rectifier, is similar to the larger one except only three single-phase plate transformers giving six-phase rectification are employed, this being considered sufficient for the voltage and power desired. These plate transformers are each provided with four high-voltage windings and four stars are formed which are connected to a similar number of rectifier tubes, as in the case of the twelve-phase rectifier, but only six-phase rectification is obtained as no effort was made to provide the special phase relations between the various stars as in the case of the larger unit.

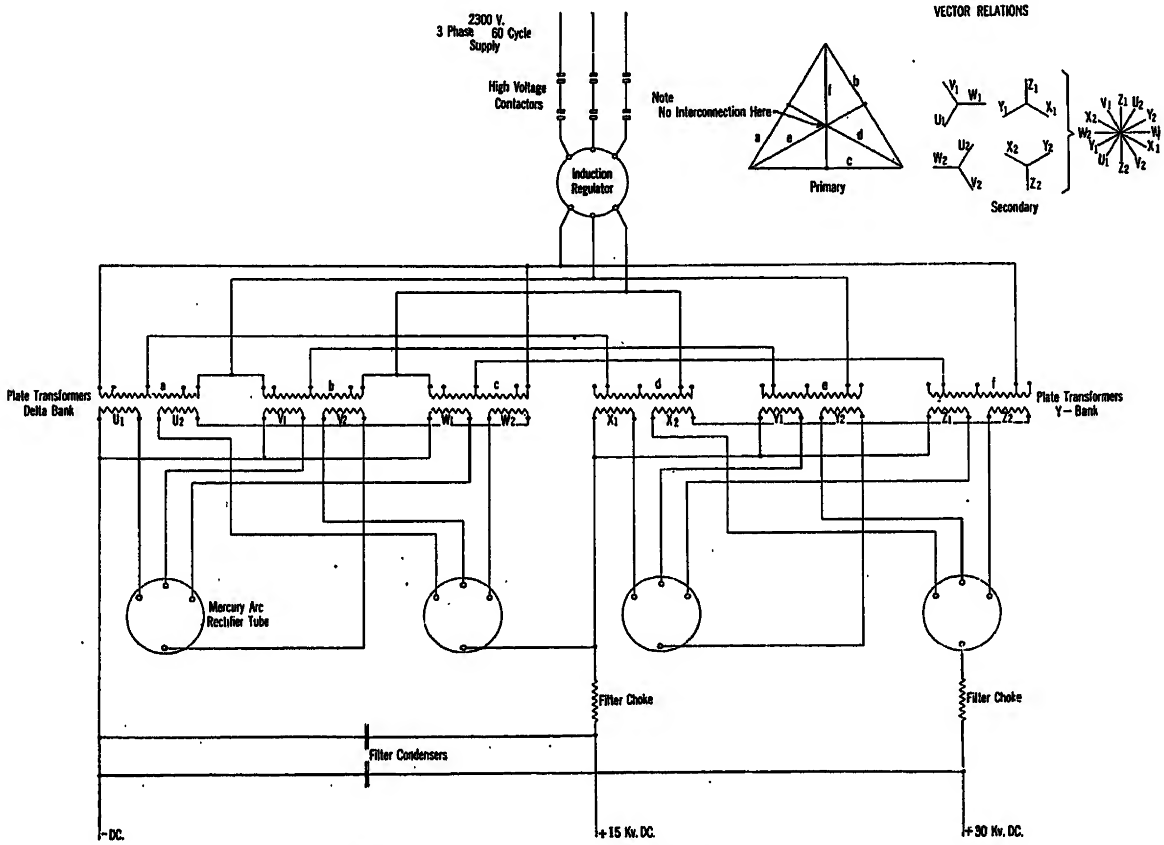

FIG. 2A—SCHEMATIC DIAGRAM OF RECTIFIERS 900 KW., 12 PHASE, 30 KV.

Connection of the transformer banks and associated apparatus for the main rectifiers is shown schematically in Fig. 2. Reference to this diagram indicates the special connections employed on the supply side of the plate transformers in the 900-kw. rectifier to provide the twelve-phase relations for the rectifier tubes. It might be thought that a simple star connection could be employed for three of the transformers to furnish the 30-deg. phase displacement between the star and delta banks. However, consideration shows that star-star connections are particularly bad for rectifiers because of the floating neutral and should never be employed, but by carrying the phase windings of the star across, like the medians of a triangle, and tying them into the sides of the delta-connected bank, stable phase relations of the desired displacement can be obtained. Also, the windings are more nearly the same voltage so that only a slight excess kva. capacity and proper taps are required to make all six transformers exactly alike, which permits connection of a single unit into either star or delta groups, an important feature in providing for reserve or spare equipment.

To smooth the rectifier output into pure continuous current, untuned filter circuits made up of large iron cored reactors and banks of high-voltage condensers are connected between the rectifier tubes and radio equipment. The rectifier plate transformers and filter reactors constitute quite a percentage of the total bulk of power equipment and are mounted out-of-doors in the rear of the building, thereby saving considerably in its size and cost. Fig. 3 shows the arrangement of this apparatus within the enclosure at the rear of the main building.

In order to control the d-c. voltage output developed by the rectifier, three-phase induction regulators are employed with which it is possible to vary the applied a-c. voltage from 0 to 4,600, the regulators being designed to give 100 per cent range both above and below the line potential. This is a greater range of voltage control than is usually provided for most radio station rectifiers, but because of the desirability, for experimental work, to have continuous variation over the

entire range of voltage without steps, it seemed advisable to install sufficient regulator capacity for this purpose. Induction regulators of the size needed, if made self-cooling, would occupy considerable floor space but since water-cooling facilities were also required for the radio tubes, it was possible to employ water-cooled regulators with considerable advantage, both in initial cost and space saved. Motor operation is provided for both rectifier regulators which makes it possible to control the high-voltage direct current supplied to the radio equipment from any one of several push-button stations located conveniently in the main transmitter and control rooms. A simple position switch on the regulator rotor, operating signal lights in the main transmitter room, gives an approximate indication of what voltage will be developed by the rectifier before the power contactors are closed. Actual power control of the rectifier is handled by air-break contactors, similar to those used in mine-hoist control, operating directly in the 2,300-volt supply to the induction regulator and plate transformers. Additional push-buttons and signal lights at the various stations around the transmitter room enable control of the power contactors and indicate their condition of operation. The contactor control circuits are interlocked for safety to the operators through switches on doors giving access to high-voltage circuits. Alternating- and direct-current overload relays are provided to shut down the equipment in event of trouble necessitating the removal of plate voltage. An oil circuit breaker, manually operated from the main switchboard, is also furnished. This circuit breaker is equipped with dash-pot overload relays set to trip on continued or extreme overload conditions.

The tubes used in the main rectifier are of the glass envelope mercury cathode type, somewhat similar to ones formerly used for operating d-c. series arc light circuits, except that they are larger and have three main anodes. Two small sustaining arc anodes and one starting mercury anode are also provided in each tube. One of these tubes is shown in Fig. 4. In operating, the tubes are fully immersed in oil, each mounted in a separate oil tank in a cradle by means of which it can be readily removed from the oil. All necessary connections are made when the cradle slides into place, thus permitting tube changes to be conveniently made with minimum effort. For interchangeability, tubes of the same rating are used in both main rectifiers. A view of the four tube tanks of the 900-kw. rectifier is shown in Fig. 5.

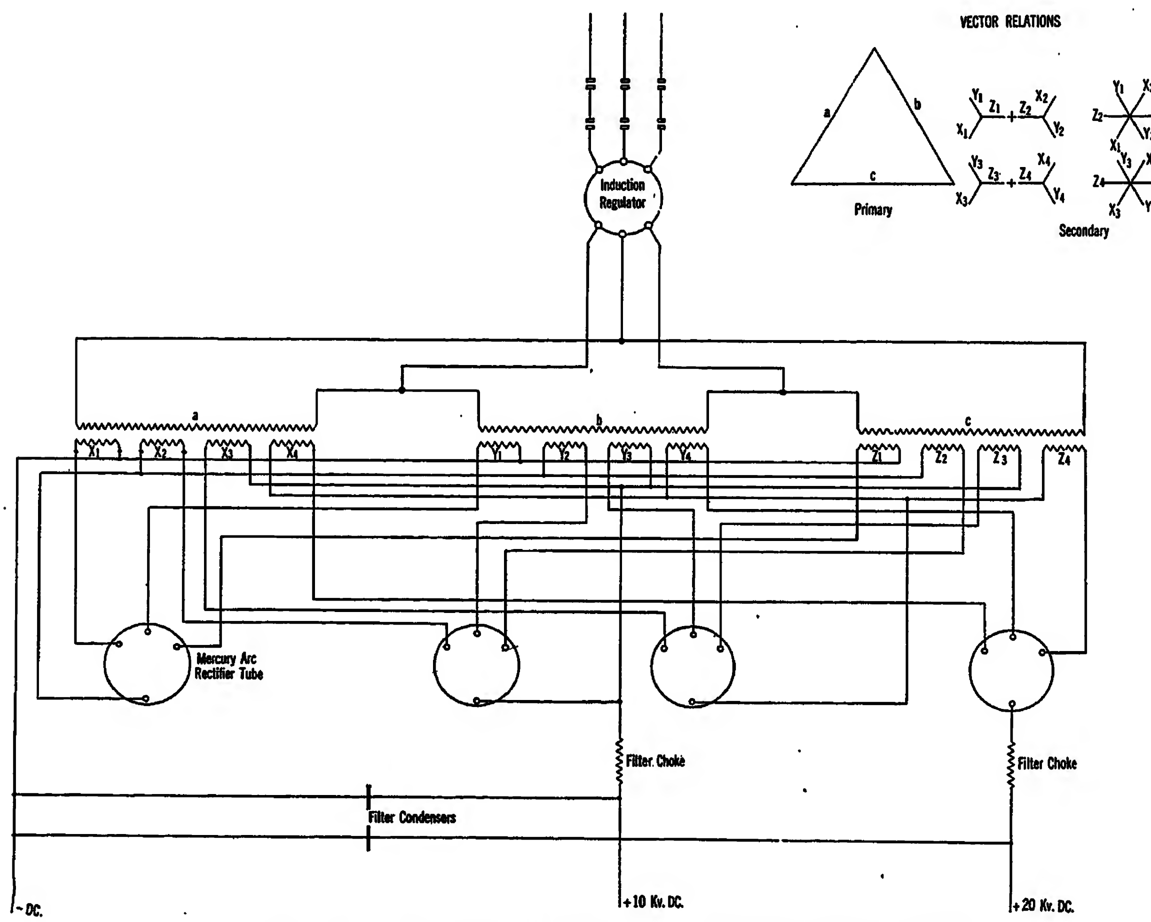

Fig. 2b—Schematic Diagram of Rectifiers 450 Kw., 6 Phase, 22 Kv.

Automatic tilting mechanisms for starting the sustaining arcs are provided. This permits the rectifier

to be placed in operation from a remote push-button station with no more attention from the operator than the switching on of the filament heating circuit of hot cathode tubes. The starting mechanism functions in

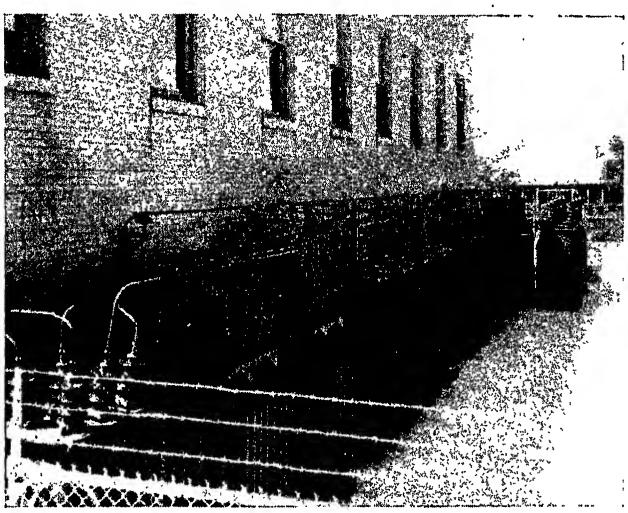

Fig. 3—Plate Transformers and Filter Reactances

one to two seconds, after which it is immediately possible to apply full operating plate potential if desired. The sustaining arcs, once established, maintain the tubes in readiness for operation when desired with-

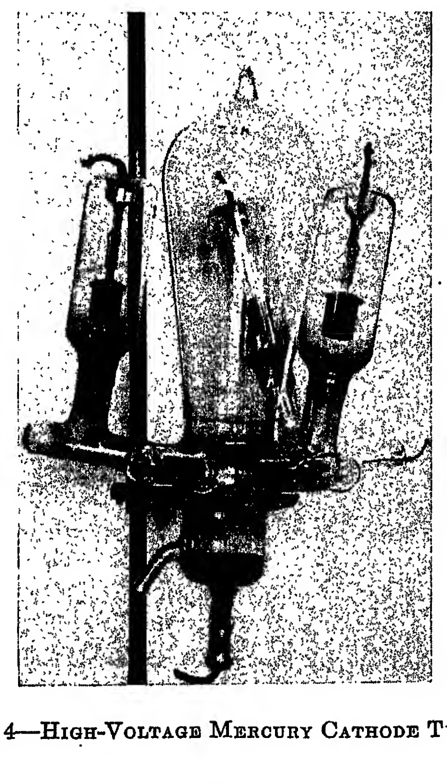

Fig. 4—High-Voltage Mercury Cathode Tubes

out further use of the starting mechanism unless the power supply circuit is interrupted. Very little energy, about 120 watts per tube, is required to operate the sustaining arcs. This is low compared to filament type tubes since the mercury cathode automatically provides emission as needed at the hot spot in the mercury pool and therefore the power for the sustaining arc does not need to maintain the maximum emission at all times. Mercury pool rectifier tubes such as these described, while not commonly employed heretofore for radio power apparatus, appear to have a number of desirable characteristics, particularly in installations of such size that the use of filament tubes would require a great number of them both in series and parallel to furnish the equivalent amount of high-voltage direct-current energy.

In all but very small telephone transmitters, direct current is required for filament heating. With even the largest sets now commercially built this can be

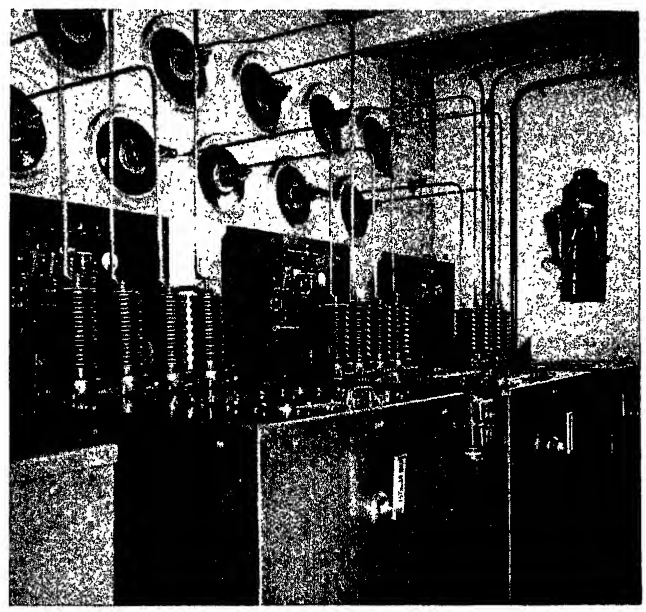

Fig. 5—900-Kw. Rectifier Tube Tanks

readily obtained from a single motor-generator set and with fairly simple control to give the necessary tube protection against insufficient cooling water, this latter being accomplished by merely stopping the whole unit or preventing its operation when there is not enough cooling water circulating through the tube jackets. Also, gradual application of filament voltage, which is essential in the starting of large tubes, is obtained by merely having the machine connected with the tubes when building up.

To operate the new Saxonburg broadcast transmitter in this manner, would require a 3,000-ampere generator. While high-current low-voltage machines are available, these are generally unsatisfactory for radio station use, being built primarily for electrolytic or battery charging work where ripple is not important. It being considered inadvisable to develop a special generator for this installation, it was felt best to arrange for the operation of standard generators in parallel. Although this is common practise in power work, it complicates

the filament supply problem by making necessary the operation of all machines in unison to obtain the control advantages of a single unit system.

In view of the filament requirements of both low- and high-frequency transmitters, a total of six 1,000-ampere generator sets was installed, three being normally assigned to the low-frequency set and two to the high frequency equipment, leaving one in reserve. A view of the filament generator installation is shown in Fig. 6. Each generator is conservatively rated at 1,000 amperes at 40 volts and will carry 1,200 amperes for several hours without excessive heating. The generator design includes special features such as large air gaps, skewed slots and a high number of commutator segments which have been found very effective in reducing commutator noise to a sufficiently low value such that no filter is required when these generators are employed in radio broadcasting station operation. Excitation is obtained from a separate d-c. circuit provided in the station for

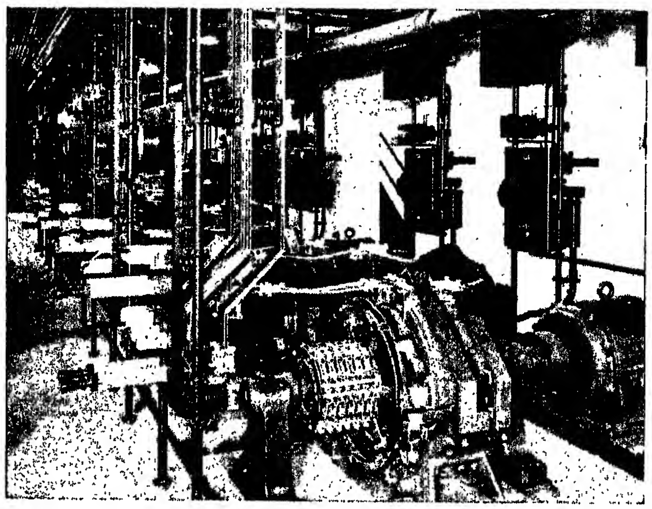

FIG. 6—FILAMENT MOTOR-GENERATOR SETS

such service, and while interpoles are used, no series field is employed, the regulation of the machines being satisfactory, and parallel operation much simpler, without any compounding. A line-start type 75-hp. motor drives each generator and operates directly from the 2,300-volt three-phase station supply with only a simple magnetic oil contactor for control. Although the generators, when in parallel, are operated in unison, the motors are started individually so as to reduce the peak power demands as much as possible.

While the filament generators were balanced to operate very smoothly it was felt advisable to insure that no vibration from these large machines was transmitted to the building structure and radio equipment and special provision was made to accomplish this. Each motor-generator is mounted on a concrete base and the whole assembly supported on cork pads. The area of the supporting cork is so designed that compression stress is carried to a point where maximum resilience is obtained, thereby producing the greatest absorption of vibration produced in the machines.

The distribution of a filament current is accomplished over a heavy copper bus structure, over 4,000 lb. of copper being used in its construction. Each filament generator is provided with a transfer panel, by means of which both field and output circuits of all generators to be operated in unison are paralleled. A master field rheostat furnishes the main filament voltage control with individual rheostats on each machine to make slight adjustments for equal distribution of load. A transfer bus is used so that machines may be grouped as desired and any machine removed from service without handicapping the station operation. Suitable low-water protection is accomplished by interlocking the main exciter circuits with the water relays so that no filament voltage can be built up until the tube water conditions are satisfactory. A special starting step of field resistance is also interlocked with the filament excitation control so that a damaging current rush does not take place when excitation is applied to cold filaments which possess only about 10 per cent of normal operating resistance.

Besides the six motor-generator units installed for filament heating service, there are four 10-kw. 3,000-volt generators which are arranged to furnish the plate circuit energy for the smaller intermediate amplifiers and also provide grid bias voltages for the radio tubes. In receivers and small transmitters, rectifiers can be used very conveniently to supply the grid voltages involved. However, in large broadcasting transmitters, particularly those employing low power modulation with linear amplification in the power amplifier stages, rectifiers are not well adapted for this service since they are inherently incapable of passing current in opposition to their generated voltage, which is an essential requirement in the operation of broadcasting transmitter power amplifiers. While a relatively large rectifier can be used with a load resistance to permit the passage of grid current, this is rather wasteful in any except small sets. Direct-current generators are, however, particularly well adapted for grid circuit service since the generator internal resistance is low and good voltage stability is maintained regardless of the fluctuation of grid current due to modulation. This latter characteristic is particularly important. The tubes used in the high power amplifier require grid potentials at approximately 3,000 volts and since the intermediate plate power circuits require voltages of the same order, similar motor-generator sets can be used advantageously for both services. Doing so permits the greatest of interchangeability and allows the minimum spare equipment for a given installation.

Of the four 3,000-volt generator units now installed, it is planned to operate one for plate power to the main broadcast transmitter and one each for grid potential for the broadcast and high-frequency transmitters. This allows one for reserve since the high-

frequency transmitter is equipped with its own self-contained six-phase intermediate plate rectifier that will be used for normal operation. The intermediate plate supply generators develop 3,000 volts using two commutators in series and are provided with filters to reduce the commutator ripple. The driving motors are of the line-start type for simplicity of control and generator excitation is taken from the station supply.

To make more simple the problem of interchanging connections, all the generators have one terminal permanently connected to the filament bus or ground circuit. This is possible since the change of polarity necessary when changing from plate to grid voltage service is very easily accomplished by merely reversing the excitation. Even with this simplification the providing of the necessary interconnections of eight terminals to four different circuits by conventional disconnect switches would be very difficult. A cable and jack board was considered but thought to be unsafe at the voltages involved because of the danger from cable insulation failure and uninserted live plugs. However, it was possible to provide all the necessary connections through the use of a bar and plug board without cables and virtually eliminate the live plug danger. This was accomplished by a special design of dead front panel with vertical and horizontal interconnecting bars in the rear to which the transmitter circuits and generator terminals are connected, the desired combinations

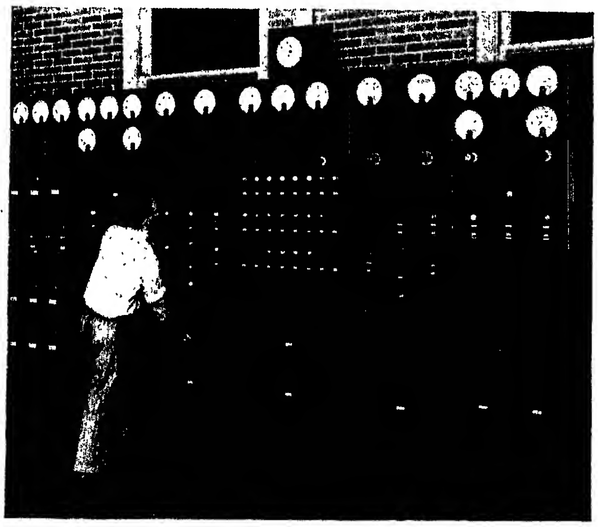

Fig. 7—Main Station Control Board

being effected by plugging through from the front of the panel. In this manner all cables were eliminated and by careful design the plugs were rendered safe on withdrawal, since they become dead before being exposed sufficiently to allow the operator's coming in contact with current carrying parts.

Adequate control of the various intermediate amplifier stages in the transmitter requires the opening and closing under load of 3,000-volt d-c. circuits and a special 3,000-volt 2-ampere contactor was developed for this service. This contactor has two air breaks in series, equipped with magnetic blowouts, and is arranged with special mechanism to provide rapid contact travel and wide openings. A severe test successfully withstood by one of these units consisted in operating it once a second for over 200,000 times, breaking 3,000 volts, 5 amperes, direct current supplied to a load consisting of a resistor and a 14-henry choke connected in series.

The control scheme, in so far as possible, has been arranged to make it unnecessary for the operator super-

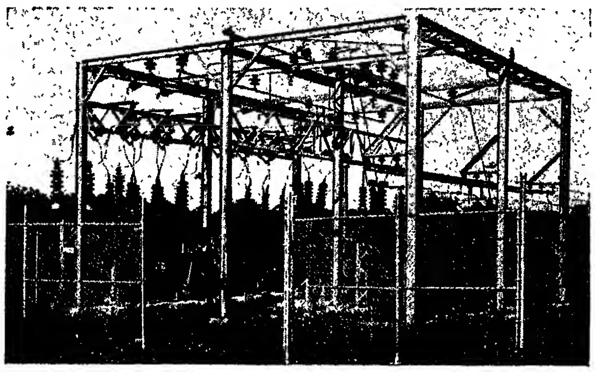

Fig. 8—Substation Feeding Station

vising the equipment to leave his position in the main transmitter room in the normal course of operation. This is accomplished by the remote control of all driving motors for generator sets and rotating equipment being centralized in push buttons on the main station switchboard, at which one of the operators is located. Similarly, the control of the power supplied to the radio equipment is effected through remote control of high-voltage contactors from the main station switchboard together with field rheostats conveniently grouped there. Relays and interlocks are provided so that only the proper sequence of operation is possible. These also function in event of severe or abnormal conditions to remove the power from the circuits in trouble including all those following in the pyramid of amplifiers. Signal lights adjacent to the push buttons indicate the state of circuit relays and response of contactors being controlled. Where circuits carrying dangerous voltages are involved, two signal lights of distinctive colors are provided to indicate open or closed conditions, as is common practise with circuit breakers. Meters enable the operator to obtain directly, or through the use of multi-point switches, the essential conditions of all important transmitter power circuits. Besides the signal lights and push-button control position on the main board, auxiliary positions are located at convenient places about the transmitter itself and in the audio control room which permits the operator on duty there to control the equipment should occasion arise. This latter is an aid to safety and is facilitated by the operating portion of the control room being elevated so as to overlook the main transmitting floor.

One of the panels on the main station control board, a view of which is shown in Fig. 7, provides control of the outdoor station which furnishes the power used. All controls and meter leads from the circuit breakers and instrument transformers are brought under ground 700 ft. to the main building so that it is possible for the operator to have control of the power line supply circuits and equipment without the delay that would be occasioned if it were necessary for someone to go to the substation structure for the operation of circuit breakers. The substation is fed by a 25,000-volt loop and the circuit breakers are arranged with relays which in event of trouble will automatically disconnect whichever end of the loop becomes faulty and enable service to be continued without interruption.

Three 500-kva. single-phase 25,000 to 2,500-volt transformers are employed to step down the power service to potentials suitable for use in the radio station. Power purchased is metered on the high-voltage side of the transformers through the use of outdoor metering equipment.

From the low-voltage side of the transformer bank power is carried over two three-phase 2,500-volt armored cables laid directly in the ground without special duct. The two cables are tied permanently in parallel but in case of failure one could be quickly disconnected allowing limited operation on the remaining circuit.

Circuit breakers such as those in the substation are normally tripped and closed by the use of 110-volt d-c. power. While only a small amount of current is needed for tripping, the closing operation requires nearly 100 amperes for coils when wound for 110 volts. To deliver this amount of current to the substation would require a large battery to be installed for this purpose alone, either in the radio station or in a special building at the substation structure. Moreover, if the battery were located in the radio station the distance would require large cables to prevent undue voltage drop. A special 110-volt battery being very undesirable, the necessity for one was eliminated by employing the 400-volt storage battery for this service. Special 400-volt coils were used in the circuit breakers which reduced the current required so that no special cables were necessary to maintain the proper voltage at the circuit breakers notwithstanding the considerable distance involved.

Besides the major power units required, particularly for the radio equipment, there is a considerable amount of conventional equipment such as exciters, tube-cooling water pumps, blowers, battery-charging equipment, as well as station service transformers for the small power units, lights and heating, which hardly need description. Two six-cell 1,600-ampere-hour storage batteries are furnished for the filament supply of the audio and synchronizing equipment in addition to the 400-volt battery already mentioned which is used for plate supply to equipment employing small tubes. Whereas one battery will suffice for plate service, since it can readily be floated or charged while in service by the employment of suitable filters on the charging equipment, two filament batteries are advisable, each capable of handling the entire load for a day, since it is very difficult to remove the noise developed by charging equipment where large currents are involved, and considerable of the amplification follows the tubes operated in this manner. These large batteries also supply certain essential indicating lamps pertaining to the substation circuit breaker control which require continuous illumination even in the event of power failure.

Although the water-cooling system may not be considered as part of the electrical power equipment, certain features of it are rather interesting. In many broadcasting stations the water directly cooling the radio tubes is in turn cooled by a system of radiators and blowers. This method if employed at KDKA's new station would require a very large and expensive installation because of the much greater dissipation involved. Instead of employing such an installation advantage is taken of the natural high level of the ground water by excavating a small pond from which sufficient water can be obtained for cooling the tube water by means of heat interchangers.

While commercial broadcasting stations as yet do not require equipment as large as that installed in KDKA's new plant, nevertheless it is only by actual construction and use of such apparatus that the development of commercial stations to give safe and economical operation can well be carried out. It is also hoped that data can be obtained which will be of value not only in the specialized field of radio engineering but to electrical science as a whole.

Discussion

R. L. Young: The paper mentions that the capacity of the power plant is considerably in excess of present use. Is the whole station on the same basis and what is its maximum rating as ratings of stations now go?

Also what is the approximate commercial cost of a station of this power?

R. L. Davis: Practically all of the power equipment at the KDKA Station has installed capacity much in excess of the present power output licensed by the Federal Radio Commission in order that experimental work involving wide variations in current and voltages may be carried out. A maximum radio-frequency power output of three to four hundred kilowatts will be possible with the station operating at full capacity of the power supply unit. This is approximately six to eight times the rated capacity of the largest stations now licensed commercially in the United States.

Inasmuch as the equipment installed at KDKA involves so many experimental features, the commercial cost of a station equipped in this manner will not compare favorably with one designed solely for commercial operation. No estimates have been made on the commercial station of this power. A number of comparative figures on the cost of broadcasting stations has recently been published.*

*See June issue of *Electronics* "The Dollar Cost of Broadcasting Stations" compiled for the National Advisory Council on Radio in Education.

Automatic Power Plants for Telephone Offices

By R. L. YOUNG[1] and R. L. LUNSFORD[2]
Member, A. I. E. E. Member, A. I. E. E.

Synopsis.—*The nature of power requirements for telephone offices is discussed, with emphasis on continuity of service. Automatic controls are indicated because of their more exact performance, with consequent reduction in variations and in interruptions and their saving in maintenance, particularly with 24-hour operation demanded. Developments are traced, showing an increasing trend toward automatic regulation and control of main power supplies, ringing and other signaling energy sources.*

The development of "unit type power plants" for telephone offices is discussed and information is given on a number of standardized plants which operate upon a full automatic or a semi-automatic basis. These furnish power supply for manual, toll, and telegraph central offices, for magneto offices and for manual and dial system private branch exchanges also for small dial system central offices.

Favorable operating experience points the way toward further introduction of automatic devices which will place most telephone power plants, except those in the larger dial system offices, in a position to operate themselves over considerable intervals of time.

THE nature of telephone service is such that continuity is regarded as important, perhaps the key-note guiding developments, and equipment is provided to furnish service 24 hours each day in the year and year after year. In times of storm or emergency when other public services may be prevented from functioning the telephone service will often be particularly important and may have to carry heavier loads.

Reliability

In order to maintain service the first essential is to have the necessary power available; and in this respect the telephone power plant has made a creditable record for reliability. It has survived fires, floods, and lightning. The basement of one metropolitan office, for example, was completely flooded by a broken street water main which resulted in the burning up of the regular 2,200-volt a-c. switching equipment, but the 240-volt d-c. service which had been provided there as an emergency continued to hold in under water and the drive motor equipment on upper floors in this case automatically transferred to this source, while the storage battery as usual provided the required low-voltage direct current. In another case an open-tank storage battery continued the office in operation though submerged by rising flood water from a nearby river. In another metropolitan office lightning struck the power service on a pole across the street, turning the transformer vault in the telephone building into a blazing furnace. The fire walls and steel door held, permitting continued operation of the power plant upon an emergency gas engine in another part of the basement.

An attempt has been made to estimate the percentage of the time during which the power plant is available for use. Complete failures such as would result from inability to provide a direct current from the battery or generators are of rare occurrence. Partial outages resulting from failure of the ringing current will become embarrassing if continued for any substantial time, such as for several minutes. Inquiry has been made as to both types of interruptions in a metropolitan division having 23 offices in 1910, 63 at present and an average of 42 offices in use during the past 20 years. The reported time out for the main power supply in all offices during 20 years averaged two seconds per year per office. The loss of ringing current on the other hand, including both complete and partial failures to certain groups of equipment, was about two minutes per year per office. These suspensions of ringing current did not interfere with conversations in progress, most of them merely resulting in a slight delay in getting through new calls. Taking the larger failure figure the power plant service is good more than 99.9996 per cent of full time.

Requirements

In order to secure reliability it is necessary that the main source of immediate telephone power may be of a type not subject to interruption or failure and that the various machines and important accessories be provided in duplicate. More than one source of primary power is also needed unless there is storage of reserve power in sufficient quantity to bridge over the times during which interruptions may occur. Provision for quick change from regular to reserve equipment is necessary. The main source of d-c. supply must be controlled within close voltage limits for certain types of plants, while the several kinds of signaling machines must be operated within established voltage and frequency limits.

These requirements point to the use of automatic regulating equipment and automatic operating arrangements to as large an extent as practicable, particularly since they must be maintained effective both day and night. For the main d-c. power a regulated reserve storage battery best meets conditions. No equipment is infallible, being subject to wear, deterioration, and accident; neither is a human attendant perfect. By care in selection of equipment, by provision of spare parts, and by sufficiently frequent testing to bring out any defects which may develop before they cause a service interruption, it is believed that a greater degree of reliability can be secured by automatic means than is possible by manual.

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Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

Historical Background

The use of automatic power plants, though recent, is, of course, not new in the electrical art, and our friends in the light and power industries, including also the electric railway field, have made very substantial progress within recent years. In fact, it would seem that a high degree of perfection has been obtained with their automatic substations and automatic hydroelectric plants.

This excellent public service power supply is used as the primary source in practically all telephone offices built within recent years, except in a few repeater installations in isolated districts. This power, however, can not be applied to telephone circuits as received, but is converted into direct current and alternating current of various voltages and frequencies, although in panel dial offices the outside service is used directly to operate small motors driving switching equipment.

Early Automatic Plants for Central Offices

Among the earliest installations of automatically-operated power equipment in regular telephone central offices were some in Pennsylvania where signaling hydrometers controlled automatic solenoid-operated switches to connect automatic starting mercury-arc rectifiers. A single battery equipment was put into service at Berwyn in April, 1917, and during the same year an installation was arranged for Donora in the Pittsburgh district, and one for Wayne with certain improvements and additions to the automatic circuit to take care of two batteries.

These plants had the batteries usual at small manual central offices, a larger 24-volt No. 1 main battery, and a smaller 24-volt No. 2 battery connected in series to give 48 volts for toll transmission. Since drain on the latter was relatively light, several days' reserve was economically provided. Either of the batteries could secure the services of the single charging rectifier if idle when a charge was needed. However, if the main No. 1 battery discharged to a predetermined point, while the rectifier was charging No. 2, the rectifier immediately switched over, and after the No. 1 charge was completed again took up the interrupted charge on No. 2. If, on the other hand, No. 2 arrived at the charging stage while the rectifier was busy on No. 1, it had to wait, but an alarm lamp visible to a switchboard operator was lighted. Alarms were given when for any reason either battery discharged 50 per cent of its capacity, also when the rectifier did not start the charge promptly after one of the hydrometers indicated that a charge was required. Some of these plants are still in satisfactory operation, one being at Oakmont near Pittsburgh. These ingenious control circuits were suggested by engineers of the Bell Telephone Company of Pennsylvania, the immediate urge being elimination of overtime attendance for charging Saturday afternoon and Sunday, since batteries were not large enough to carry the offices safely over this period.

The Modern Automatic Plant

Much of the same general arrangement can be seen in the most recent plants designed for small offices, which will be described in detail later, in which the signaling hydrometer has been replaced by an ampere-hour meter with operating and alarm contacts while the mercury arc rectifier has been replaced by a hot cathode tube type which is self-starting without any mechanical equipment other than that necessary to turn on the power. The newer rectifier is lower in cost, and two or more are provided as necessary to avoid switching between batteries.

The ampere-hour meter is more sturdy, is cheaper than the earlier control device and is equipped to provide some overcharge each cycle, which can be adjusted to the amount needed by the operating cycle used. This eliminates the periodic overcharges formerly given under manual supervision.

In the case of larger automatic plants it is necessary to have a number of the machines controlled automatically in their several characteristics as well as to provide equipment to start, stop or continue them in operation as required. It is believed that it will be of interest to mention some of the equipment developed to accomplish this. In most cases it was found that regulating equipment commercially available was not suitable for the special conditions involved in the telephone power plant and the modifications made will be described.

Regulation of Main Central Office D-C. Voltage

Storage Batteries and Charging Machines

With the more extended use of panel type dial equipment by the Bell System inaugurated about a dozen years ago,[3] it appeared that voltage control of the main central office batteries by use of counter e. m. f. cells then employed was not economical in large sizes. The plan at that time was to provide several counter cells, one or more of which was normally in circuit to reduce the battery voltage to required limits. During discharging of the battery, the counter cells were cut out, while for charging additional counter cells were introduced.

The Floating System and Manual Regulation

A duplicate battery system was developed and additional "emergency cells" of the regular type were provided.[4] The duplicate batteries were so connected with switches that either one could be disconnected from the circuit for charging as might be necessary and afterwards reconnected without interrupting the circuit, the remaining battery, in the meantime, being floated at a constant voltage from charging generators which

3. Semi-mechanical at Newark in 1915; full mechanical at New York toll tandem, 1920, local offices at Omaha, 1921, New York, 1922.

4. U. S. Pat. 1,468,096, Sept. 18, 1923.

were also connected to the office load. This eliminated the increased voltage upon the office circuits during freshening charges so no counter cells were needed, while the emergency cells were also available for connection in the office supply circuit to maintain the entire battery voltage at its usual value during an emergency discharge.

This system used the economical plan of "floating" the storage batteries, preferably as near to 2.15 volts per cell as the arrangements provided allowed, at which value they would absorb just the proper amount of current to keep them permanently in good condition. With an office load fluctuating widely during a day's time it was, of course, found difficult to maintain a reasonably constant voltage and trickle current into the battery by manual adjustments of the charging generator output, so various systems of automatic regulation were studied. The storage battery voltage is inherently critical at this point, a 24-volt battery changing more than 1.5 volts or 6 per cent as the current is increased from zero to 2 per cent of normal charging rate.

Office Load Curves

The difficult conditions of variable current which must be met by either manual or automatic control are shown in Fig. 1 by load curves representing several types of offices.

Group *A* is a typical small manual office with 24-volt currents from 5 to 130 amperes, the peak being at 11 a. m. with an auxiliary peak at 8 p. m. caused by residential social calls. The 48-volt load for toll transmission is very light. Automatic power plants for loads of this character are in satisfactory operation in both manual and dial offices, the larger sizes, however, not being available yet as coded plants.

Group *B* shows a typical panel dial unit in a residential district, in which it will be noted that the current demands as far as voltage is concerned are quite reversed. In such an office the main operating power is taken at 48 volts, while the local talking or transmission load is at 24 volts.

Group *C* shows an entirely different load, a through repeater central office with nearly constant currents to light the filaments and supply the plate circuits. Most of such repeaters are kept in operative condition continuously. Later, descriptions will be given of automatic plants to supply load of this type with current at regulated voltage.

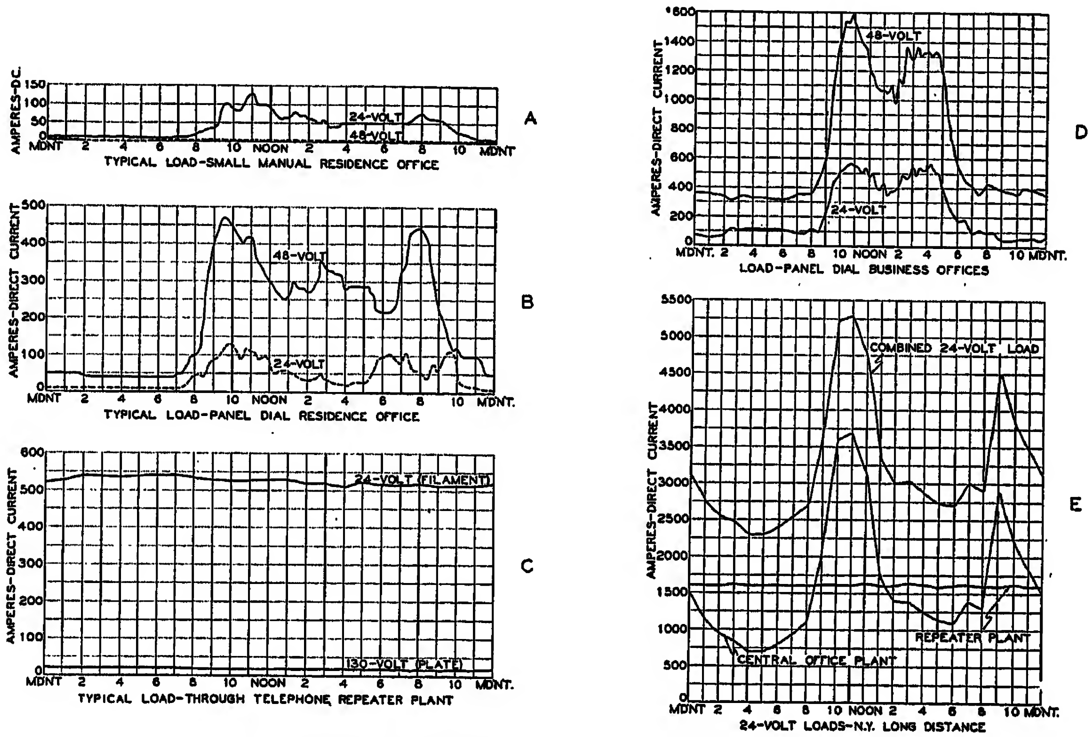

Fig. 1—Telephone Office Load Variations in 24 Hours

Group *D* shows the load curves read on a large metropolitan panel dial power plant in a business district with more than 2,000 amperes combined peak in the daytime reduced almost to 400 at night, the latter load being accounted for largely by float-charging of storage batteries at private branch exchanges. This plant at the time was supplying three offices carrying two-thirds of a million daily calls, not including interconnecting calls at tributary private branch exchanges.

Group *E* shows the 24-volt load of the New York Long Distance Office with switchboards distributed over 10 floors supplied by a power plant in two sections, one section installed first now being assigned largely to supply repeaters nearby. It is commonly

known that the power used to carry voice currents over a telephone line is very small but it is not always appreciated that considerable equipment must be operated in the central office to generate, supervise, and control this. The peak load of an ordinary day for long distance is about 5,300 amperes at 24 volts with another 4,500-ampere peak in the evening when social long-distance traffic is encouraged by lower rates. Over 10,000 amperes in 24-volt generators and a reserve battery of 34,880-ampere hours weighing 111 tons are now provided, also 1,200 amperes additional in 130-volt generators. In addition to the reserve batteries,

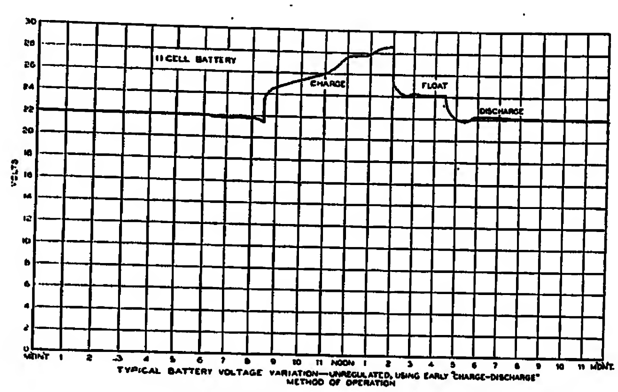

FIG. 2—TYPICAL BATTERY VOLTAGE VARIATION

Unregulated, using early "charge-discharge" method of operation

reliability is insured by duplicate a-c. feeders from one generating system and by several d-c. feeders from a different generating system.

Over 1,000 motor hp. is required to drive these charging machines and a vacuum tube system to handle tickets, part of this being reserve. The New York Long Distance Building, including the addition now under construction, also contains a toll and some local offices and is believed to be the heaviest powered building in the entire city, with a-c. and d-c. services provided for a total office and building connected load estimated at 10,000 hp. in 1934.

For the load illustrated by groups *D* and *E* automatic voltage regulating equipment is in use, but automatic power plants are not yet developed, results on smaller installations being awaited.

VOLTAGE REGULATION BY MANUAL CONTROL

Under the system of battery and generator operation formerly in common use in manual central offices a range of about 7 volts or 28 per cent was not uncommon when no effort to regulate was made, as shown in Fig. 2.

More than a dozen trials of the continuous floating method of regulating voltage manually were put into operation during the year 1919 in New York, Boston, Philadelphia, and Chicago, following earlier tests. The regulation normally obtained with frequent manual adjustments reduced the variation to 2 volts, equaling 8 or 9 per cent as shown in the recording voltmeter chart, Fig. 3.

IMPROVEMENT WITH THE CAM TYPE REGULATOR

Several types of commercial automatic regulators were tried but were not found satisfactory. Because of the time lag of the storage battery in arriving at a stable voltage after the input current was changed, regulators which worked upon a rapid basis such as the well-known solenoid type gave rise to severe and continuous hunting. A Wheatstone bridge type of voltage recorder modified to act as a regulator, in conjunction with a separate motor-driven rheostat, was found to be the most satisfactory. Following this a so-called "cam type" automatic voltage regulator was developed and has given very satisfactory results over several years. It is usually adjusted to maintain the voltage within one-half volt total variation or slightly less on the 24-volt battery, holding voltage within 1 per cent of mean value as shown in Fig. 4 which covers a section of chart on the earliest experimental model.

This is close enough for the purpose, and does not involve the more or less continuous operation which may result from a narrow setting which responds to minor battery voltage changes caused by fluctuating loads. A duplicate installation of the latest regulators of this type, together with their switching equipment to control generators on two batteries is shown in Fig. 5 from which the regulator will be seen to consist essentially of three parts:

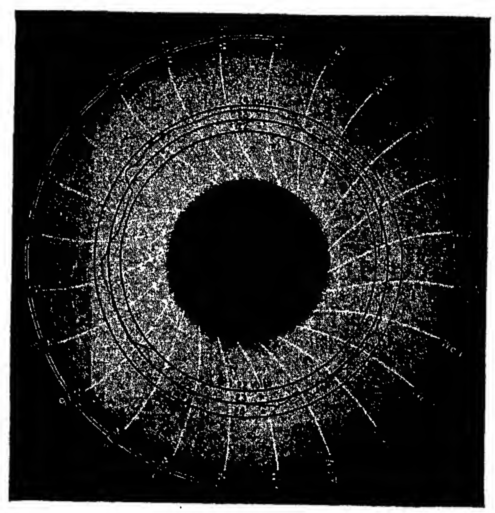

FIG. 3—VOLTAGE OF CONTINUOUSLY FLOATED BATTERY

With intermittent manual adjustment of generator, or "hand regulation"

(1) A sensitive solenoid contacting device working in conjunction with (2) a motor-driven cam delay equipment which at intervals is in position to energize contactors to control (3) a drive motor geared to a rotary face plate resistance box.

These are mounted in a steel cabinet with doors on the front and on the back, and arranged for assembly in the power board lineup or upon a separate framework.

The cam motor is usually of the synchronous clock type taking very small energy and giving a uniform speed. The contacting cam is so shaped and positioned that no contact is made when the battery voltage is correct but when the voltage is high or low, contact for a second, more or less, is made at intervals of ten seconds.

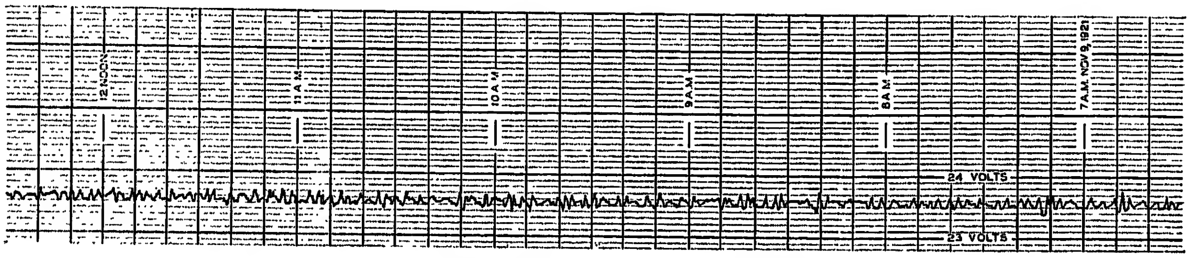

FIG. 4—VOLTAGE CHART EARLY MODEL ROTATING CAM-TYPE REGULATOR

Automatically adjusting a motor-driven generator field rheostat, variation ± 0.15 volts

The duration of contact varis direectly with the amount that the main battery voltage deviates from normal, so for slight deviations very short corrective impulses are given. When variable 24-volt and 48-volt loads are taken from the same string of battery cells a regulator is used across each section. This device for the first time satisfactorily solved the problem, which had engaged our attention for several years,[5] of regulating battery voltage closely and automatically.

Typical Unit Type Power Plants for Telephone Offices Arranged for Automatic or Semi-Automatic Operation

As previously outlined the need for automatic equipment in telephone and telegraph power plants to meet the increasingly more stringent requirements for local and long distance communication led to the development of several small power plants which in normal operation are fully or partly automatic and require practically no attendance other than routine maintenance. The following descriptions and illustrations briefly picture some of these plants. Larger plants using the same general principles are on trial and under development.

Plants for Small Private Branch Exchanges

For many years storage batteries located at private branch exchanges have been charged over cable pairs or by local equipment upon an unattended basis, current being supplied at approximately constant rate. This system has certain limitations, however, not being well adapted to considerable variations in load caused by unusual business conditions or seasonal variations, and it is difficult to select a charging rate which will meet all conditions without being excessive during part of the time.

With the introduction of the dial private branch exchange operated mechanically without an attendant, the power requirements as compared with manual operation were increased and more complicated arrangements were needed for automatic ringing, for providing tones and other signals and in larger units for charging the battery economically under the load variations likely to be encountered.

In selecting equipment for this service advantage was taken of recent developments, particularly the small sealed-in type batteries, while dry-disk type chargers using the copper-oxide rectifying principle, and further development of the earlier hot cathode tube rectifier, placed at the disposal of telephone engineers charging equipment better suited to automatic operation than the motor-generator sets and mercury arc rectifiers previously available.

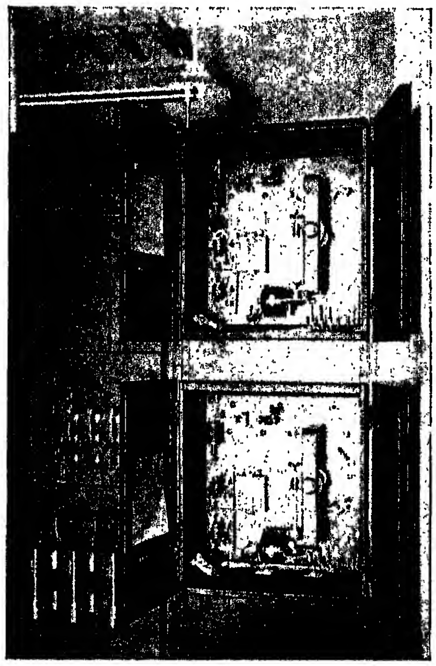

FIG. 5—CAM-TYPE AUTOMATIC VOLTAGE REGULATORS FOR LARGE MACHINES AND BATTERIES

Assemblies of automatic plants developed for small dial private branch exchanges are illustrated in Fig. 6 and Fig. 7.

These plants consist essentially of two steel cabinets, the lower cabinet containing the battery equipment while the upper cabinet mounts the control equipment

5. U. S. Pat. 1,586,569, June 1, 1926.

including fuses, meters, switches, relays, ringing machine and also automatic control for maintaining the voltage range within six volts.

Rectifiers on the small plants are inside but on the larger plant are mounted on top of the upper unit as shown in Fig. 7. Motor-generators instead of rectifiers are used in d-c. districts.

In order to minimize installation work, plants are shipped as units, it being necessary only to mount the battery and charging equipment and connect the leads furnished with terminals attached. A plug and flexible cord are provided for connection to any nearby receptacle providing commercial power. The active cells of the battery are shipped, *charged and filled* ready for interconnection, and are equipped with varicolored wax ball charge indicators which show at a glance the approximate condition of cell charge. The 32-45-volt plants use 18 cells and the 44-50-volt plant 23 cells with four counter e. m. f. cells.

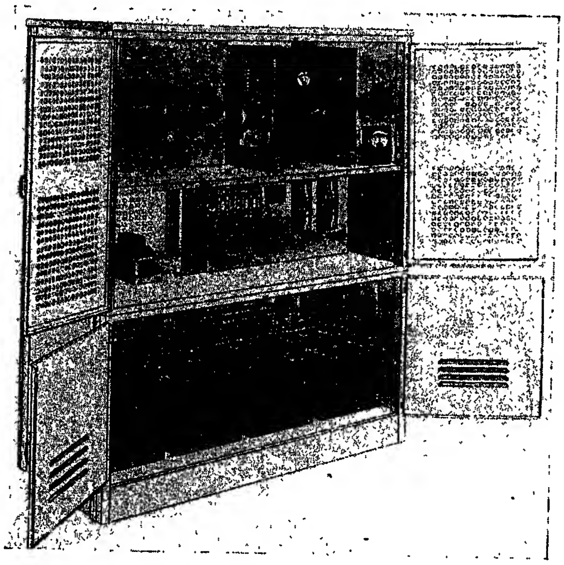

FIG. 6—POWER PLANT FOR SMALL PRIVATE BRANCH EXCHANGE

Dial type No. 102-B, 44-50 volts, 2.5 amperes

Charging equipment is of capacity to carry a large percentage of the busy hour load and return to the battery during the light load period whatever discharge has been taken, plus additional current to offset battery losses. In these plants the rectifiers operate continuously. Voltage regulation is obtained automatically by means of counter cells cut in or out by solenoid type switches under the control of a voltmeter relay.

Use of one or two 3-ampere hot cathode type Tungar rectifiers allows a flexible arrangement to meet economically load conditions from the initial to the ultimate stage of plant growth.

Ringing current is generally obtained over cable pairs from the nearest central office with tones provided from local relay equipment. Where such supply is not satisfactory a llams inverted rotary converter operating on the battery is provided as shown, this also furnishing tones.

This machine has automatic control, starting when a receiver is lifted from the switchhook and stopping after completion of the call. It is equipped with jack and plug connections for ready replacement by a spare machine when maintenance is required. This mounting arrangement permits some economy to be realized by providing a small central stock where there are several nearby installations.

SMALL POWER PLANTS FOR VACUUM TUBE PLATE AND FILAMENT SUPPLY

Several plants have been designed to supply economical energy to plates and filaments of repeater tubes and to other equipment, when such energy is not available from regular central office power plants. Unit plants up to 80, 200 and 800 milliamperes, respectively, at 130 volts are described, followed by larger plants from 5 to 20 amperes. A small 24-volt plant up to 8 amperes is covered, which is supplemented by a line of larger low-voltage plants for various purposes.

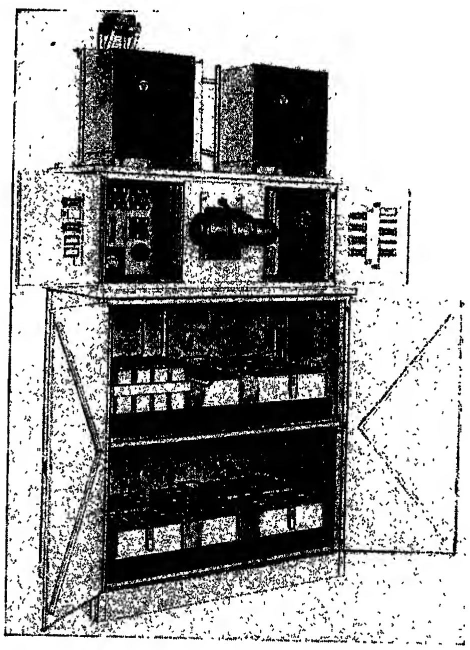

FIG. 7—POWER PLANT FOR LARGER DIAL PRIVATE BRANCH EXCHANGE

No. 102-B, 44-50 volts, 2.5 and 5 amperes

In order to match more nearly in appearance the repeater equipment with which they are usually associated the small plants are arranged for mounting on floor-supported vertical channels.

In the illustrations shown the equipment is mounted open upon shelves or panels. In addition to this arrangement, enclosing sheet steel covers are being prepared for several of these plants for use in locations where it is desired to provide further protection or to improve the appearance of the plant.

130-Volt Plants

The 603-A 50-80-milliampere 130-volt plant, not illustrated, consists of a copper-oxide rectifier, filter section and reserve batteries of block type dry cells automatically connected into circuit upon failure of the regular power service. An unusual arrangement is employed for this purpose, the dry cells being across the supply rectifier in series with a copper-oxide rectifier disk assembly which acts as a valve because of its pronounced unidirectional current characteristic. Under normal conditions a small current trickles through the dry cells, this having the desirable effect of tending to overcome local action, but being too small to electrolyze seriously the water in the cells. If the voltage drops by failure of the primary power source, the dry cells automatically pick up the load by sending current through the Rectox valve in the reverse direction at which time it interposes very little resistance.

Fig. 8—Small Plant for Supplying Vacuum Tube Plates

No. 604-A, 125-135 volts, 50 to 200 milliamperes

The next larger, 50-200-milliampere 130-volt plant shown in Fig. 8 consists of two 66-cell couple-type enclosed glass jar storage batteries, a half-wave Tungar rectifier and the necessary control equipment. The voltage at the bus bars is automatically controlled by means of a voltmeter relay adjusted for 126-134 volts. Automatic voltage control of the output is obtained with the aid of tapped resistors, alternately cut in and out of the charging circuit by a voltmeter relay, this varying the trickle charge current going into the battery to maintain its voltage within the stated limits.

A 200-800-milliampere, 130-volt plant coded 604-B operates in substantially the same manner as the 604-A plant differing only in the use of more equipment to care for the additional load.

Type 400 plants from 5 to 20 amperes at 130 volts are under development, the 5-ampere rating now being available and so arranged that it can be extended later to a 20-ampere capacity by the addition of equipment which will make it fully automatic both during normal floating and during replacement of a charge following an emergency discharge. In the larger plants one battery is connected across the generator, and another battery across the load, with a choke coil between, providing a very effective noise filter to reduce any tendency toward disturbance of the telephone circuits by the charging equipment.

Small 24-Volt Plant

The 10-ampere plant now available is shown in Fig. 9. It employs an 11-cell sealed glass jar storage battery and a 12-ampere 75-volt Tungar rectifier. Automatic control equipment is mounted on unit panels and the batteries on shelves.

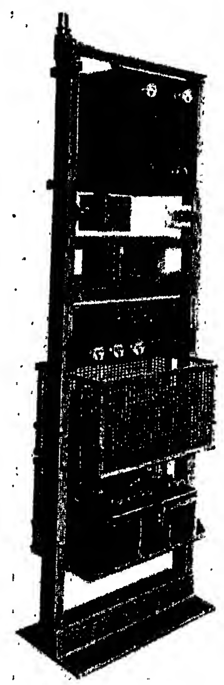

Fig. 9—Small Power Plant for Supplying Vacuum Tube Filaments

No. 602-A, 20-28 volts, 10 amperes

To provide close voltage regulation when required another bay of equipment may be added, consisting of a motor-driven rheostat, automatic control equipment and a filter comprising a choke coil and electrolytic

condensers.[6] The filter is necessary to prevent noise and crosstalk which would otherwise result from introduction of the regulating resistance. A finely adjusted voltmeter relay, connected across the points to be regulated, actuates the control circuit of the motor rheostat, through other relays and contactors, to drive the rheostat in either direction automatically keeping the voltage within the desired range.[7] The voltmeter type of relay has been used extensively for voltage regulation and for alarm purposes and many thousands of them are in service in the Bell System.

Suitable visual and audible alarms are included as a part of all these power plants to notify the maintenance forces of the failure of a fuse, a Tungar bulb, the primary source of power or occurrence of high- or low-voltage at the equipment supplied. As with the other plants, batteries have charge indicators to simplify maintenance.

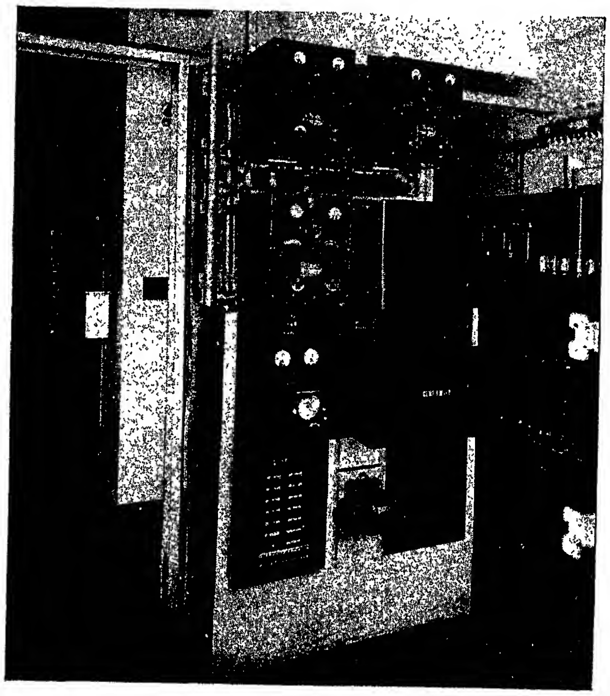

FIG. 10—LOW VOLTAGE PLANT FOR SMALL CENTRAL OFFICES
No. 203-A, 44-50 volts, 15, 30, or 50 amperes

Among considerations in the mechanical design of the equipment for unit type plants generally were ease of access for adjustment, additions of other units to care for growth, and applicability to telephone equipment of a variety of types.

6. For discussion of condenser and power plant filters see H. O. Siegmund "The Aluminum Electrolytic Condenser," *Bell System Tech. Jl.*, p. 41, Jan. 1929, or *Trans. Amer. Electrochemical Soc.*, p. 203, 1928, *Bell Laboratories Record*, p. 276, April 1927, together with U. S. Pat. 1,725,377, Aug. 20, 1929.

Also R. L. Young "Power Plants for Telephone Offices," *Bell System Tech. Jl.*, p. 708, Oct. 1927.

7. For use of these relays in early automatic control system for telephone batteries see U. S. Pat. 1,520,793, Dec. 30, 1924.

Larger Low-Voltage Plants Controlled by an Ampere-Hour Meter

To meet the requirements for automatic battery supply plants larger than the sizes already described, the following have been developed to furnish from 5 to 100 amperes at nominal voltages of 24, 34, and 48. These plants are coded as the 200 type.

The larger the telephone system served the more necessary it is to furnish a reliable and continuous source of electric energy and the more savings may be gained in the efficient operation of the power equipment, especially the batteries. A considerable improvement in the life of the battery equipment together with a reduction in charging energy may be obtained by accurately measuring the amount of discharge and providing for return to the battery of this amount plus sufficient to make up the internal losses. Investigation has shown that for larger plants of the size herein considered, particularly when the load varies from day to day, the ampere-hour meter gives good results. The 200-type plants, therefore, have been designed on this basis, the meter automatically starting or stopping the chargers at predetermined points on the cycle of battery discharge and charge. An assembly of this plant for one of the lower current ratings is illustrated by Fig. 10.

The Ampere-Hour Meter and Alarms

The features included in ampere-hour meters for telephone plant control include start-charge, stop-charge and alarm contacts and, of course, the adjustable overcharge shunt usually provided. Addition of a thermocouple was also found necessary to increase light-load accuracy. The very great variations in discharge current with small drains over prolonged periods at night gave a service cycle not usual in commercial work and with the ordinary type meter resulted in discharges not compensated for during the shorter, heavier current charging period.

The meter controls the power supply to rectifier chargers through contactors. If, due to power failure, bulb failure or otherwise, the meter continues to record a discharge for a certain period after the charge is supposed to start, an alarm consisting of lamp and bell is operated. If the office is unattended this alarm as well as others for blown fuses, failure of ringing current, etc., is transmitted over cable pairs to the nearest attended office. Independently of alarm calls, however, it will ordinarily be the practise to have a maintenance man visit the unattended office periodically to inspect equipment and perform routine maintenance.

Batteries

On the 203-A, 44-50-volt plants the battery consists of 23 cells and four counter e. m. f. cells, while for the 205-A, 45-50-volt plants it consists of 24 cells and five counter e. m. f. cells. The counter e. m. f. cells together with a voltmeter relay and control equipment maintain the bus bar voltage automatically within proper range.

Two types of batteries with only one capacity in each type have been selected; a 3-cell sealed glass jar pasted-plate unit of about 105-ampere-hour capacity, and a larger sealed glass jar single cell of 480-ampere-hour capacity. From one to four sets of batteries in parallel may be used, providing a flexible arrangement to handle growth in economical steps.

Chargers

The 12-ampere Tungar rectifier including two, three or four in parallel meets varying load conditions from the initial to the ultimate growth. Where only d-c. service is available, a motor-generator set will be furnished.

Ringing Equipment

Two types of ringing equipment are available, namely "a-c.—d-c." and "superimposed," the latter required in connection with four-party selective ringing.

The a-c.—d-c. ringing current is obtained from a small ringing machine mounted on the power board, operating under the control of the switchhook as previously described, generating alternating current which is passed through a battery to add a d-c. component, this being helpful in tripping off the automatic ringing when the called subscriber answers.

In the superimposed ringing system, alternating current from a machine is sent through two batteries oppositely poled to furnish both positive and negative superimposed current.

Unit Type Ringing Plants

It is, of course, essential to have a continuously and automatically controlled supply of signaling equipment for ringing subscribers' bells and furnishing various signals to properly supervise and complete telephone calls. Individually coded ringing and signaling power plants have been standardized as described and illustrated below for use with the battery supply plants previously covered.

Small Ringing and Signaling Plants, Including One Plant Which Also Supplies Battery Current

To meet a considerable demand for a small, compact, economical, and automatic ringing and signaling plant suitable for use in outlying districts often served by magneto type central offices, the 800-type plants were designed. The equipment is arranged for mounting upon a standard floor-supported channel rack the assembly of which is illustrated by Fig. 11.

The capacities and voltages of power plants of this series are as follows, some being automatic and some being arranged for manual transfer to the reserve source.

The 801-A plant, and the 801-C plant illustrated, intended principally for magneto offices having carrier system installations, supply ringing power at a 20-cycle frequency from a motor-driven magneto during normal operation. The circuits are so arranged, however, that they will be automatically connected to an emergency vibrating interrupter operated from dry cells in the event that the primary source of power should fail.

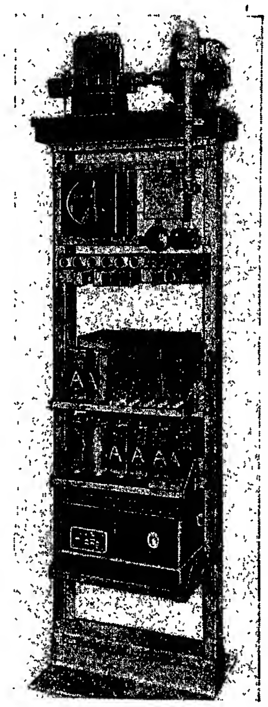

Fig. 11—Small Plant for Supplying Ringing and Transmitter Current

No. 801-C, 75-110 volts, 20 cycles, 0.19 amperes a-c. 8.2-9.2 volts, 20 or 40 ampere-hours d-c.

When the 801-C plant is used in a magneto type office power supply for the operator's telephone circuits is provided by a small 8-volt storage battery continuously supplied by a Rectox rectifier, making this a double-current plant.

Referring to Fig. 11 it will be noted that the battery equipment for the interrupter consists of dry cells and radio block type *B* batteries. The case on the lowest shelf houses the 8-volt battery supply unit.

The 802-A plant supplies power at 135 cycles from

Type No.	Capacity	Nominal a-c. voltage	Nominal frequency	Service
801-A	0.19 ampere	75–110	20	Ringing
801-C	0.19 ampere	75–110	20	Ringing
	20 or 40 ampere-hours	8.2–9.2 (d-c.)		Transmitter battery supply
802-A	40 watts	35	135	Composite telegraph-telephone ringing
802-B	0.015 ampere	6	1000	Toll line signaling
	0.025 ampere	4.25	1000	Toll line signaling

24-volt battery-driven motor-generator sets for signaling on long telephone toll lines which are composited to provide channels for telegraph circuits. Similarly, the 802-B plant provides signaling current at 1,000 cycles, sometimes referred to as "voice-frequency ringing" for use on long toll lines.

Large Ringing Plants

In addition to the ringing power plants just described there are others with increased capacities ranging from one to eight amperes. This series is illustrated by one of the smaller sizes in Fig. 12.

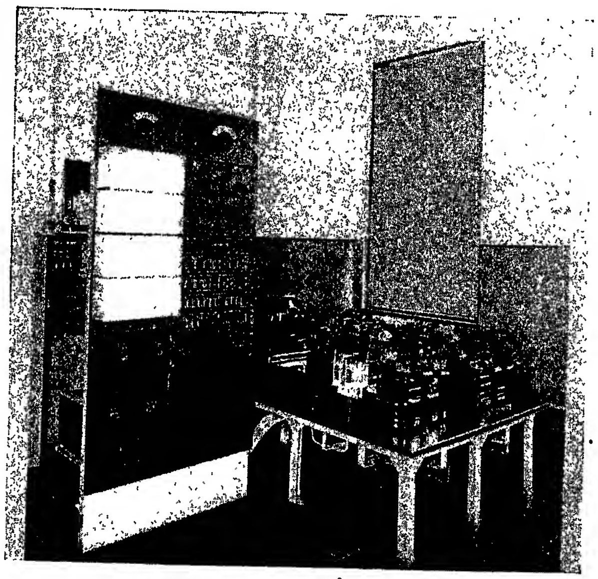

Fig. 12—Power Plant for Ringing and Coin Control

No. 803-A, 105-100-85-77 volts, 1 to 8 amperes, 20 cycles a-c. also + 110 and − 110 volts, 0.25 to 0.50 ampere d-c.

The larger plants of this group also supply positive and negative 110-volt direct current for use in collecting and refunding coins in connection with coin box service. This current is obtained from a double-current generator with an associated transformer equipped with several secondary taps. Two generators with their control equipment are supplied for each central office, one of which is ordinarily driven from an a-c. motor and the other from a battery motor, the former, under normal conditions, being continuously operating. Each machine has directly coupled to it by means of the shaft and gears equipment to supply tones and signals used in the supervision of telephone calls and to provide interrupted current for automatic "machine ringing." Fig. 12 illustrates the segmental drum interrupters used for many years, and also shows the radio-frequency choke coils, contained in rectangular boxes, provided to prevent interference from the current interruptions.

Fig. 13 pictures one of the newer machines equipped with the recently developed tone alternator and mercury type drum interrupters. This machine is shown provided with both line and battery-driven motors which have been used under some conditions.

Automatic Transfer to Reserve

In order to insure practically continuous ringing service the circuit for these machines is arranged to automatically transfer the office load from the line-driven to the battery-driven generator in case an interruption of any sort in the ringing current is encountered, alarms indicating such a condition. If the interruption has been caused by a failure of the commercial source of power to the driving motor, the transfer to the battery-driven set will remain effective only while the commercial power is off, automatically restoring to normal operation upon the return of this power. This conserves the battery energy and insures the most efficient operation of the plant.

The transfer feature uses a relay permanently connected across the output leads of the line-driven generator. Several 12-pole double-throw contactors are provided in the leads from the two generators to the load which is connected to the mid-point of these solenoid-operated switches.

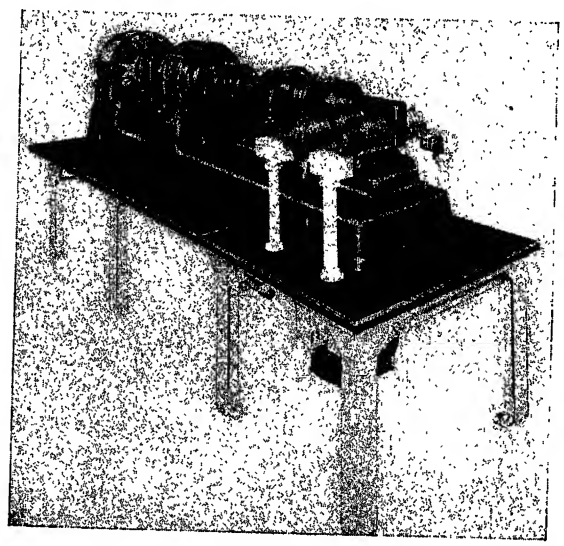

Fig. 13—Ringing Motor-Generator Set for Large Offices

Including tone alternator and mercury interrupters

In case the ringing current is interrupted or reduced materially in voltage, the above mentioned relay releases and closes a circuit to the automatic starter of the battery motor, starting the reserve set. The voltage on the reserve generator builds up very rapidly and at a predetermined point operates the relay again which closes a circuit to the operating coils of the contactors, these making the proper transfer of the office leads to the emergency machine. The operation, from failure of ringing current on one generator to its complete restoration on the other, is accomplished in from 5 to 10 seconds. This transfer merely delays such calls as are awaiting ringing service at the time and,

since transfers are infrequent, has a negligible reaction upon the service as a whole. Transfers for routine maintenance are made without the delay.

Automatic Voltage and Speed Control

A solenoid-type voltage regulator of the continuous vibrating type which opens and short-circuits a resistance in the generator field circuit has been provided for several years past to control automatically one of four a-c. voltages which are available. It indirectly controls the other three and also the two d-c. voltages. This regulator takes care of variations in load and in supply voltage conditions.

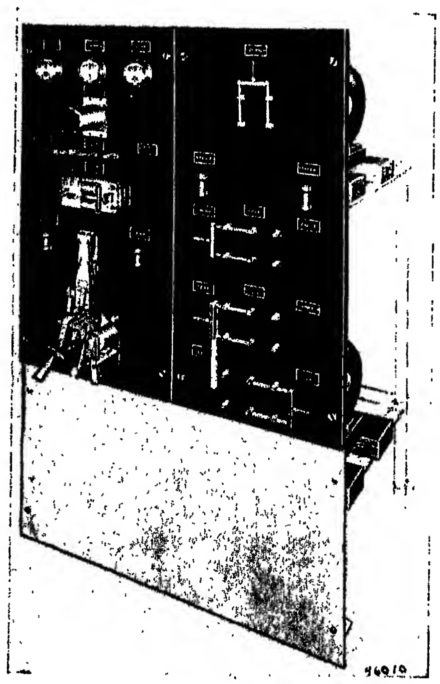

Fig. 14—Ground Potential Compensator for Telegraph Offices

No. 705-A, 0-115 volts positive or negative, 5 amperes

When driven by an induction motor inherent speed regulation is sufficient to give satisfactory frequency. When, however, the generator is driven by a d-c. motor operating upon the reserve storage battery with variable voltage, a centrifugal governor is provided which operates rapidly to open and then short-circuit a resistance in series with the motor field, the relative intervals open and closed being such as to maintain substantially constant speed.

In addition to the automatic battery, ringing, and signaling plants covered above it may be of interest to describe a rather unusual plant provided for use in some telephone offices to maintain continuous "grounded telegraph" service under very unfavorable ground voltage conditions.

Ground Potential Compensator for Telegraph Circuits in Telephone Offices

A plant known as a "ground potential compensator" is illustrated by Fig. 14. It consists essentially of two motor-generator sets, one line-operated for regular use and the other for reserve driven by a motor operated from the battery. The control equipment is mounted on impregnated asbestos panels supported by a framework upon which the motor-generator sets are mounted at the rear of the panels. Up to the present time it has not been considered necessary in this service to arrange for automatic transfer from one set to the other, reliance being placed upon alarms and the manual transfer of the sets since the offices where these sets are in use are continuously attended. The compensator is capable of neutralizing automatically ground potentials between zero and + 115 or − 115 volts.

The object of this plant is to compensate[8] for the varying ground potentials sometimes encountered between telegraph central offices when of a magnitude to interfere seriously with telegraph transmission. A geological formation with a rather high ground resistance between stations, especially when this condition occurs near a parallel run, d-c. street railway, accounts for the most serious fluctuating ground potentials.

The armature of the operating generator is connected in series with the ground lead of the telegraph batteries and the field excitation is controlled indirectly by means of the ground potential fluctuations between the local ground and a ground of approximately zero potential with respect to the distant telegraph station or office.

Such a neutral ground is usually obtainable within a few miles of the affected office over a pilot wire the current in which operates a sensitive pilot relay which causes positive or negative battery as required to flow through the fields of the generator to buck or boost the local ground. In order to make the pilot relay exceedingly sensitive, an auxiliary circuit is added which causes it to vibrate at from 100 to 200 cycles per second.

Operating Experience with Automatic Power Equipment

No trouble of consequence has been reported on automatic starting or voltage regulating equipment in two or three years' operation.

With regard to the dry type copper-oxide rectifiers used in small offices the failures thus far reported appear to be less than 1 per cent. The oldest rectifiers, however, have been in service for less than four years so it is possible that aging may in time increase the mortality, though there is little sign of this at present. Taps to restore output in the event of loss by aging are provided.

The supervisory indicator ball system provided in the batteries has been found helpful in warning of

8. U. S. Pat. 1,126,956, Feb. 2, 1915.

abnormal conditions before the reserve capacity has become exhausted.

The larger automatic plants where the charge is controlled by an ampere-hour meter have had some trouble reported in about 10 per cent of the installations where the meters have been slowed up by dirt or mercury dross accumulating inside the sealed mercury chambers, so that the meters did not record the small discharges at night and, therefore, put back insufficient quantities during the shorter charging periods to keep the batteries properly charged. In most cases upon which records are at hand the difficulties have been discovered and corrected before the batteries ran down completely. In one division upon which detailed data are available six such troubles developed on a total of 68 installations made during a three-year period. All these troubles were discovered and corrected in time to prevent loss of service.

The trial installations of the automatic transfer of ringing machines operated satisfactorily on a few actual occasions of power failure and on numerous tests during some three years, with no failures yet recorded in the comparatively small number of offices so equipped.

One trial installation in a large building with fully automatic central power plant for private branch exchanges, using a motor-generator set, has been in operation for over one and one-half years without any failures. Two similar trials in central offices have been in operation for well over a year without failures reported. As these installations are still considered experimental they have not yet been coded and are not described in this paper, though most of the automatic equipment used is covered.

Future Developments

Experience to date with automatic control equipment in telephone offices and with completely automatic plants has been so gratifying that it seems probable the use of such equipment will be extended, particularly to include larger types of offices. Alarms are, of course, provided to secure help in the event that any of the automatic equipment should fail to perform, or unexpected emergencies should develop. The ideal arrangement seems to be a completely automatic plant which will normally function by itself, both during normal times and as far as practicable during emergencies, requiring of the attendant infrequent supervision and routine cleaning, adjustment, and maintenance.

Discussion

For discussion of this paper see page 971.

Telegraph Power Plants

By E. W. GRIFFITH[1]
Non-member

Synopsis.—This paper mentions some of the early methods of providing electrical energy to operate telegraph circuits and contrasts these with a description of the telegraph power plant in the Western Union Telegraph Company's new building in New York City. It also describes the present general methods of distributing power to telegraph circuits and the equipment recently designed for maintaining continuity of power including engine generator sets for standby service.

AS the term "power plant" generally conveys to an engineer's mind a station where electric energy is generated, the expression, "telegraph power plants," would have had more significance forty or fifty years ago than it does today, for prior to 1888 electric energy for the operation of telegraph circuits was generated, in all stations, on the premises either by primary batteries or dynamos driven by steam engines. Today, there are only two plants in the United States in the Western Union service where power is generated by a private plant, one being at the Punta Rassa, Fla., cable station and the other at Camp Douglas, Wis. At all other points the Western Union contracts for electric energy from public utility companies and transforms the energy to the potentials and type of current necessary for the operation of a telegraph office by means of transformers, motor-generators, storage battery or rectifiers.

The first installation of a dynamo plant in a telegraph office was placed in service in September, 1880 when three groups of 5 dynamos each, belt driven by three steam engines, were installed by the Western Union at 195 Broadway, New York. A reproduction of a sketch showing this installation as it appeared in the Scientific American of Jan. 31, 1880 is shown herewith. Each machine was wound to deliver 40 amperes at 70 volts. Four potentials were used 70, 140, 210 and 280. The fifth machine served to excite the other four in each group. One group furnished positive potentials, the other negative and the third was a spare for either potential. The higher potentials were obtained by connecting the machines in series and making taps at each machine for the potentials mentioned.

This dynamo plant replaced all of the gravity battery cells then in service for main line operation in that office, totaling almost 15,000 cells. The local circuits were still operated from 4,600 cells. This was the first marked improvement in telegraph power plants. Practically an entire floor of 10,000 sq. ft. was released for other purposes as only about one-tenth of the space occupied by the batteries was necessary for the dynamo plant. The difference in weight of the two plants was 60 tons.

Extensive replacement of primary batteries with dynamo plants was not started until 1888 when the New York dynamos were removed and installed at Pittsburgh, Pa., where another group of machines was also installed for providing 15-volt potential for local circuits. The three groups of dynamos were belt driven by three 110-volt d-c. motors. Power for operating the motors was obtained from the Allegheny County Electric Light Company. This dynamo plant replaced 13,000 gravity cells, and was the first telegraph office in the world where there was not a single chemical cell used. It was also the first telegraph power plant obtaining electric energy from an outside source, and the first station where the power company's 110-volt supply was used directly to operate telegraph circuits.

The removal of the New York machines to Pittsburgh was made after a new plant was furnished at the Western Union office at 195 Broadway. This new plant consisted of three groups of machines larger than the initial machines. Machines for three additional potentials were installed in 1892: 7 volts for locals, 23 volts for loops, and 45 volts for short lines. At the same time three steam engines of 75 hp. belted to three Edison 50-kw. dynamos were installed for lights and power and for operation of a carrier system conveying telegrams to and from various points in the office.

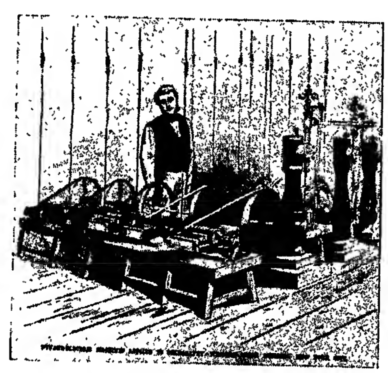

FIG. A—FIRST TELEGRAPH DYNAMO PLANT, NEW YORK CITY, 1880

Dynamo plants were installed by the Western Union at Chicago, Albany, Boston, Buffalo, Philadelphia, and several other stations from 1889 to 1894 releasing thousands of cells of primary battery.

The storage battery, although used in England around 1890, was not considered developed to a point to give sufficiently reliable service for operation of telegraph plants, by engineers in the United States, until 1893.

1. Telegraph Power Engr., The Western Union Telegraph Co., New York, N. Y.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

During that year the first important application of the storage battery in this country was made by the Postal Telegraph Company in its Baltimore office where 430 cells were installed replacing 2,000 gravity batteries. Storage battery was first used by the Western Union in 1893 for local circuits in its office at 21 Cortlandt Street, New York where one 100-ampere hr. cell replaced 30 primary cells The advantage of the storage battery for telegraph operation was soon realized. Only a few main line circuits could be operated from a gravity battery because of its high resistance, whereas any number of circuits could be fed from a battery of storage cells depending upon the capacity of the cells. In view of the steady voltage obtainable with storage battery, its reliability, flexibility, and economy, numerous gravity plants were replaced starting in 1895.

The first large installation was made early in 1895 by the Western Union at Atlanta, Ga. where 700 cells were placed in service releasing 8,000 gravity batteries. Only half of the 700 storage cells were in use at any one time. The battery was charged by two dynamos driven by motors operated from the Electric Light Company's 500-volt d-c. service. During the next two years storage battery plants were installed at most of the larger offices replacing 30,000 primary cells in that period.

The advent of the storage battery for the operation of telegraph circuits brought out numerous charging schemes. In some of the Western Union's branch offices where no charging facilities were available the main line current operating the circuits at such offices was diverted through a single storage cell before connecting to ground and served as a charging current for the battery. This installation, consisting of one 100-ampere hr. storage cell, replaced 30 gravity cells formerly used for local circuits in the office. In the Western Union's office at Allentown, Pa., where a small storage battery installation was made in 1895, 16-cp. carbon lamps were used for a charging resistance and at the same time to light the office.

Notwithstanding the replacements of gravity battery with dynamos and storage cells the number of gravity cells in Western Union service rose from 120,000 in 1877 to 300,000 in 1895. This increase in the use of gravity cells was due in part to the opening of many new small offices where no facilities were available for charging storage batteries or operating dynamos and in part to the addition of new circuits in existing offices where gravity cells were already in service. The peak of the gravity battery installation was reached in 1903 when the Western Union had 528,000 cells of batteries in service. From that time on primary batteries were rapidly replaced by motor-generators or storage cells. At the present time there are only 65,000 cells of gravity battery in the Western Union service, 20,000 of which are in use for main line operation and 45,000 for local service. These cells are principally at isolated railroad stations and in groups of less than 150 cells.

The most modern and probably the largest telegraph power plant in the world was recently placed in service in the Western Union's new building at 60 Hudson Street, New York City. For the operation of telegraph circuits a very steady and reliable source of power is necessary. Every precaution must be taken to keep the lines fed with electric energy and to maintain a steady potential on the telegraph circuits to prevent delays in the handling of telegrams. A power interruption is serious in the telegraph service not only because of message delays for the period of interruption but also because of the time required to reestablish service particularly in those methods of operation requiring manual attention, such as in trunk line multiplex systems. Therefore in the important offices of the Western Union where the service of the power company is subject to interruption, the telegraph plant consists of equipment to maintain continuous service.

Telegrams at the present time are conveyed from one wire to another or from one department to another by means of systems of belt conveyors and pneumatic tubes which systems are operated by electric motors. It is essential that such apparatus also be kept in operation continuously to avoid delay in the handling of telegrams.

In the Western Union's new building in New York facilities are provided to maintain continuous power for telegraph operation under any conditions. Direct-current, 120-240 volts, three-wire power obtained from the New York Edison Company is used for a major portion of the building load and the telegraph load. This d-c. service is obtained over two sets of feeders directly from two separate substations. These feeders follow different routes from the substations and enter the building at opposite sides as a precaution against interruptions due to local disturbance in the streets. A tie to the street network is provided from each entrance room in addition to the direct feeders.

Since a small amount of a-c. power was required for use in the laboratory and in order that the service connection for this a-c. power would be of sufficient size for use also as a reserve power supply, a portion of the building equipment and pneumatic tube compressors was connected to the a-c. service. The use of alternating current was restricted to definite sections of the building so as to avoid duplication of wiring and possible confusion between services. A 300-kw. motor-generator as shown in Fig. 1 was provided to convert this a-c. service to three-wire direct current for emergency power purposes.

The Western Union is greatly indebted to the New York Edison Company and The United Electric Light & Power Company for their cooperation in arranging for the services mentioned. They realized the importance of reliable power service for telegraph operation and provided every facility available to maintain continuity of service. With all of these facilities it would seem practically impossible to have a complete power interruption However, it was considered advisable to provide some source of emergency service independent

of any outside source to guard against catastrophes and explosions in the streets which might destroy both a-c. and d-c. feeders, and also to protect against major troubles in generating stations.

To provide an independent emergency power source a 250-kw. steam turbine generator set as shown in Fig. 2 was installed. This turbine is driven by steam from the regular building boiler plant. A turbine set was selected rather than an internal combustion engine in view of the fact that the boiler plant for building steam supply is operated at 80 lb. pressure and of ample size. Without the necessity of providing a special steam supply considerable economy and saving in space was effected by the use of the turbine set. The turbine selected was of a type which could be started quickly.

Fig. 1—300-Kw. Emergency Motor-Generator Set, New York

When a power failure occurs it is evident that some time will elapse before the emergency a-c. to d-c. motor-generator or turbine set can be started and placed in service. To avoid any interruption in the operation of telegraph or ticker circuits facilities are provided to maintain continuity of power to such circuits until the emergency units can be substituted for the regular power supply. Two banks of storage battery are provided, a view of which is shown in Fig. 3. Each bank provides 120-240-volt power, the same as the Edison d-c. power and each bank is of sufficient size to carry a load of 400 amperes for a period of 45 minutes with a drop of not over 6 volts. One bank is for ticker service and the other for telegraph service. These batteries are connected to the loads mentioned by means of automatic switches which operate when the voltage of the Edison service falls to 114 volts on either side.

Fig. 2—250-Kw. Emergency Steam Turbine Generator Set, New York

The reserve motor-generator and the turbine generator are intended to carry only the essential loads such as telegraph and ticker circuits, belt conveyors, a portion of the pneumatic tubes, about 15 per cent of the lighting circuits on operating floors, and one elevator.

The main power switchboards located in the basement are separated into two sections: one as shown in Fig. 4, for controlling building power and general lighting, and one as shown in Fig. 5 for telegraph power including belt conveyors, compressors, a portion of the lights and controls for the emergency units and battery. The telegraph section of the switchboard provides facilities for transfer of all loads connected thereto to regular or emergency services.

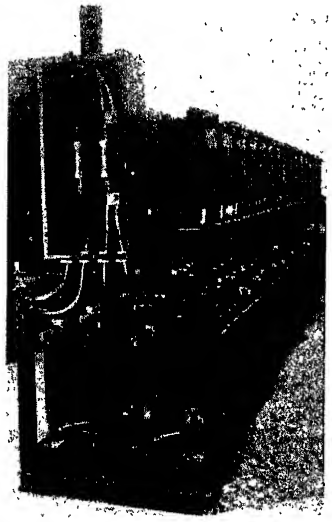

Fig. 3—Emergency Storage Battery, New York

Fig. 4—Main D-c. Building Power Switchboard, New York

Duplicate sets of feeders are provided through separate shafts in opposite sides of the building from the telegraph switchboard to the 11th floor power room, where are located the motor-generators for providing the necessary line and local potentials. A switchboard is

provided in the 11th floor power room for distributing the local and line potentials and the 240-volt d-c. service for belt conveyor power to the various operating floors. A view of this board is shown in Fig. 6. The telegraph and ticker power feeders terminate on the four double-throw switches on the left hand panel.

A signal system is installed to indicate to the engine room attendant which of the four sets of telegraph and ticker feeders from the basement to the 11th floor power

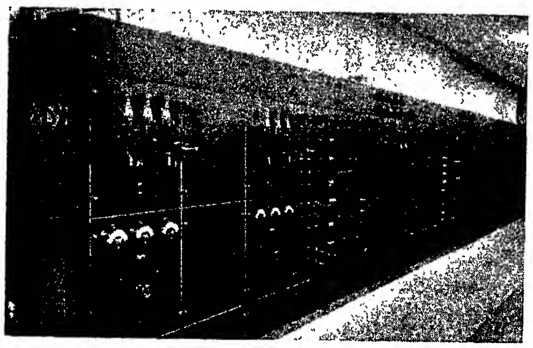

FIG. 5—TELEGRAPH POWER MAIN SWITCHBOARD, NEW YORK

room, are in use. Specially designed auxiliary contacts are installed on the four feeder switches on the 11th floor switchboard. These contacts control lights in a signaling cabinet placed above the basement switchboard as shown in Fig. 7. A single line diagram of the feeder connections to the 11th floor switches is marked on the face of the cabinet. The completion of the circuit whenever a switch on the 11th floor switchboard is closed is indicated by the illumination of a line on the cabinet representing that switch. It may be seen, that

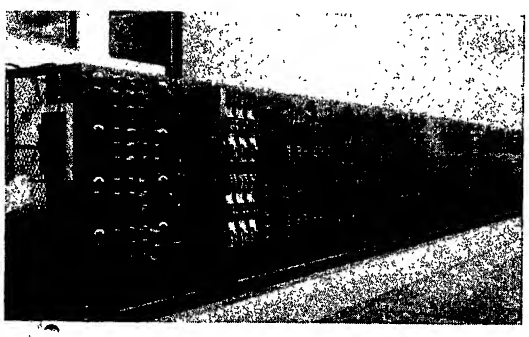

FIG. 6—TELEGRAPH POWER SWITCHBOARD, 11TH FLOOR NEW YORK

telegraph feeder No. 2 and ticker feeder No. 2 were in use for their respective loads when this picture was taken. Special independent telephone circuits and alarm systems are also provided for communication between these two centers and to give warning of the failure of any motor-generator set or operation of the automatic transfer switches. These automatic switches, which will be described later, are located in the 11th floor power room close to the telegraph load. Two separate feeders run from the batteries through the basement switchboard to the emergency side of the automatic switches and provide against failure of power to the telegraph and ticker service whether such services are being fed from Edison power, the emergency motor-generator or the turbine set.

Distribution of power for the operation of telegraph circuits and belt conveyors from the 11th floor power room is made through separate sets of feeders to each floor in two shafts. From each of these shafts the power is distributed to approximately one-half of each floor. In event of the blowing of a fuse on the distribution panel only half of one floor is affected.

An additional precaution, which is an innovation in so far as the Western Union is concerned, is the use of no-voltage release circuit breakers located in the power shafts on the belt power feeders and telegraph local feeders. Both loads consist of a large number of small motors and much trouble had been experienced in the past as a result of power being restored after a short

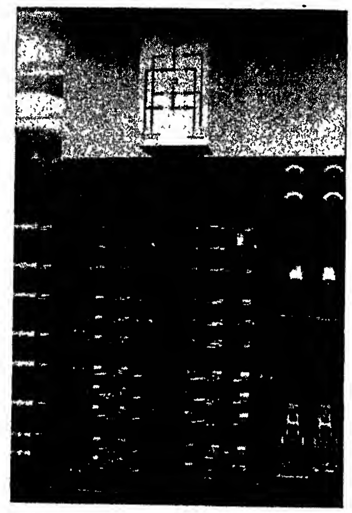

FIG. 7—SIGNAL SYSTEM ON TELEGRAPH FEEDERS, NEW YORK

failure, with these motors stopped but still on the line. The usual result, then, was the blowing of a main fuse or opening of a circuit breaker and a further delay affecting, perhaps, the entire office.

The various telegraph potentials are fed through a distribution cabinet in each shaft. From the shaft cabinet, feeders run to several zone cabinets on each floor. The belt power is fed through two or three circuit breakers in the shafts. Each circuit breaker feeds to a distribution cabinet on the operating room floors. A view of the telegraph distribution cabinet and circuit breaker in a shaft is shown in Fig. 8.

The entire arrangement of distribution is such as to cause a minimum of delay in event of accidental short circuits or power failures.

The maximum demand required in the Western Union's New York office for handling telegrams at present totals 295 kw. divided as follows:

For telegraph circuits..	75 kw.
For ticker circuits......	50 kw.
For belt conveyors.....	40 kw.
For pneumatic tubes...	130 kw.

The monthly power consumption for these services totals very nearly 150,000 kw-hr.

There are 37 motor-generator sets in service in the 11th floor power room totaling 190 hp. the maximum size of any one set being 10 kw., for the operation of 1,123 automatic and 145 Morse telegraph circuits and about 500 ticker circuits. A view of a group of these motor-generators is shown in Fig. 9.

For the operation of message belt conveyors 15,722

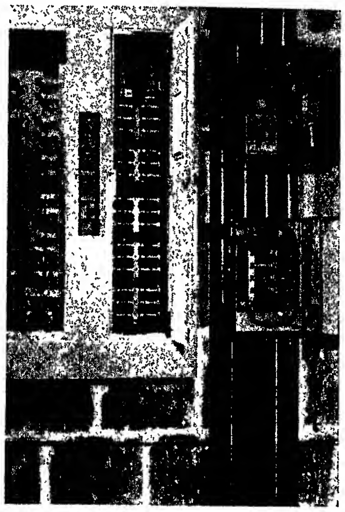

Fig. 8—Typical Installation Cabinets and Circuit Breakers in Shaft, New York

ft. of belt driven by 274 small motors totaling 76 hp. are installed. A section of such equipment with driving motors is shown in Fig. 10. In Fig. 11 is shown the individual table belt motor drives.

There is installed in the building 50,000 ft. of tubing

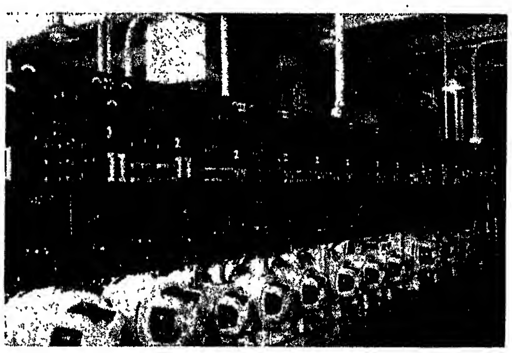

Fig. 9—Group of Motor-Generators, 11th Floor Power Room, New York

for handling telegrams to 60 branch offices and 48 stations, requiring 356 hp. in blowers and compressors to operate them. Fig. 12 is a view of the 60-hp. compressor units and the high-speed blowers are shown in Fig. 13.

It is now the general practise, with the Western Union, when a telegraph office is moved to a new location or a new repeater station is established, to obtain electric energy from a local power company and to install motor-generators, for operation of its telegraph circuits as the initial cost and maintenance is less than storage batteries and a considerable saving in space is effected. There have been no installations of storage battery plants, except at cable stations and for emergency service, for several years. In some cases, particularly in the south or southwest where thunderstorms are prevalent, a small portion of an existing storage battery plant is used in the new location in conjunction with automatic switches to maintain continuity of service over the short periods of interruptions. There is, however, a number of telegraph stations using

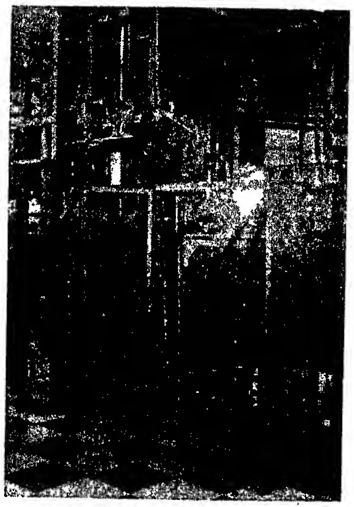

Fig. 10—Belt-Conveyor Driving Motors, New York

storage battery for operation of the circuits. At most of the offices so equipped the storage battery will be replaced with motor-generators when the office is moved or enlarged. Storage batteries have been replaced by motor-generators in all existing plants except cable stations when it is found economical to do so. The Western Union has 120 stations including cable offices

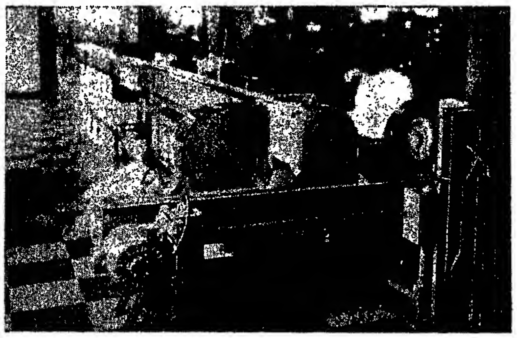

Fig. 11—Direct-Drive Belt Motors, New York

where 33,000 storage cells are installed; the largest at any one point being at Bay Roberts, N. F. cable station where 2,581 cells are in service. The largest size is 400-ampere hour.

The first installation by the Western Union, of storage battery, automatically operated in conjunction with

motor-generators to maintain continuous service, was made at the time a new office was installed in New Orleans in 1923. Considerable trouble had been experienced in the old office due to many short power failures interrupting the service and particularly the circuits emanating from the Cotton Exchange there. At the Exchange in New Orleans cotton orders are sometimes executed with Liverpool, England, in a minute and a half so that the importance of continuity of service can readily be seen.

The installation of such plants was extended to other large stations where numerous automatic circuits were in use and where power interruptions were frequent. Difficulty was experienced, however, in obtaining an automatic switch which would operate fast enough to maintain synchronism on the automatic circuits. Several types were tried, and it was not until 1927 that a switch was developed by a manufacturer, in conjunction with Western Union engineers, to meet the requirements.

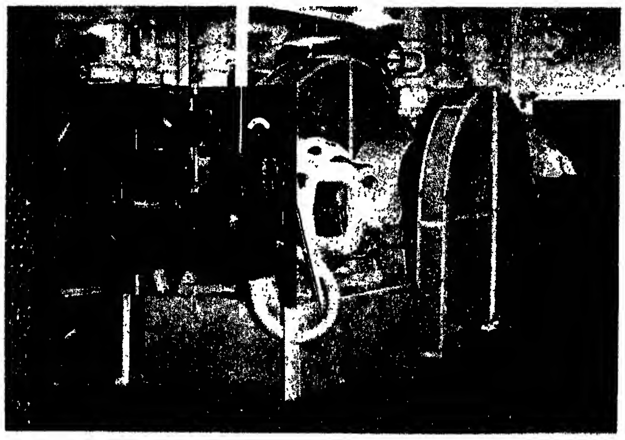

Fig. 12—60-Hp. Pneumatic Tube Compressor, New York

Fig. 13—High-Speed Pneumatic Tube Blowers, New York

In Fig. 14 two of these switches which are in service in New York are shown. These are solenoid type with the blades or moving contacts assembled as one unit on a rocker arm. The current is carried by copper-to-copper contacts but the breaking contacts are copper to carbon. Special relays were designed for operating the switch on a slight drop in potential of the regular power source. A resistance in series with the switch coil is cut in and out by auxiliary contacts on the switch. The resistance is short-circuited when the switch is closed on the bottom or reserve contacts, thereby subjecting the coil to full voltage when the relays are closed. When the switch is closed on the regular side the short circuit is removed thereby reducing the current in the switch coil to just enough to hold it. This reduced current also reduces the lag when the power fails and the switch drops to the reverse side. The two groups of contacts are arranged so that during the operation of the switch there is only one-sixteenth of an inch clearance at the time the regular contacts open and contact is made on the reverse side. Consequently the interval of no current is very small. The change from one source of power to the other is accomplished in less than one-tenth of a second. The switch is arranged so that it can be restored to regular position only by closing the relay contacts manually. The air gap on the relays is adjusted so that the relays will not close automatically. This was done purposely so as to prevent rapid operation of the switch in event of fluctuations of the regular power source. The switch is kept on the battery until the service is restored and steady.

Fig. 14—Automatic Throw-Over Switch, New York

The switches shown in Fig. 14 are the largest of this type in service. They are 800-ampere capacity. The back contacts of the operating relays are utilized for audible signal when the switch operates.

Storage battery or motor-generators are in service in over 1,200 Western Union offices. Ninety per cent of such installations are motor-generator plants.

Motor-generators for Western Union service are constructed by various manufacturers under special specifications. Very close regulation is necessary not only from no load to full load but at any load as the load of telegraph circuits varies. The maximum temperature rise allowed for field coils and windings is 35 deg. cent. and commutators 40 deg. cent. which insures obtaining liberally rated machines. The manufacturers are also required to specify the minimum guaranteed efficiency.

As practically all telegraph circuits are energized at all times the efficiency of the machines is a factor in determining the value of machines offered by various manufacturers.

In all except very small offices positive and negative polarities are required as most of the telegraph circuits are operated on the duplex system. For each potential

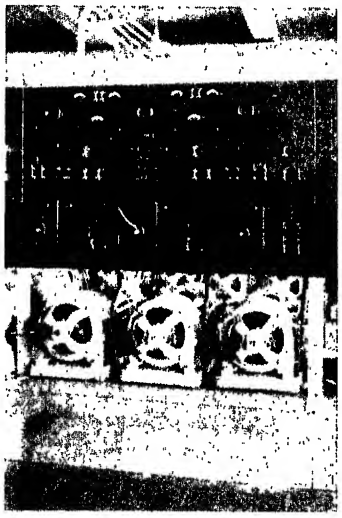

Fig. 15—Group of D-c.—D-c. Motor-Generators for Furnishing One Positive and Negative Polarity, New York

a group of three machines is provided, one for positive, one for negative, and one spare for use of either polarity. Such a group is shown in Fig. 15. These are d-c.—d-c. sets with hand starters. Equalizer connections are provided on the generators to permit paralleling while the load is transferred from one generator to another.

The machines are mounted on benches of angle iron

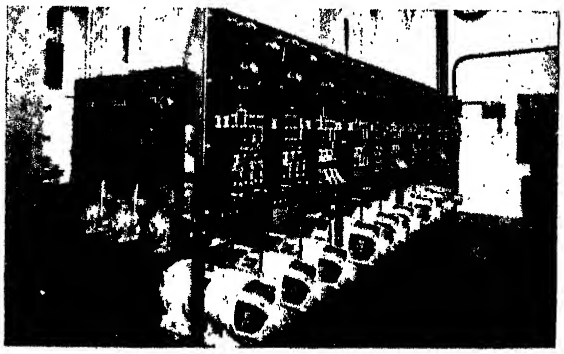

Fig. 16—Group of A-c.—D-c. Machines, Syracuse, N. Y.

construction with ¼-in. sheet metal tops. The motor-generators are mounted on springs to prevent the transmission of any vibration. Ebony asbestos panels are now used exclusively in preference to slate, as trouble was experienced due to conductive veins in slate and breakage in shipment.

A group of nine a-c.—d-c. motor-generators installed in the Syracuse office is shown in Fig. 16. The motors are started by means of double-throw switches placing the machines directly on the line for starting. Otherwise the panels and benches are similar to the d-c.—d-c. type. A group of small motor blowers for operating pneumatic tubes is shown at the left.

Motor-generators of 500 watt size and smaller are generally mounted on what are termed "wall type benches" which are specially designed to conserve space. A view of two such benches installed in the Buffalo office is shown in Fig. 17. They are of angle iron and sheet steel construction and are self-supporting, and can be mounted back to back if necessary. As these benches are designed primarily for use in small offices during the advent of the simplex automatic system, safety type switches are provided. The motor-genera-

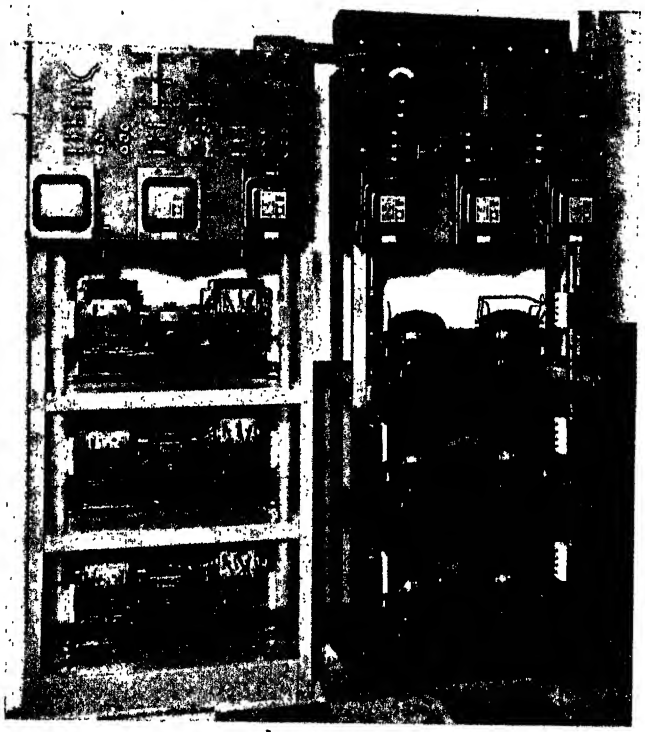

Fig. 17—Wall-Type Motor-Generator Benches

tors shown in the photograph are metallic circuit machines. These sets are also of special design with a double commutator generator. Each machine provides both polarities of 160 volts and is used on an individual metallic circuit. By means of double-throw switches, jacks, and cords and plugs, spare machines can be transferred to any circuit in event of trouble with the regular machine.

When the wall type bench is used in small offices it is equipped with a small panel with simplified apparatus to change machines similar to the larger motor-generator installations.

There are 1,085 stations of the Western Union equipped with a total of 4,230 motor-generators; the largest number at any one station being in the New York office where 37 are installed. The largest size used exclusively for the operation of telegraph circuits

is in Philadelphia where three 20-kw. 110-volt potential machines are in service.

In most of the large offices special motor-generators are installed for synchronizing clocks. These machines are automatically started 20 seconds before each hour and stopped about 20 seconds after the hour. The starting and stopping are controlled by contacts on special clocks through relays.

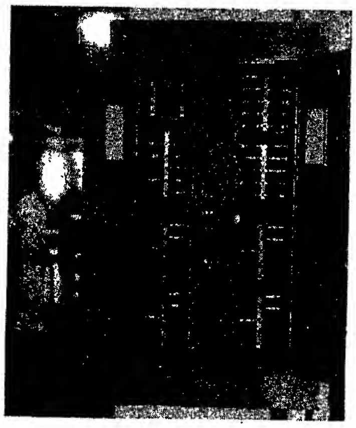

Fig. 18—Typical Zone Cabinet

Power is also supplied for the operation of time stamps. At each automatic operating position a stamp is installed for the operator to time the telegrams when received. The stamps are operated every minute by master relays.

Specially designed cabinets, termed "zone cabinets," located near the operating equipment, are used to distribute the power to sections of operating apparatus.

Fig. 19—Belt-Power Distribution Cabinet

Several cabinets are located on each floor. A photograph of a typical installation is shown in Fig. 18. This cabinet provides ten feeders of each polarity of 160- and 240-volt potential, twelve for the 110-volt negative, 10 for the 110-volt positive, ten for time stamp circuits, and twelve for grounds. Two copper straps are installed on the inside of the barrier in the upper left hand corner, one being connected to the 110-volt bus in the cabinet through a lamp and the other connected to the ground bus. This arrangement is for testing fuses. In the center a special compartment is provided for storing fuses. A framed chart is installed on the inside of each cover to properly designate the feeders.

Zone cabinets are also provided for belt conveyor motors and are generally mounted on columns similar to the telegraph zone cabinets. Such a cabinet as installed in the New York office is shown in Fig. 19. This cabinet provides for feeding ten 220-volt d-c. circuits and is equipped with a fuse testing device and fuse container. The box at the bottom of the cabinet contains a special starting device for controlling the motor on a "pick-up" belt and the motors on "table belts" feeding into that pick-up belt. It was found desirable to stop all table belts in event the pick-up belt stopped for any reason, otherwise telegrams would accumulate in large numbers on the latter belt. The

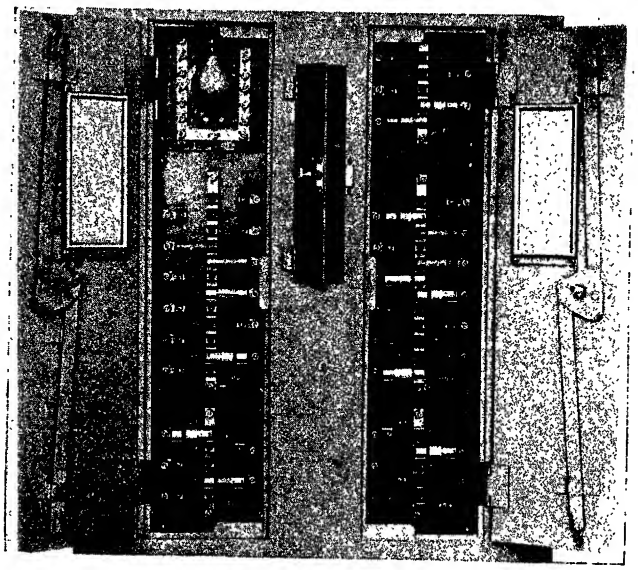

Fig. 20—New Type Zone Cabinet

coil of the starter is connected across the pick-up belt motor and the starter controls a maximum of 12 table-belt motors. This installation at New York was the first of this kind and has proven a great benefit to the service.

In order to reduce the number of styles and sizes of zone cabinets to a minimum and to economize on materials and space a new type of zone cabinet has recently been standardized. A photograph of this cabinet is shown in Fig. 20. The cabinet has been designed to use boxes of three sizes, all of the same width and depth but of three different heights to provide for variable distribution requirements. The fuse clips are mounted directly on the main buses thereby saving two inches in width. Two sizes of panels are used, the larger being twice the height of the smaller. With this arrangement greater flexibility will be effected and at the same time the costs will be reduced since the parts

can be ordered in larger quantities and stocked and assembled as desired.

At all important offices where an interruption to the power source would seriously affect telegraph operation an emergency plant is installed. The first type of engine used for emergency power was a single-cylinder slow-speed gas engine belted to a generator. Several of these single-cylinder sets were installed but most of them have been replaced by multi-cylinder higher speed gasoline engine units. The first multi-cylinder type was installed in 1915. However, the multi-cylinder type was not used exclusively until about 1920. They were 40 to 50 hp. engines running at 600 to 900 r. p. m. and operated mostly on illuminating gas. Considerable trouble was experienced in operating on gas due to the fact that generally during a failure of the electric service an abnormal use of gas occurred which reduced or varied the pressure sufficiently to cause unsteady operation of the engine. Subsequently a gasoline-kerosene type was developed and used. This type, although designed for normal operation on kerosene, was started and warmed up on gasoline. With these engines the amount of gasoline was limited to the amount necessary for starting purposes only. Troubles were also reported in connection with such sets due to the rapid collection of carbon and the difficulty in maintaining the duplicate carburetors and fuel supply systems. With one exception all emergency units installed since 1923 have been gasoline engine driven.

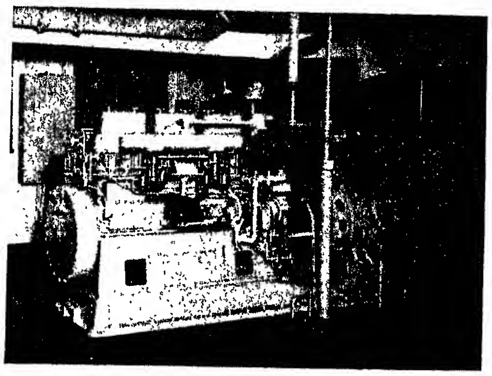

Fig. 21—Emergency Unit, Buffalo, N. Y.

There are 114 emergency engine-generator units installed in Western Union offices in the United States. Sixty-seven operate on gasoline, 23 on gasoline and kerosene, 16 on gas, 7 are semi-Diesel and 1 is full Diesel. The smallest unit is 8 hp. and the largest at the present time is 120 hp. although 200-hp. units will be installed in the near future.

The prime requirements for an emergency plant for telegraph service are simplicity of operation, quick starting, capability of handling a load very soon after starting, very close speed regulation on constant load, close regulation from no load to full load and continuous operation once started.

A photograph of a recent installation in the basement of the Rand Building in Buffalo is shown in Fig. 21. This unit consists of an 80-hp., 900-r. p. m., 6-cylinder gasoline engine driving a 40-kw. three-wire 220-volt d-c. generator.

To insure steady operation only engines with four or more cylinders are specified.

All engines of 25 hp. and larger are equipped with two entirely separate ignition systems including separate magnetos with impulse couplings and separate spark plugs. Separate switches are provided so that both sets of ignition systems can be operated simultaneously or either set alone.

An overspeed cutout device is provided to ground the magnetos when the engine reaches a predetermined maximum allowable speed. This device is so arranged that once it operates the magnetos will remain grounded until reset by hand.

Fuel pumps are specified in duplicate and may be either two direct-driven pumps or one direct-driven pump and a hand-operated pump.

A one pint gasoline tank is provided on the engine for gravity feed to the carburetor. This tank is equipped with a gage showing the quantity of fuel also a quick-vent float-air valve. The latter valve will close the vent if the tank is completely filled and is provided to prevent flooding of the room with gasoline in event the main tank is filled beyond its capacity.

Most of the units are not provided with radiators for cooling water. However, each engine is equipped with a water pump for future use in conjunction with a radiator if there appears to be a possibility of water failure. At a few stations where the water supply is unreliable the sets are supplied with radiators initially. At one or two points radiators are provided as a separate unit driven by a motor.

A regulator is specified to automatically open the cooling water supply when the temperature reaches a certain point, to maintain the water at a determined temperature, and shut off the supply when the engine stops.

A control panel is specified for mounting the tachometer, starting switch, ignition switches, oil-pressure gage, thermometer, throttle and choke controls, and a frame for brief operating instructions. In Fig. 21 this panel is shown mounted over the flywheel. From this point the engine can be started or stopped and its complete operation such as speed, oil pressure, and temperature can be observed.

The engines are direct connected to the generators or alternators by means of flexible couplings, and are started with automobile type starters.

The entire assembly of the engine and generator is made on a substantial cast iron base. To lessen the noise and prevent the transmission of vibration to the building the entire sub-base is mounted on springs. All connections such as water inlet and outlet, fuel suction and return lines and exhaust pipe are made with flexible

piping. The spring mounting also lessens the wear of the moving parts of the engine due to the cushioning effect, thereby reducing the maintenance and prolonging the life of the unit.

All moving parts are protected by guards and the exhaust manifold is water jacketed.

A maxim silencer is usually installed in the exhaust line. In the installation at Buffalo it is on the roof at the 20th floor set back.

As considerable condensation occurs in the long run of exhaust pipe a drain cock is placed at the bottom of the exhaust line in the engine room. This can be seen just to the right of the generator.

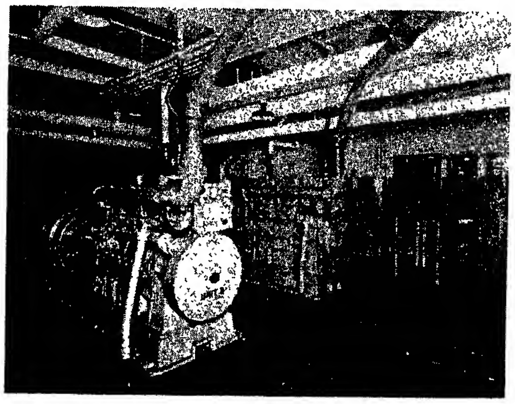

Fig. 22—Emergency Units, Pittsburgh, Pa.

In some of the larger offices two smaller emergency units are provided instead of one large unit because it is frequently found to be more economical, it affords greater protection than the single unit installation and the replacement of parts on the smaller engines can be more readily handled by the local force. Two units are installed in Pittsburgh, Pa., a view of which is shown in Fig. 22. These units are similar to the one at Buffalo except the engines drive 220-volt three-phase 60-cycle 50-kva. alternators. At the right can be seen the storage batteries used for starting together with the charging panel. These batteries are furnished in duplicate to insure ample capacity in event of trouble in starting. The batteries are also used in connection with a relay to automatically light a few 12-volt lamps in the engine room when the power fails. Engine units of the type shown can be started very quickly. In some cases after a power failure they have been started and loaded in less than 2 min. However on trial test this has been accomplished in 30 sec.

Where two engines with alternators are installed no attempt is made to parallel them. The various loads are divided and switching facilities provided so that all or any part of the loads can be placed on either unit.

Only one Diesel-engine set is in Western Union service. This is installed at San Francisco and is shown in Fig. 23. It has not been found economical to install Diesel engines instead of gasoline engines for telegraph emergency service as the initial cost of the Diesel engines is much higher and the sets are run so seldom that economy of operation is comparatively a small item. The Diesel set was installed in this case as the storage of gasoline, except in a small quantity, was prohibited. This set is equipped similarly to the gasoline sets; that is, with overspeed cutout device, water regulator, duplicate fuel supply pumps, engine fuel supply tank, control panel, and spring mounting. It was necessary to use two electric starters on this engine as well as to provide compression relief valves for use in starting. This set is a 70-hp. 800-r. p. m. 4-cylinder engine, direct connected to a 40-kw. 3-wire 220-volt d-c. generator.

In a few cases it has been found advisable to install the emergency units above the ground floor because of the danger of high water or floods. One such installation is at New Orleans where two 60-hp. 900-r. p. m. engine-generator sets are installed on the third floor. In this case a motor-driven pump was mounted in a fireproof enclosure a few feet above the ground level for pumping fuel from the underground supply tank to a small reservoir in the base of the engine. This is a rather expensive method of installation and is subject to troubles in operation.

A semi-automatically operated gasoline engine generator set was recently installed in the Western Union office at Shreveport, La., on a trial basis. If proper operation can be obtained it will save considerable time in restoration of service where the unit is several floors from the operating force, as it is in this case.

Such a semi-automatic unit is shown in Fig. 24. In addition to the standard accessories with which this set is equipped there are also the necessary electrically-operated attachments for starting or stopping the set at a remote station. These consist of a contactor for closing the starting motor circuit, the solenoid-operated choke on the carburetor and the fuel pump control circuits. The set is also equipped with a radiator and fan for cooling purposes in order to make the unit independent of the local water supply. One set of control buttons is mounted on the engine control panel and another or duplicate set is mounted on a power panel in the operating room in another part of the building.

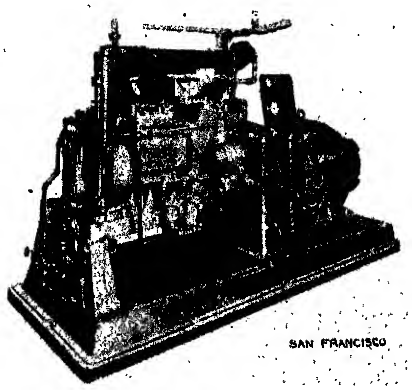

Fig. 23—Diesel Emergency Unit, San Francisco

In case of a failure of the regular power an attendant in the operating room may start the emergency unit and transfer the load to it without leaving the room.

In the past five years marked improvement has been made in the design of internal combustion engines prompted by the demand of the automotive and aviation industries for higher powered engines of less weight. This brought about the design of higher speed industrial type engines which proved more reliable and satisfactory in operation. With the economic advantages in the use of higher speed engines for telegraph emergency plants in view, tests were recently made on engines operating at 1,200 r. p. m. which resulted in a decision to make a few trial installations. The first set will be an 80-hp. 4-cylinder engine running at 1,200 r. p. m. and will be installed at the Western Union's office at Jacksonville, Fla. where a saving of about one-third in the initial cost of the plant as well as a saving of space is expected.

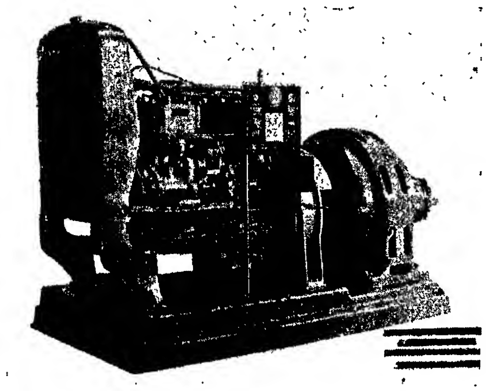

Fig. 24—Semi-Automatic Emergency Unit, Shreveport, La.

Two units similar to the one shown in Fig. 25 consisting of a 200-hp. 8-cylinder 1,200-r. p. m. gasoline-engine direct connected to a 125-kva. 208-volt three-phase 60-cycle alternator will be installed in the new Western Union office at Boston. A considerable saving in space as well as in initial cost will also be made in this case.

Fig. 25—200-Hp., 1,200-R. P. M. Gasoline Engine-Generator Unit

To provide for the quick restoration of service at offices which may be completely destroyed by fire, flood, earthquake, tornado or hurricane several portable engine-generator sets of 5-kw. capacity each mounted on wheeled trucks together with similar trucks containing motor-generator sets, telegraph switchboards, and operating sets are distributed throughout the country available for quick transportation by rail or truck to any point.

Perhaps there may be some question as to whether the emergency plants described have been of any service. One instance of their use was during a calamity which is remembered by all. When Miami, Fla. was hit by a hurricane in 1926, two small engine units provided power to operate the telegraph plant and provide light for over 72 consecutive hours or until the power was restored.

To provide for the satisfactory operation of new circuits, methods, and apparatus in the most economical and reliable manner development and design of power plant equipment are carried on continually and there is no doubt that the telegraph power plant of 1980 will show as much improvement compared to the present plant as the present day plant compared to 1880.

Discussion

AUTOMATIC POWER PLANTS FOR TELEPHONE OFFICES

(Young and Lunsford)

TELEGRAPH POWER PLANTS

(E. W. Griffith)

J. L. Woodbridge: Possibly a brief mention of certain developments in the design and application of the storage battery for the service described in the first paper may be appropriate.

The function of the storage battery in this field is threefold,—first as a reservoir of energy to insure continuity of service, second as a means for meeting the increasingly exacting requirements for a constant d-c. voltage and third, in many cases, as a means of reducing crosstalk and machine noise which would be disturbing without some type of filter. It has been the constant effort of the battery manufacturer, in cooperation with the engineers of the telephone industry, not only to perfect the storage battery for carrying out these functions but also to simplify its operation and reduce to a minimum the item of attendance and maintenance.

The improvements and refinements in manufacturing methods and processes which have been and are constantly being introduced are of real importance but are not obvious from a superficial examination and can hardly be reviewed in a discussion of this kind. Certain developments in cell construction and operating methods may, however, be of interest.

One of these is the introduction of the sealed glass jar cell, shipped assembled, sealed, and charged. This practise was introduced over 15 years ago for the smaller sizes of cell used in farm-lighting plants and its success has warranted its extension to the larger cells required for telephone installations. One such installation in Denver is shown in Fig. 1, the enclosed cells being on a rack in the foreground and the large batteries in lead-lined wood tanks appearing in the rear.

It is hardly necessary to point out the important advantages of the enclosed cells in respect to convenience of installation and subsequent inspection and maintenance. Reference has been made to the pilot balls or charge indicators which are now provided in these cells. Hollow-glass pilot balls were furnished with the farm-lighting batteries referred to above. Early experience with the wax balls led to some skepticism in regard to the permanence of their indications but improvements which have been introduced by the manufacturer have strengthened confidence in them, provided their limitations are understood.

To avoid misunderstanding on this point, it is well to point out that owing to the tolerances required in practical production methods both in the specific gravity of the balls themselves and in the nominal full charge specific gravity of the electrolyte as well as the variations in the latter with change of level and with time in service, the indications of these charge indicators must be considered as only approximate, but with these limitations in mind they are undoubtedly a useful adjunct. There appears, however, to be a general sentiment to the effect that in view of these limitations two pilot balls in a cell will serve all practical purposes, and probably in most cases the operation is guided by the indications of one of these. A recently-developed three-cell unit in a glass case is shown in Fig. 2, while a two-cell unit of smaller capacity provided with a rubber tray is shown in Fig. 3.

Fig. 1—Battery Room, Denver Telephone Office

Referring to the counter e. m. f. cells as a means for voltage control, a most important and radical improvement has been introduced, in the substitution of nickel electrodes in an alkaline electrolyte for the lead-alloy-acid combination. Such a cell is shown in Fig. 4. Several distinct advantages have been secured. The nickel electrodes are not subject to corrosion in service, being practically permanent in the absence of impurities, provided air is excluded, this latter being accomplished by the layer of oil on the surface of the electrolyte. No capacity is developed in service, permitting the cells to be short-circuited without excessive flow of current through the short-circuiting switches. No sediment is deposited, thus eliminating periodic cleaning. The momentary high voltage sometimes observed when current was first passed through the lead-acid type of cell after a prolonged stand on short circuit does not occur with these nickel-alkaline cells. They also require less space for a given current capacity.

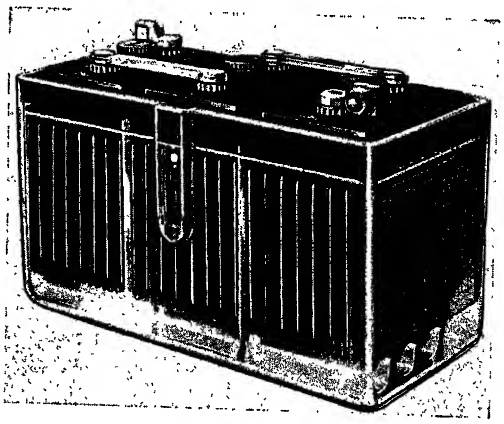

Fig. 2—Three-Cell Battery Unit with Rubber Tray—2-B1-9, 4 Volts, 40 Ampere-Hours

In respect to operating methods, important improvements have been introduced. The substitution of the single battery in full float operation for the double battery operated on alternating cycles of charge and discharge has been mentioned, as well as the use of the ampere-hour meter for the control of partially worked batteries. One of the limitations of the ampere-hour meter has been referred to in the fact that it requires occasional overhauling on account of the fouling of the mercury.

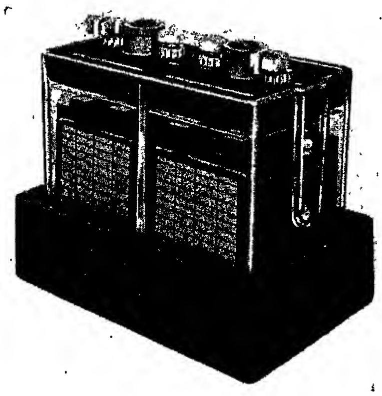

Fig. 3—Two-Cell Battery Unit, with Rubber Tray—2-B1-9, 4 Volts, 40 Ampere-Hours

There is another limitation which should also be kept in mind. The losses in the storage battery are of two kinds. First, that due to the inefficiency of the charging current, a certain portion of the charging energy being necessarily wasted in gassing. In accordance with Faraday's law, that portion of the charging current which causes gassing has no effect in charging the plates. In the case of a battery which is subjected to an appreciable amount of daily discharge, this loss is roughly proportional to the amount of discharge, and can therefore be approximately compensated for by adjusting the ampere-hour meter to run slow by a certain percentage in the charge direction. The other loss in the battery is that due to local action, and is proportional to the elapsed time, varying also with the cell temperature, but bears no relation to the amount of discharge. In the case of a worked battery, this loss is small compared with the charging loss and may be included in the overcharge provided by the adjustment of the ampere-hour meter. In the case of a very

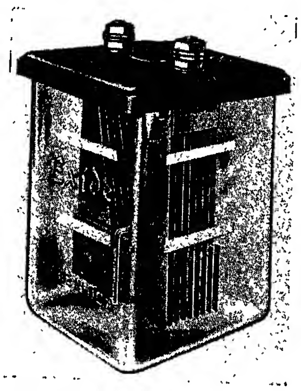

Fig. 4—Nickel-Alkaline Counter Cell—NAK, 2 Volts, 30 Amperes

lightly worked battery, the local action loss, although actually small, may be a very considerable proportion of the total loss, exceeding the limit of adjustment provided in the meter. This condition may be met by supplying to the battery a small constant trickle charge, not subject to control by the ampere-hour meter, but sufficient to compensate for the constant losses.

Much study has recently been given to the control of battery charging by means of a voltage responsive cut-out. In the early history of the storage battery many attempts were made to terminate the charge at a predetermined battery voltage. This method is no more successful now than it ever was, on account of the variation in the full charge voltage of the battery under varying conditions of charging rate, temperature, age of the battery, etc. It has, however, proved quite satisfactory to employ a two-rate charge, the rate being reduced from the higher to the lower rate in response to a battery voltage well below the final value, the lower rate being maintained continuously thereafter and fixed at a value which will not prove injurious. Various modifications of this principle have been successfully applied to meet varying operating conditions.

I. R. Smith: The development of dry disk copper oxide or Rectox rectifiers for the various applications described by Messrs. Young and Lunsford began over four years ago. Since that time I have had the pleasure of cooperating with Mr. Lunsford and his associates of the Bell Telephone Laboratories in the development of a number of different units for these power applications. It may be pertinent to say a few words about these units and their characteristics.

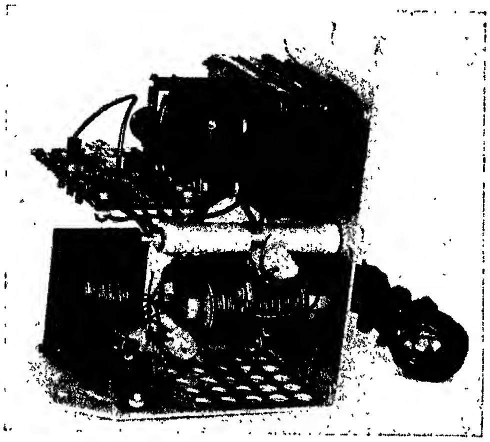

Fig. 5—Rectox Charger for PBX Batteries. Cover Removed

Rectifiers for telephone use differ somewhat from those designed for other applications. In the first place, as pointed out by the authors, reliability is paramount. Operating costs, flexibility, size, and weight are also of importance, but not to the same extent. In the second place, an extensive system of specification and inspection has been built up, based on the long experience of the telephone company in the use of rectifiers. This system, of course, is also applied to the other types of equipment used and I think has a great deal to do with the success of the automatic plant, for it assures interchangeability of parts and duplication of results previously obtained.

For such an application the Rectox rectifier is well suited, as it excels in reliability any other available rectifier, having the ability to run for years with little or no attention, assuming that it has been properly designed and properly applied. Hence, it is well suited for application to telephone work.

The first unit developed was that used for charging 17-20-volt PBX batteries, a view of which is shown in Fig. 5. This unit has a maximum rate of ½ ampere, but it must also be able to charge as low as 0.2 amperes, and of delivering either the maximum or the minimum rate at rated battery voltage and for line voltages varying plus or minus 10 per cent. Furthermore, the rate must be adjustable between the maximum and the minimum in steps of not more than 55 milliamperes each. The resulting unit then is extremely flexible. Adjustment is made by means of transformer taps, for the sake of efficiency, as this rectifier runs continuously. And, as it is used while the battery is furnishing talking current, the normal d-c. wave delivered from the rectifier must be smoothed out somewhat, which is accomplished by means of a small reactor in the d-c. output circuit. Mechanically the unit is designed to be portable, to mount on wall or shelf, and to harmonize in appearance with other equipment in the station.

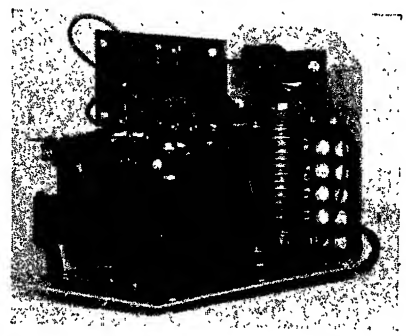

Fig. 6—135-Volt Rectox Charger for Plate Supply Batteries. Cover Removed

The next development was that of the 80-milliampere 130-volt unit for charging repeater plate supply batteries, as illustrated in Fig. 6. Mounted in the same case as the PBX charger, this unit is also provided with a wide range of adjustment for flexibility and to assure its being able to handle any application within its range. On lines of secondary importance where outage due to power failure is not serious, this rectifier with a suitable filter is sometimes used to excite the plate of the tube directly, without the battery.

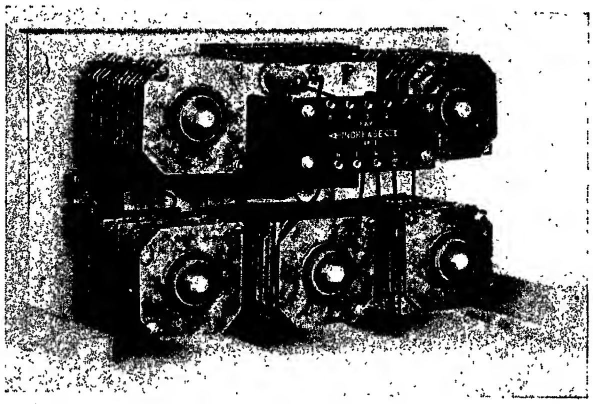

Fig. 7—2¼-Ampere 20-30-Volt Rectox Charger for Filament Supply Batteries

Repeater tube filament supply battery charging called for a somewhat larger Rectox shown in Fig. 7, capable of delivering 2¼ amperes at 20-30 volts. The same provision for wide range of adjustment to meet many conditions has been made. The unit being much larger than the preceding rectifiers was arranged for relay rack mounting.

Several other developments have been carried on, including the magneto power unit, (a combination battery and trickle charger) the superimposed ringing battery charger, and various units used as valves. The latest development is a rectifier which delivers 120 volts and 100 milliamperes with a ripple of less than 3 per cent in the output wave. This rectifier is shown in Fig. 8.

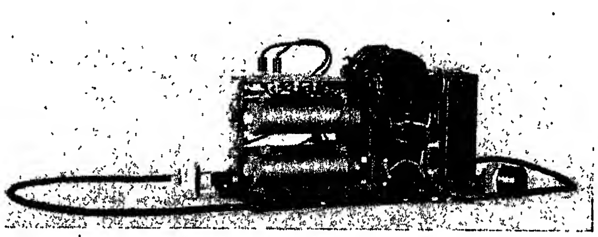

Fig. 8—Power Unit, Case Removed. Rated at 120-Volt 0.1 Ampere D. C.

There are two characteristics of importance. First is the matter of reliability. I think it is of interest to view the life test curve, Fig. 9, showing the average performance of a number of units, with a reduction in charging current of 22 per cent in 3½ years of continuous operation. Those who have had experience with rectifiers know that such operation, absolutely without maintenance or replacements, is really remarkable.

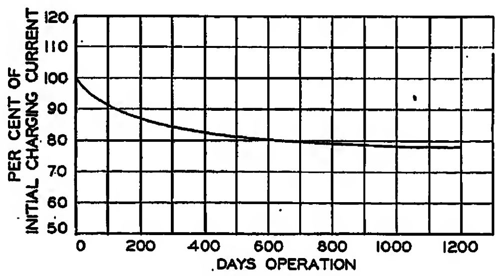

Fig. 9—Rectox Performance on Life Test

On all telephone type rectifiers excess transformer capacity is provided to permit the voltage to be raised when necessary due to aging, thus extending the useful life to some indefinite future date.

The other characteristic is efficiency. Fig. 10 shows the efficiency of a 4-disk bridge-connected rectifier rated at 5 d-c. volts and delivering 0.5 amperes. This illustrates how the efficiency is affected by a change in the operating voltage. In the operating range which would be from 3 to 5 volts, efficiency is quite satisfactory.

K. C. White: It may be interesting to mention that the Pennsylvania Railroad Co. has applied several of the power plants described in this paper to a rather extensive dial telephone system in the Western Pennsylvania and Eastern Ohio area. This system is illustrated schematically in Fig. 11.

There are 9 fully automatic, 7 semi-automatic and 8 manual telephone P.B.X.'s in this system with rather elaborate interconnecting tie lines. Automatic power plants have contributed greatly in making it practicable due to the very much lessened maintenance required resulting in a considerably smaller operating personnel. In addition the better-voltage practically foolproof operation, and longer battery life make for a much higher efficiency than that obtainable in manually-operated plants.

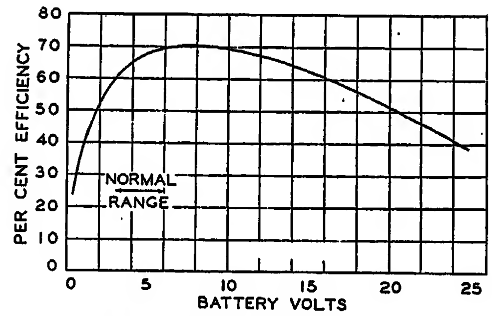

Fig. 10—Efficiency of Four-Disk Rectox Unit at 0.5 Ampere

Some earlier types of automatic power plants of somewhat different characteristics were placed in service in this territory between 1917 and 1927 and formed a basis for the present design which has proven most satisfactory. In fact I can recall no power failure in any of these plants installed since the latter date. This result is particularly gratifying since there is hardly a service where reliability and dependability are more important than that required in the operation of a railroad especially in the central region of the Pennsylvania Railroad Co.

It has been especially pleasing to us to receive numerous favorable comments on the automatic dial telephone P.B.X. system outlined above in which the automatic power plants are contributing their share. These comments have come from superintendents of our operating and traffic departments as well as the rank and file of our yard men.

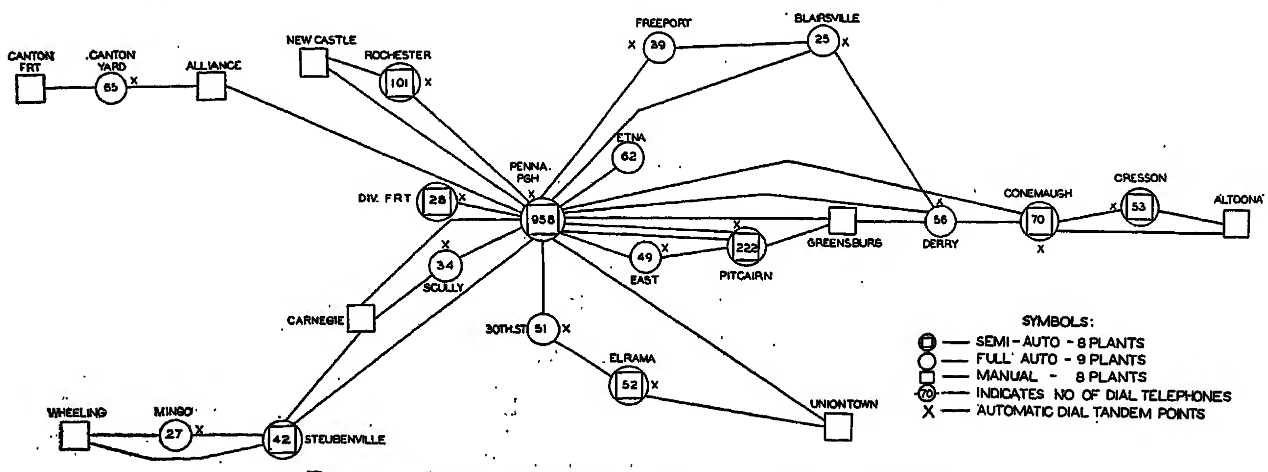

Fig. 11—The Pennsylvania Railroad Central Region

Private branch exchange showing type of power plant and number of dial telephones

G. H. Kendricks: When fire destroyed the office of the Western Union in Pittsburgh in 1885 the Alleghany County Light Company set aside two 500 light d-c. machines for its exclusive use, for it was unable to secure quarters large enough to replace the gravity battery destroyed. It was necessary in order to use this plant to ground the positive or negative polarity of these machines in order to give the proper current to the Company's lines. Up until recently the telegraph service was a single-wire system, being either grounded or "batteried" from the distant end. In order properly to distribute the correct amount of current to the lines, artificial resistance was introduced in the line next to the battery. This was done for various reasons. First, to protect the instruments in case of a short circuit or a nearby ground on the line. At this particular time tho 16-candle power 110-volt carbon filament lamp was used. Gravity battery was still maintained for local and sounder service. In order to supply current to the multiplex circuits it was necessary to build up with gravity battery on top of the 110-volt generators.

When the First National Bank Building was rebuilt the company installed the old Siemens-Halske generators used in New York and in addition two Western Electric 15-volt generators for local sounder service. This, it is understood, was the first telegraph office operated exclusively with generators. At this time there were new ways of adding different types of resistance for battery leads. The one in question in the new office was a plaster of Paris cylinder about 15 in. long by 3 in. in diameter and wrapped with No. 36 German silver wire. This was abandoned on account of the space occupied. Then the resistance lamp of various resistances was introduced, which is used at the present time.

With the introduction of the generators it became necessary to provide some contrivance for dividing current for quadruplex operation which was commonly used at that time. In the gravity battery all that was necessary was to tap the battery at the required voltage, but with generators with only one voltage it was necessary to use a rheostat with German silver wire so as to give a proportion from three to one or four to one as the proper amount of current for the polar side of the quad and for the common side of the quad. Quadruplexes have all been discarded since the introduction of high-voltage power plants. The Western Union operating department is now located on the 15th and 16th floors (approx. 25,000 sq. ft.) of the Chamber of Commerce Building and today practically 85 per cent of its service is worked by printer operation. The power plants today are composed of two 25-volt motor-generators, six 110-volt motor-generators, three 160-volt motor-generators, and three 240-volt motor-generators, occupying a space about 6 ft. by 40 ft. (approx. 2,400 sq. ft.) in the main operating room.

The telegraph company has also introduced in the larger cities, pneumatic tubes for carrying messages from the main office to the branches. This is done in Pittsburgh by seven 5-hp. motor-operated blowers to the branch office, and three 1-hp. motor-operated blowers for tubes from one branch office to another in the near vicinity of the larger branch offices.

C. S. Alt: The company with which I am associated has pioneered in the development of emergency engine sets for telephone and telegraph service assisted in a major way by the engineering staff of both systems.

Although our public service supply of power today is of a high order, emergencies do arise which leave us without power, hence the call for an emergency power unit. Primarily such a power unit must of necessity be easily and quickly started and put into service at full load without delay, therefore our development has been carried on with this idea always foremost.

Accomplished accurate speed regulation and governing must be adhered to especially so in the a-c. power plants. Although it has not as yet been necessary in the communication service, a regulation has been accomplished to meet the needs of parallel operation of two or more units.

Freedom from vibration and its attendant results are also a very important item in the operation and development of such units. Extreme caution in balance of all moving parts is necessary. No power demand is so exacting in its requirements in this respect as the telegraph and telephone service, and the years of development bring to one's mind a number of experiences dealing with period and harmonic vibration in buildings peculiarly susceptible to such problems.

The supply and handling of fuel for such a unit is also a problem and the telephone and telegraph company experiences have proven that gasoline as a fuel is most dependable. Gasoline in an office building in the crowded business section of our large cities might seem like playing with a great hazard but experience has shown that this is not the case as the Laboratories of the National Board of Fire Underwriters are continually working hand in hand with us in this field and today a gasoline emergency power unit can be installed in any building with the full approval of this Board. All plants used by the telephone and telegraph service are approved and listed by the Underwriter Laboratories. Simplification and centralization of engine and fuel controls are also very necessary in order that an experienced operator is not required to start the unit. Condensed starting instructions consisting of as few operations as possible are supplied attached to the control panel permitting starting and application of load in a matter of seconds.

Quietness of starting and operation are also needed. Backfiring of an engine in this service must be guarded against. Exhaust silencing is absolutely necessary. The silencer must be located and be of sufficient strength to silence and hold exhaust-pipe explosions and after firing.

Guarding of all moving parts on the unit is necessary from a safety angle. It is surprising to know the number of people who still insist on wiping off, with a piece of waste or rag, a revolving shaft or gear. The remedy is to enclose all these dangerous items.

R. L. Dunlap: Four years ago the first automatic power plant was put into service in this section at the Pitcairn, Pennsylvania Central Office. Since then 72 of these plants have been installed in the Pittsburgh territory. About one third of the number is being used at central offices in the smaller towns and the rest serve private branch exchanges. Among the branch exchanges are department stores, railroads, newspaper offices, insurance companies, banks, a telegraph office, a jail, a court house, private estates and industrial concerns.

Before the automatic plants were developed department stores were a constant source of worry. Whenever the maintenance man read in the newspaper that a large store in the center of town was about to have a special sale, he connected a few more cable pairs to feed battery to the branch exchange in that store. The sale generally ended some time before he remembered to disconnect the pairs, and consequently the battery at the branch received a most thorough overcharging.

It has been found that the automatic power plant at a private branch exchange provides better voltage regulation and longer battery life with less maintenance effort and a saving in power consumption. Another factor has been that cable pairs formerly used for battery charging can now be used for telephone conversations.

At the same time that automatic charging equipment was being developed improvements were made in storage battery design. The closed model of today with its built in charge indicators and large sediment space differs as greatly from the open type of 5 years ago as the 1931 model automobile differs from cars of 1925-6.

R. L. Lunsford: "Two of the interesting features of the ringing and signaling equipment, namely the mercury switch interrupters and the inductor alternator, furnish the tones and signals

necessary in the supervision of all telephone calls and some are heard every time a call is made.

The mercury interrupter has been developed because of the fact that the low-speed interrupters make and break substantial currents in circuits having inductance and capacity and it has been found impossible to avoid sparking at the brushes with the well-known segment type rings. This has resulted in pitting of the interrupter segments and has required an undue amount of maintenance.

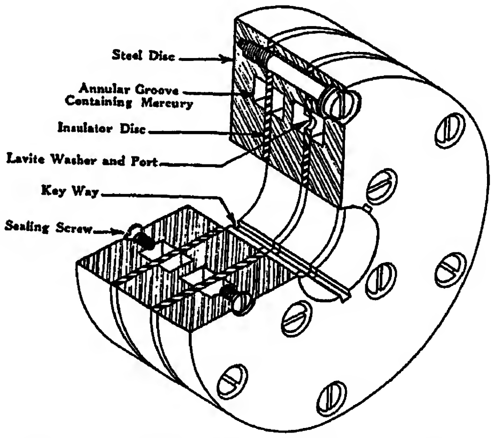

FIG. 12—MERCURY INTERRUPTER UNIT

The interrupter units (see Fig. 12) though old in theory are believed to be new in the form in which they are used. They consist of either two or three steel disks separated by sheet insulators provided with ports or openings and the whole clamped tightly together. An annular channel is cut in the side of each disk and a small amount of mercury is placed therein. The disks and the interrupter units are so arranged that the channel of one disk is adjacent to the channel of the next disk with the insulator separating the two.

When the interrupter units are partially filled with mercury and rotated on a horizontal axis, contact is made from one ring to the other ring by means of the mercury only when a

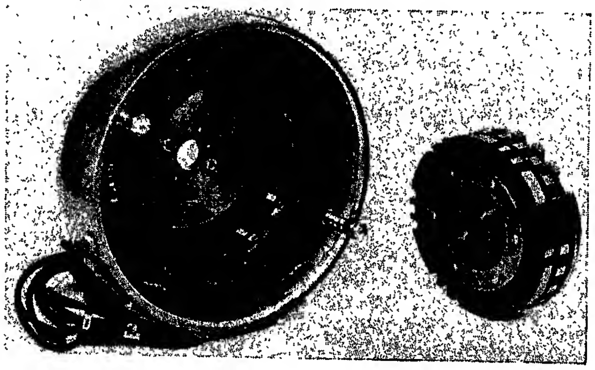

FIG. 13—INDUCTOR ALTERNATOR FOR GENERATING TONES USED IN TELEPHONE POWER PLANTS

port of the insulator dips into the pool of mercury. The ports are provided with Lavite washers to withstand the arcing which occurs when the contact is made or broken. The number of ports, their spacing around the insulators, the amount of mercury in the channels, and the speed of rotation determine the timing of the interrupter.

A rather novel feature of these interrupters is the method used in providing an inert gas within them. Several gases were tried out but it finally developed that the simplest and best arrangement was to assemble and hermetically seal them and place them in operation. The sparking inside does the rest and provides atmospheric nitrogen as soon as the oxygen is consumed.

The brush rigging is similar to that used on existing interrupters. Metallic brushes with Baylis brush holders are mounted so that they slide on the rims of the disks to maintain connections with external circuits. The type of brush used requires very little maintenance and is longer wearing than the softer carbon brushes used on the old segment type interrupter.

The tone alternator is the new device mentioned, the development of which has just been brought to a satisfactory conclusion. This produces the "high tones" and "low tones" formerly obtained by segment type interrupters such as shown in Fig. 12 of the paper. On the older type, brushes were used riding on segmental rings to interrupt battery current, thus furnishing current having about 480 and 153 interruptions

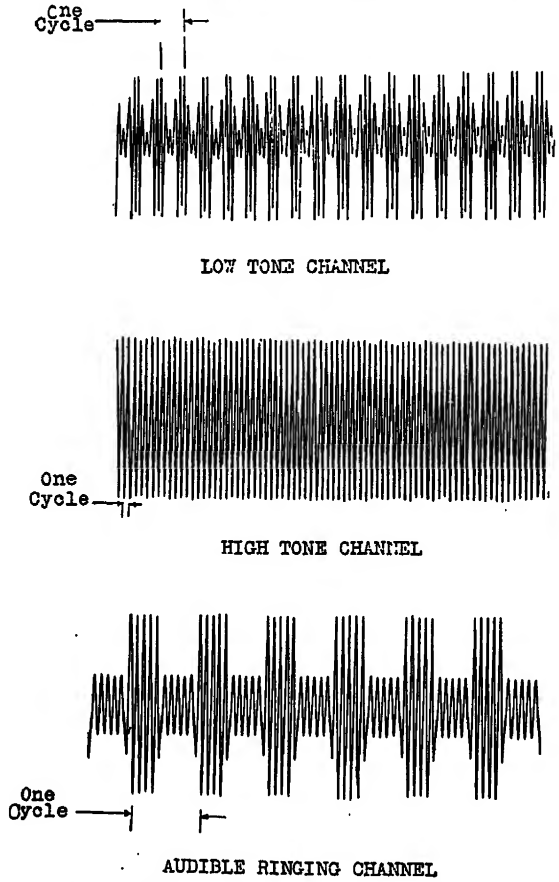

FIG. 14—TONE ALTERNATOR—SHOWING WAVE SHAPES OF THREE CHANNELS

per second. Attempts to reduce maintenance on these rings and improve quality of the tone by introducing spark killers made the tone practically disappear.

A multi-channel inductor-type alternator was, therefore, developed as shown in Fig. 13, which has some features not used to our knowledge in commercial applications. The central disk of the rotor generates a continuous high-frequency alternating current for the high-tone channel. When, however, it was attempted to use a low-frequency tone generated in the same manner, it was found that this tone had insufficient energy to go through the repeating coils and other parts of the circuit. This explained the results from spark killers on the earlier interrupters and indicated that the tone was being transmitted by its harmonics rather than its fundamental. Combination rotors were, therefore, designed for the alternator which delivered a few cycles of high-frequency at a relatively higher voltage

alternated with a few more cycles of the same frequency at a reduced voltage. The rotors for these channels are shown in the figure.

Fig. 14 shows oscillographs of the wave shapes obtained with this machine, the high tone being the continuous channel in the center. The low-tone channel shown in the upper curve has the same satisfactory penetration as the old sparking interrupters with high harmonics but it has the same effect upon the ear as a simple sine wave of lower tone. The bottom curve shows a composite wave delivering a still lower effective frequency provided for audible ringing, which is a tone placed upon the circuit of the subscriber being rung and transmitted back to the calling party to notify him that the connection has been established and ringing is in progress. The manner of mounting both the tone alternator and mercury interrupters with their associated ringing machine is shown in Fig. 13 of the paper.

Although these machines are just now being introduced into the telephone system, several trial installations in operation during the past year or more have shown that the tone is more uniform and more satisfactory than anything formerly provided. This result is also obtained over long periods without machine maintenance from the attendant, whereas the older type machines had to be cleaned several times daily in some cases to give satisfactory performance.

Recent Developments in the Operation of Overseas Radio Telephone Service

By F. A. COWAN[1]

Member, A. I. E. E.

JUST a little more than four years' time has elapsed since the first commercial transatlantic radio telephone service was inaugurated. During this period the scope of overseas service has grown so that today a telephone user in North America may talk with users in Europe, South America, Australia, one point in Africa, and on ships at sea. The establishment of these extensive transoceanic ties has been discussed in a number of papers[2] presented at meetings of the Institute. It is the purpose of this paper to consider some of the more recent developments in the operation of the overseas services.

Those portions of the world which could be reached by telephone from the United States, Canada, Cuba, and Mexico at the end of 1930 are indicated by the shaded areas on the map shown in Fig. 1. This map also shows the radio telephone links which make the overseas connections possible. All of these overseas telephone ties are effected by means of radio operating on short wavelengths in the range between 12 and 100 meters (25,000-3,000 kc.), with the exception of the original circuit between New York and London, which operates at a wavelength of about 5,000 meters (60 kc.). The extensive use of short wavelengths for radio telephone service was occasioned by the particular conditions which were met in the extensions of the overseas services. Such factors as availability of wavelengths, susceptibility to noise in transmission across equatorial regions, available space on ships, character of transmission variations, as well as economic considerations, usually favor the use of short wavelengths. In the case of New York-London circuits, which traverse the North Atlantic region, however, the greater stability of the transmitting medium at long wavelengths together with relatively shorter distances and more favorable locations for the long-wave receiving stations make them somewhat more reliable than short-wave circuits. It has been found, however, that conditions which affect transmission adversely on long-wave circuits rarely occur simultaneously with those factors which affect the short wavelength circuits, so that continuity of service is greatly improved by using the two in conjunction.

Aside from the North Atlantic region, the short wavelength circuits appear to offer the most promising means of extending the overseas service. This is substantiated by the excellent performance of the New York-Buenos Aires, Argentina, circuit which was put in service in the spring of 1930. In view of these considerations, extensions of service now proposed to Bermuda and Hawaii will utilize short wavelength radio.

Since the circuit from New York City to Buenos Aires employs short wavelength radio and is representative of circuits recently established and contemplated, this particular circuit will be considered in some detail. Fig. 2 shows, in schematic form, the layout of the New York-Buenos Aires circuit.

The short-wave transmitting and receiving stations in this country for the South American service, as well as for the European service, are located near Lawrence-

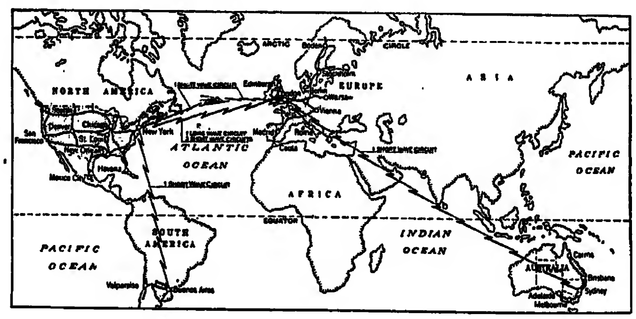

FIG. 1—INTERCONNECTION OF TELEPHONE SYSTEMS OF NORTH AMERICA WITH THOSE OF OTHER CONTINENTS

ville, N. J., and Netcong, N. J., respectively. These transmitting and receiving centers are about 50 miles apart; are about the same distance from New York; and are connected by cable with New York City where the terminals of all of the overseas and ship-to-shore circuits in the Atlantic region are concentrated. In South America, the transmitting and receiving stations are located at Hurlingham and Platanos, Argentina, respectively. Both of these points are within 25 miles of the circuit terminal at Buenos Aires.

The equipment and arrangements at Lawrenceville, Netcong, and New York City have been discussed in considerable detail in the papers previously referred to. The equipment used in South America is practically identical with that used in this country. At both transmitting points piezo-electric crystal controlled short-wave transmitters are used. These transmitters employ high-power water-cooled vacuum tubes with an unmodulated power output of between 15 and 20 kw. which, when modulated 100 per cent, corresponds to a

1. Engr. Trans. Long Lines Dept., American Tel. & Tel. Co., New York, N. Y.

2. K. W. Waterson and O. B. Blackwell, A. I. E. E. JOURNAL, April, and May 1928, respectively.

T. G. Miller, A. A. Oswald and Ralph Bown, A. I. E. E. JOURNAL, February, April, and May 1930, respectively.

Lloyd Espenschied and W. Wilson, A. I. E. E. JOURNAL, July 1930.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

peak power output of between 60 and 80 kw. Directive antenna arrays are provided for three separate wavelengths to care for the changing transmission conditions with seasons of the year and hours of the day. The directive characteristics of these arrays increase the power radiated in the desired direction by an amount equivalent to that which would be obtained by increasing the transmitter power from 20 to 50 fold, depending upon the wavelength being used.

At the receiving points, directive antenna arrays are also employed. These arrays, by virtue of their directive characteristics, improve the receiving conditions by strengthening the signal received from the desired direction and reducing the effect of noise or signals coming from other directions. The receiving sets proper are of the double detection type and are equipped with automatic gain controls to compensate for moment-to-moment variations in the strength of the received carrier wave. The complete receiving system is capable of utilizing satisfactorily a signal having a field strength as low as one microvolt per meter when radio noise conditions are satisfactory. This performance has been made possible by the selection of a receiving site which

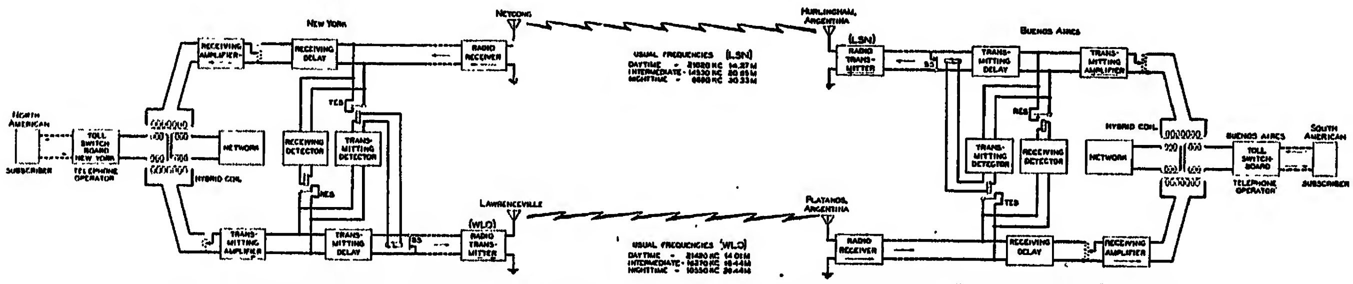

Fig. 2—Schematic Layout of New York-Buenos Aires Radio Telephone Circuit

is sufficiently isolated to minimize local interference and by the careful design of the component parts of the receiving system.

At the terminal points voice-operated relay switching equipment is provided which disables the radio path in one direction while speech is traveling in the opposite direction. It also maintains the transmitting side of the circuit in a disabled condition when no speech is being transmitted. This automatic switching simplifies the transmission problem by resolving the two-way circuit into two one-way circuits. The circuits which actuate the switching relays are carefully adjusted in order to compensate for changing transmission conditions. This minimizes interruptions due to false operation of the relays caused by abnormal conditions on either the radio channel or the land line.

Once circuits of the type under consideration have been established, their performance is controlled largely by the condition of the transmitting medium at any particular time. Since the control of the medium is beyond the scope of human endeavor, the operating problem becomes one of maintaining the component parts of the system in good condition and operating the system as a whole to best advantage.

One index of the effectiveness with which radio telephone circuits are operated is the amount of circuit time which is lost. For this reason, this phase of the circuit performance is carefully scrutinized. Fig. 3 gives a comparative analysis of the lost circuit time for one month on the New York-Buenos Aires circuit and one of the New York-London short-wave circuits. It will be noted in each case that the major percentage of lost time is due to atmospheric conditions.

The equipment used in the short-wave radio circuits is so designed and constructed as to minimize the likelihood of trouble. In addition, spare units of equipment have been provided for replacing equipment which would be most subject to trouble or which would require considerable time to repair. These precautions, together with a systematic testing routine, make it possible to operate the overseas circuits with the small amount of lost time, due to equipment troubles, that is indicated by Fig. 3.

The over-all operation of the radio circuits is carried out under the supervision of technical operators located at the circuit terminals. These operators coordinate the activities at their local transmitting and receiving stations with those at the distant terminal. In this country the technical operators receive advice as to what wavelengths are most effective at any time, from a channel observing station which is equipped with special apparatus for maintaining a continuous check on radio transmission conditions. This check on transmission conditions is made by measuring the field strength of foreign stations operating on various wavelengths.

Very accurate measurements of the frequency of the local radio telephone transmitters and of associated foreign transmitters are also made, periodically, by the channel observing station. In case there is any appreciable deviation from the assigned frequency an investigation is made immediately, the cause determined, and the deviation is corrected.

Fig. 4 shows the results of a series of frequency measurements made during August, 1930, on transmitter WLO, which is used in the South American service. It will be noted from the chart that the frequency deviation was in all cases less than $\pm$ 0.01 per cent whereas the limit set by the Federal Radio Commission allows a deviation of ± 0.05 per cent of the carrier frequency. In cases of interference to the telephone channels from

radio stations assigned to neighboring wavelengths, the frequency measurements aid in the correction of the difficulty.

The foregoing describes the steps that are taken to secure the most effective operation of the radio telephone circuits in so far as controllable factors are concerned. As previously stated, the variation of the transmitting medium is at present the major factor limiting the usefulness of short-wave radio channels. These variations in the transmitting medium result in

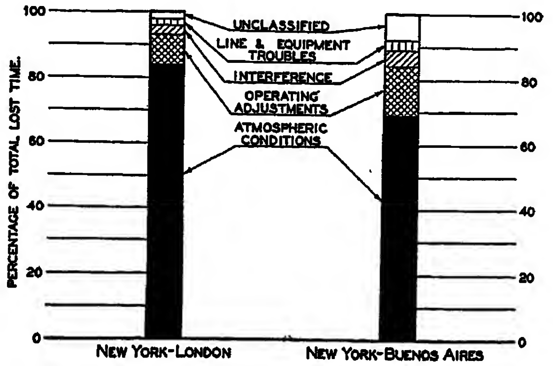

Fig. 3—Analysis of Lost Circuit Time on Short Wave-Length Radio Telephone Circuits for August 1930

fluctuations of the momentary values of the signal strength as well as in a variation of the average signal strength throughout the day. The former effect manifests itself either as ordinary or as selective fading, depending upon whether or not all frequency components of the signals are affected simultaneously to the same degree. Ordinary fading is compensated for by means of the automatic gain control incorporated in the receiver and the only effect which is produced when this type of fading occurs is an apparent variation of

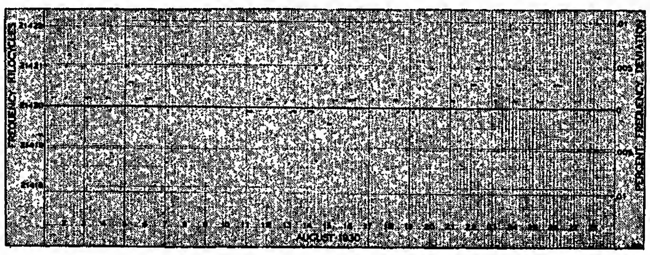

Fig. 4—Frequency Measurements of Transmitter WLO for August 1930 Assigned Frequency 21,420 Kc.

the noise level. Selective fading, however, which usually accompanies ordinary fading to varying degrees, gives rise to distortion effects and may be sufficient at times to render a channel unsuitable for commercial transmission on a given wavelength.

Periods of abnormal activity in the earth's magnetic field, known as "magnetic storms," occur at irregular periods and are accompanied by decreases in the signal strengths at the receiving points to unusable values at times. When these storms occur, transmission on all frequencies is affected to a greater or lesser degree and the time during each 24-hour period when reliable transmission can be obtained is reduced materially.

Fig. 5 shows the average effect of the major magnetic disturbances which occurred during 1930, on the available circuit time of a representative New York-London short-wave radio telephone channel. It should not be construed from this curve that magnetic storms have occurred regularly every two weeks, but rather that when they have occurred the available circuit time, for several days preceding and following the day on which the storm was at its height, has usually been affected as shown.

In view of the adaptability of short-wave telephone systems to transmission over great distances and the fact that rapid progress is being made in improving, simplifying, and reducing the cost of these systems, they are of outstanding importance in the development of world-wide telephonic communication. The brief

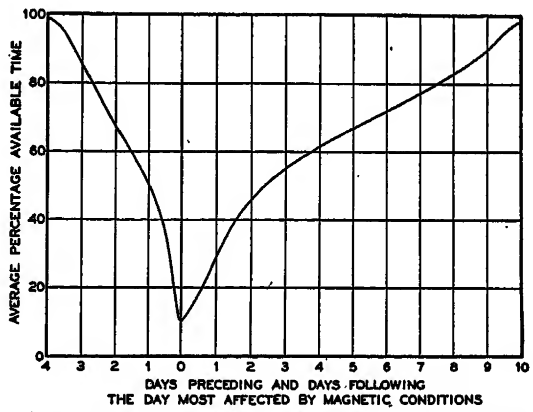

Fig. 5—Average Effect of Major Magnetic Disturbances During 1930 on the Available Circuit Time of a Representative New York-London Short Wave Radio Telephone Channel

period which has elapsed since establishing the first short-wave radio telephone channel has been one of rapid progress in the development and application of short-wave systems and they are, at the present time, generally accepted as a medium for extending the scope of overseas telephone service.

Short-wave radio telephone circuits have been used for the transmission of international broadcast programs, for conducting joint meetings of international conventions, and for telephone interviews. Numerous accounts of examples of the latter type of usage have been published from time to time by American newspapers. One of the most notable examples, perhaps, was recorded by *The New York Times* when a personal interview with General Uriburu was obtained after the recent revolution in Argentina. Services of these types undoubtedly promote mutual understanding between the people of various nations and are helpful in the cementing of international good will.

Discussion

J. P. Scott: This paper presents an analysis of the lost circuit time by causes (Fig. 3) and indicates the number of days that a typical magnetic storm is troublesome (Fig. 5). What is the total per cent lost time due to all causes? How frequently are magnetic storms usually experienced?

F. A. Cowan: As pointed out in the paper the chief cause of lost circuit time is the condition of the transmitting medium. The medium varies considerably from day to day, season to season, and from year to year. The average percentage lost time for one period of several months that I have in mind was about 10 per cent for the New York-London long-wave circuit, about 25 per cent for the short-wave New York-London circuits, and about 5 per cent for the New York-Buenos Aires short-wave circuit. The higher percentage lost time on the New York-London short-wave circuits than on the New York-Buenos Aires circuit is accounted for by the proximity of the transmission path to the north magnetic pole of the earth in the case of transmission between New York and London. In view of the fact that the factors which affect long- and short-wave transmission rarely occur simultaneously, the over-all reliability of the service is materially enhanced by using the two types of facilities in conjunction.

Magnetic storms do not occur with any definite regularity. The number varies from year to year. During 1930 there were about 12 major magnetic storms.

W. C. Hecht: How are the practical operating hours for the several radio circuits affected by differences in time of day at the terminals?

F. A. Cowan: The load distribution curves for the London and Buenos Aires service are not materially different from the load curve for New York-Chicago service.

N. S. Hoff: What steps are involved and how much time is required from the conception of a radio circuit to the establishment of service?

F. A. Cowan: Definite arrangements have to be made with a foreign correspondent, frequency assignments must be obtained, buildings must be designed and constructed, unless already available, and equipment and antenna systems must be engineered, constructed, installed, and tested before commercial service can be established. This will require anywhere from one to two years, depending upon the particular problems involved.

W. L. Shafer: The paper mentions the large gains that have been obtained by the use of directive antenna arrays at the transmitter and receiver. Are there possibilities of further substantial gains by such means? What limits the gain obtainable in practise?

F. A. Cowan: The gain of the antennas is obtained by arranging the elements in such a manner as to direct the energy in the desired direction. The method is analogous to the use of a reflector behind a light. In view of the fact that the transmitting medium is not homogeneous the transmitted waves do not always proceed to the distant point by the shortest straight line paths, but are sometimes reflected from their course. Consequently, if the angle of radiation of the transmitting antenna is made too small the waves might miss the receiving antenna at the distant station. The gain which it is practicable to obtain by the use of directive antenna is limited by this as well as economic considerations.

L. R. Huggler: What over-all length of circuit route is involved when radio circuits are used in tandem with wire extensions at the radio terminals?

F. A. Cowan: One of the longest calls, from the standpoint of distance, was a call from Los Angeles, California to Melbourne, Australia. This call involved a distance of 18,000 or 22,000 miles, depending upon the direction of transmission between London and Sydney. There have been other calls involving only one radio link between the West Coast of this country and the West Coast of South America, which is roughly 10,000 miles. One of the longest calls for the ship-to-shore service was between San Francisco and the *S. S. Belgenland* when the ship was in the vicinity of Japan. This call was established through the Ocean Gate, N. J. ship-to-shore transmitting station and hence the circuit length was about 10,000 miles.

R. T. Griffith: In the answer to the previous question Mr. Cowan states that the transmission between London and Sydney might be in either direction over the great circle route. How is it determined which direction the transmission is in? What arrangements, if any, are made to use the longer great circle route?

F. A. Cowan: The antennas are ordinarily set up to transmit over the shorter great circle route. If it is desired to transmit over the longer great circle route, it is necessary to use a different antenna arrangement.

Temperatures in Electric Power Cables
Under Variable Loading

BY ELWOOD A. CHURCH*
Associate, A. I. E. E.

Synopsis.—Considerable need has been felt for accurate methods of solution of the problem of temperature rise under variable loading in order that the maximum use may be made of the large investment in power cables.

In this paper a rigorous solution of the problem of temperature rise from sheath surface to conductor is attempted, making use of Bessel functions. The heat flow cycle is resolved into harmonics, and each harmonic solved separately for temperature at the conductor. The various harmonics of temperature are then combined in their proper phase relation to obtain the temperature cycle. For purposes of assigning emergency ratings a solution is arrived at for suddenly applied steady loads, making use of the Fourier integral.

The problem is solved rigorously for single-conductor cables and three-conductor cables of shielded (type "H") construction. Modifications of the constants of the cables are described which will allow the theory to be applied to cables of standard belted construction with reasonable accuracy.

The probable errors involved in the assumptions necessary in the solution are discussed. It is believed that knowing the temperature of the air at the sheath surface, the temperature of the conductor can be calculated within 4 or 5 per cent of the correct value if the constants of the cable are known within this accuracy.

The method can also be applied to solution of the temperature rise of the sheath surface provided the constants of the duct bank are known with sufficient accuracy. This temperature can be added to the temperature rise of the conductor above the sheath temperature to obtain the total temperature rise of the conductor above the assumed base temperature.

INTRODUCTION

FAIRLY accurate methods are now available for calculating the temperature rise of the conductor above the assumed base temperature when the load is steady. These give results which are as accurate as the known constants of the cable and duct bank.

Until recently no attempt was made to obtain even a moderately rigorous solution for the case of variable loading.

A recent Institute paper[1] describes a point-by-point method of solution for variable loading which gives fairly accurate results. This point-by-point solution is based on the assumption that the temperature of the conductor rises exponentially with time which is only approximately true.

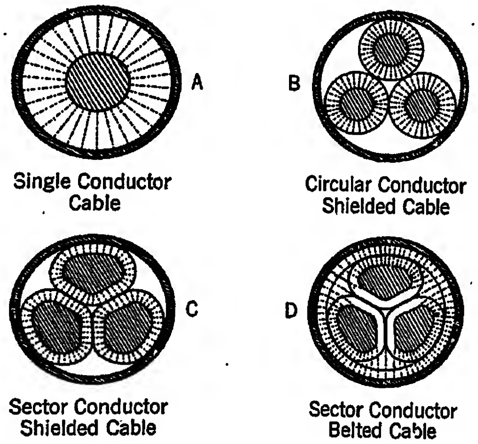

FIG. 1—CROSS-SECTIONS OF CABLES SHOWING DIRECTION OF HEAT FLOW

The theory presented in this paper tends to show that the temperature curve rises faster at first and then slower, say one hour after the load is applied, than the exponential law would allow. Solutions based on the exponential law are thus likely to give lower maximum temperatures and greater time lags than actual for ordinary load cycles.

The present paper solves the problem rigorously for cables with configuration shown in Figs. 1A and 1B. The errors involved in applying it to the configuration shown in Figs. 1C and 1D are discussed in a subsequent paragraph.

SOLUTION FOR DAILY LOAD CYCLE

The general expression for heat flow in the conductor of either a single-conductor or three-conductor cable, assuming constant resistance, circular cross-section of conductor, and uniform current distribution, as derived in Appendix A is as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{S_c I^2 R}{\pi r_1^2} = C_c \rho_c S_c \frac{\partial T}{\partial t}. \quad (1)$$

The general expression for heat flow in the insulation is the well-known Fourier's equation, and is as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = C_i \rho_i S_i \frac{\partial T}{\partial t}. \quad (2)$$

where

T = Temperature at any point in the conductor or insulation.
I = Total current in the conductor.
R = Resistance of the conductor, at the frequency of the current, if alternating current, and the average temperature of the conductor.
r = Radial distance measured from the center of the conductor.
r_1 = Radius of the conductor.
C_c = Specific heat of the conductor material.

*Electrical Engg. Dept., Edison Electric Illuminating Co. of Boston.

1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

C_i = Specific heat of the insulation material.
ρ_c = Specific gravity of the conductor material.
ρ_i = Specific gravity of the insulation material.
S_c = Heat resistivity of the conductor material.
S_i = Heat resistivity of the insulation material.
t = Time.

When T varies sinusoidally with time, the solution of equation (1) is:

$$\bar{Q}_1 = -\frac{2\pi q_c r_1 \gamma}{S_c}\left(\bar{T}_1 + j\frac{\bar{Q}_0 S_c}{\pi q_c^2 r_1^2}\right). \quad (3)$$

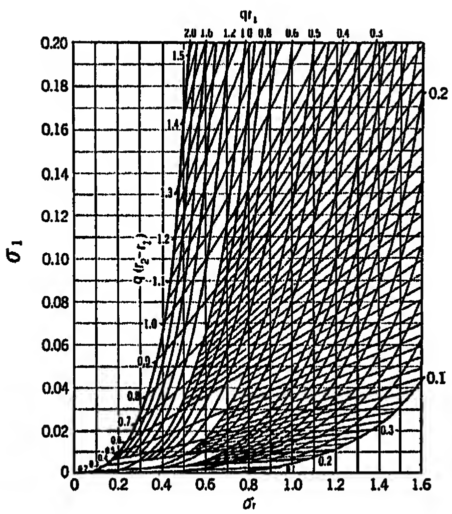

FIG. 2—CHART OF THE HEAT-FLOW CONSTANT: $\sigma = i_2 k_1 - i_1 k_2$

$i_1 = I_0 \sqrt{j}\, q\, r_1$, $i_2 = I_0 \sqrt{j}\, q\, r_2$.
$k_1 = K_0 \sqrt{j}\, q\, r_1$, $k_2 = K_0 \sqrt{j}\, q\, r_2$.

or when $q_c r_1$ is small, as in cases met with in solution of power cables:

$$\bar{Q}_1 = \bar{Q}_0 - j\frac{\pi q_c^2 r_1^2 \bar{T}_1}{S_c}. \quad (4)$$

The solution of equation (2) when T varies sinusoidally with time is:

$$\bar{Q}_1 = \frac{2\pi}{S_i \sigma}(q_i r_1 \tau \bar{T}_1 - \bar{T}_2), \quad (5)$$

$$\bar{Q}_2 = \frac{2\pi}{S_i \sigma}(\bar{T}_1 - q_i r_2 \psi \bar{T}_2). \quad (6)$$

Neglecting the heat storage in the lead sheath, the expression for heat flow in the sheath is for a single-conductor cable:

$$\bar{Q}_2 = \frac{\pi D}{K}(\bar{T}_2 - \bar{T}_3). \quad (7)$$

and for the three-conductor cable:

$$\bar{Q}_2 = \frac{\pi D}{3K}(\bar{T}_2 - \bar{T}_3). \quad (7a)$$

where
$\bar{T}_1$ = Vector temperature at surface of conductor.
$\bar{T}_2$ = Vector temperature at surface of insulation.
$\bar{T}^3$ = Vector temperature of the air at the surface of the sheath.
$\bar{Q}_0$ = Vector heat flow per conductor, equal to the $I^2 R$ loss.
$\bar{Q}_1$ = Vector heat flow per conductor at inside surface of the insulation.
$\bar{Q}_2$ = Vector heat flow per conductor, at outside surface of the insulation, or at outside surface of the sheath if sheath losses can be neglected.
r_2 = Outside radius of the insulation.
$q_c = \sqrt{\omega C_c \rho_c S_c}$.
$q_i = \sqrt{\omega C_i \rho_i S_i}$.
D = Diameter of the sheath.
K = Surface heat resistivity of the sheath.

γ, σ, τ, and ψ are complex constants, depending on the constants and the configuration of the cable. They are analogous to the general circuit constants in the solution of a transmission line for voltage and current at the terminals and may be designated the heat flow constants of the cable. Their derivation is given fully in Appendix A. For ease in computation, charts of σ, τ, and ψ have been prepared and are illustrated in Figs. 2 to 4. Curves of the Bessel functions from which these charts were computed are shown in Fig. 5. Tables of these functions are given in Bibliography Nos. 2 and 15.

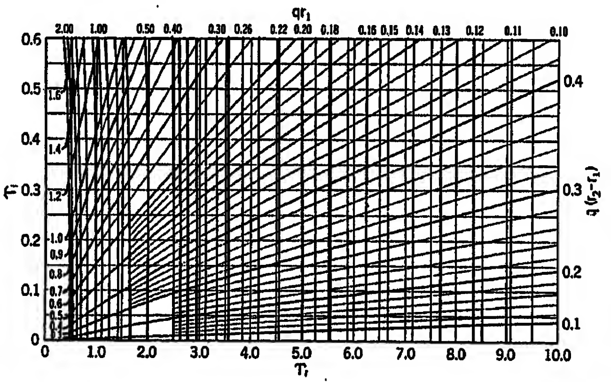

FIG. 3—CHART OF THE HEAT-FLOW CONSTANT: $\tau = i_1' k_2 - i_2 k_1'$

$i_1' = \sqrt{j}\, I_0' \sqrt{j}\, q\, r_1$, $k_2 = K_0 \sqrt{j}\, q\, r_2$.
$k_1' = \sqrt{j}\, K_0' \sqrt{j}\, q\, r_1$, $i_2 = I_0 \sqrt{j}\, q\, r_2$.

Combining equations (4) to (7), the solution for a single-conductor cable is given by the following simultaneous equations:

$$\frac{\bar{Q}_0}{\pi} - j\frac{q_c^2 r_1^2 \bar{T}_1}{S_c} = \frac{2}{S_i \sigma}(q_i r_1 \tau \bar{T}_1 - \bar{T}_2) \quad (8)$$

$$\frac{D}{K}(\bar{T}_2 - \bar{T}_3) = \frac{2}{S_i \sigma}(\bar{T}_1 - q_i r_2 \psi \bar{T}_2) \quad (9)$$

In the solution for a three-conductor cable, equation

(8) remains the same and equation (9) is written as follows:

$$\frac{D}{3\,K}\,(\bar{T}_2 - \bar{T}_3) = \frac{2}{S_i\ \sigma}\,(\bar{T}_1 - q_i\,r_2\,\psi\,\bar{T}_2)\,. \qquad \text{(9a)}$$

Where sheath losses are present in appreciable amounts a correction can be made to the values of $\bar{Q}_2$ in equations (7) or (7a), when an accurate solution is desired.

A sample calculation, showing in detail the method employed in the solution of a specific problem is given in Appendix D. Fig. 6 shows the results of this calculation in the form of a curve of temperature against time. Briefly, the method is as follows:

The watts loss in the cable is plotted for the load cycle assumed. This curve is analyzed for its principal harmonics by well-known methods. The solution for temperature is obtained for each harmonic of heat flow and the results combined in their proper phase relation to give the resultant temperature cycle. Usually three or four harmonics will be all that is necessary to

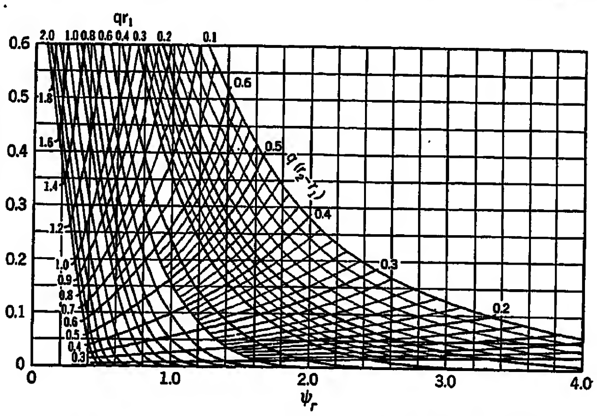

FIG. 4—CHART OF THE HEAT-FLOW CONSTANT: $\psi = i_2'\,k_1 - i_1\,k_2'$

$i_2' = \sqrt{j}\,I_0'\,\sqrt{j}\,q\,r_2$, $k_1 = K_0\,\sqrt{j}\,q\,r_1$,
$k_2' = \sqrt{j}\,K_0'\,\sqrt{j}\,q\,r_2$, $i_1 = I_0\,\sqrt{j}\,q\,r_1$.

solve for unless the rise of watts loss is very rapid, since the effects of the small hourly variations in loading on the cable have very little effect on the resultant temperature cycle. A proper balance should be obtained between the labor of solution and the accuracy with which it is desired to obtain it.

The value of $\bar{T}_3$, the temperature of the air at the outside of the cable, can be obtained in various ways. If the cable is installed in a duct and only approximate solutions are required, this can be assumed constant at the maximum value obtained over a twenty-four hour load cycle. Usually $\bar{T}_3$ will vary from 20 to 30 per cent of the total variation in temperature at the conductor, when the cable is installed in an ordinary duct bank.

If $\bar{T}_3$ is considered constant its value becomes zero in equations (9) and (9a), merely adding to the steady state component of the total temperature at the conductor which is computed by well-known methods. The value of $\bar{T}_1/\bar{Q}_0$ will then be constant in both phase and magnitude for any given cable. Tables of $\bar{T}_1/\bar{Q}_0$ for the principal harmonics may hence be made up for any given cable, as shown in Table I in Appendix D. These may be combined in their proper phase relation to obtain a solution for any load cycle.

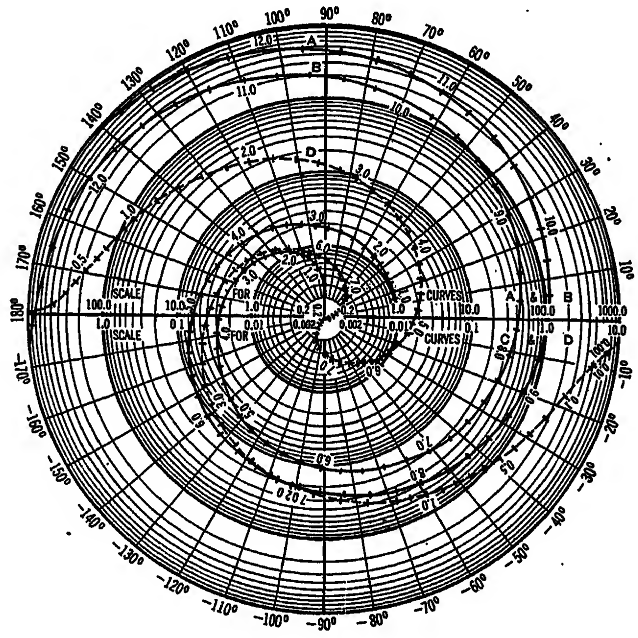

FIG. 5—CURVES OF BESSEL FUNCTIONS OF ZERO ORDER

Curve A—Bessel function of first kind, $I_0\ \sqrt{j}\ x$
Curve B—First derivative of $I_0\ \sqrt{j}\ x$ with respect to x, $\sqrt{j}\ I_0'\ \sqrt{j}\ x$
Curve C—Bessel function of second kind, $K_0\ \sqrt{j}\ x$
Curve D—First derivative of $K_0\ \sqrt{j}\ x$ with respect to x, $\sqrt{j}\ K_0'\ \sqrt{j}\ x$
Examples—$I_0\ \sqrt{j}4.2 = 3.86\ \underline{/146.4^\circ}$; $K_0\ \sqrt{j}\ 3.4 = 0.060\ \underline{/-159.0^\circ}$

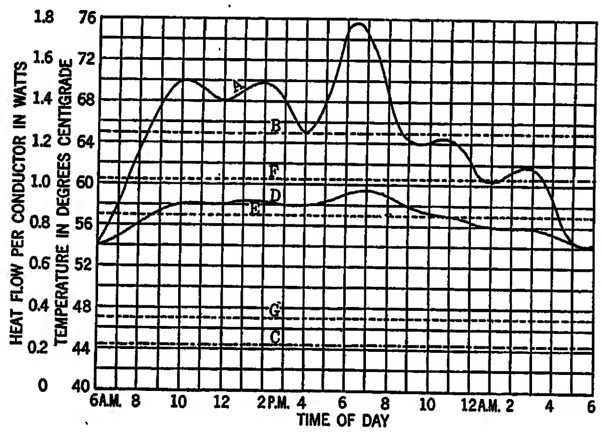

FIG. 6—HEAT FLOW AND TEMPERATURE CURVES FOR THREE-CONDUCTOR, 400,000-CM., 28-KV. CABLE

Curve A—Heat-flow cycle in watts per conductor; $I^2\,R$ loss
Curve B—Average $I^2\,R$ loss per conductor over 24-hour period
Curve C—Dielectric loss per conductor for the average temperature
Curve D—Temperature cycle for load cycle of curve A; temperature at surface of conductors
Curve E—Average temperature at conductor surface
Curve F—Temperature for constant load equal to the maximum
Curve G—Measured maximum temperature of air at sheath surface

If some judgment is used in selection of constants of the duct bank equation (5) may be used for the solution of the air temperature in the duct where $\bar{T}_1$ in the equation represents this temperature and the base temperature is considered zero.

The air temperature then can be expressed as follows:

$$\bar{T}_3 = \frac{\bar{Q}_2 \, S_e \, \sigma_e}{\pi \, q_e \, D' \, \tau_e}, \qquad (10)$$

Where

S_e = heat resistivity of duct structure.

$q_e = \sqrt{\omega \, C_e \, \rho_e \, S_e}$

ρ_e = specific gravity of the duct structure.

C_e = specific heat of the duct structure.

D' = inside diameter of duct structure, assuming an equivalent cylindrical structure.

σ_e and τ_e are computed in the same way as σ and τ in equation (5).

Various approximate formulas are given in Appendix C for obtaining the functions σ_e and τ_e which are outside of the range of the charts given in Figs. 2 and 3.

On account of the effect of adjacent cables and the uncertainty of the constants of the duct bank, the best way of obtaining the air temperature at the sheath surface is to actually measure it. Opinions differ as to the proper method of measuring this temperature in the duct outside the lead sheath. It is generally conceded that this temperature is nearly the same throughout the space between the sheath and the duct and can be considered so in the solution for steady heat flow. Since the air has a very low thermal capacity its temperature will follow very closely in phase the variations in the lead sheath. Hence if the maximum value of this air temperature, throughout the load cycle considered, can be obtained it can be added directly to the maximum temperature rise between this air and the conductor to obtain the total rise. The error caused by these temperature components being out of phase will be small since the total variation in the air temperature is only about one quarter of the total.

Maximum reading thermometers inserted between the cable and the duct wall seem to be the best method provided some means are used to prevent the thermometer from being in direct contact with either the sheath or the duct wall.

A thin shield of heat-insulating material surrounding the thermometer seems to be the best method of accomplishing this result. The drop in temperature through the shield will be negligible compared to the error obtained if the thermometer is in direct contact with the lead sheath or is enclosed in a metal shield which is in direct contact with the sheath.

Measurements taken in occupied ducts indicate that eight to ten feet in the duct is sufficiently far from the manhole to obtain an accurate reading. In ducts where the temperature was between 50 and 60 deg. cent., the difference in temperature between three and six feet was only three degrees, and from six to ten feet, only about one degree. The air in occupied ducts does not have the freedom of circulation it does in unoccupied ducts so the error due to manhole temperatures is minimized.

If the cable is suspended in air the greatest temperature which the air attains should be used with proper allowance for air movement if there is such. A very slight air movement has a very great effect in lowering the sheath temperature. This case is subject to even greater uncertainty than the case of cables installed in the earth so it is exceedingly difficult to obtain an accurate solution. The best method of attack is to assume a constant maximum for T_3 and compute T_1 from this base temperature.

Solution for a Steady Load Suddenly Applied

The solution for a steady load suddenly applied is as follows:

$$T_0 = \frac{2 \, Q_0}{\pi} \int_0^\infty \delta_r \, \frac{1}{\omega} \sin \omega t \, d\omega. \qquad (11)$$

where

T_0 = The temperature of the conductor at any time t after the steady load is applied.

δ_r = The real part of the expression,

$$\delta = -j \left\{ \frac{\gamma}{\pi \, q_e \, r_1} \right\} \times \left\{ \frac{1}{\dfrac{q_i \, r_1 \, \tau}{S_i \, \sigma} - \dfrac{\eta}{S_i \, \sigma \, (S_i \, \sigma \, D + \eta \, q_i \, r_2 \, \psi)} - \dfrac{q_e \, r_1 \, \gamma}{S_e}} \right\}$$

where

$\eta = 2\,K$ for a single-conductor cable in still air,

$= 6\,K$ for a three-conductor cable in still air,

$= 2\,(K + D\,\lambda)$ for single-conductor cables in ducts, and

$= 6\,(K + D\,\lambda)$ for three-conductor cables in ducts.

where

$$\lambda = \frac{N \, S_e \, \sigma_e}{q_e \, D' \, \tau_e}.$$

N = Number of cables in the duct bank all assumed to be equal and loaded to the same amount.

The expression $\frac{1}{\omega} \delta_r \sin \omega t \; d\omega$ is much too complicated to integrate by formal means, but it may readily be done by graphical methods if some means such as a planimeter is available for obtaining the area under a curve. The value of the function is plotted for different values of t and integrated with respect to ω. Integrating graphically from 0 to ∞ offers no difficulty here since the function converges rapidly as the value of ω is increased. Usually integrating from 0 to 15 will give sufficiently accurate results with t less than three hours, and from 0 to 8 with t greater than three hours. It will seldom be necessary to go beyond six hours in computation of the temperature rise curve unless overload ratings for long periods are desired.

The above method is very powerful in the solution of all transient problems where the impedance function, δ, is very complicated as it is in this case. Where δ

is a simple function the method offers no advantage over other better known methods. The philosophy of the method is described very fully elsewhere[13,14].

Results of a calculation of the temperature rise of the conductor of a 400 m. c. m. shielded cable are shown in Fig. 7. The calculation is carried through in brief form in Appendix D.

A chart showing allowable overloads for various time limits and initial loads, computed from these curves, is shown in Fig. 8. As an example; suppose four cables each rated at 100 amperes continuous, are carrying a continuous load of 80 amperes each, a load of 134 amperes each may be applied for two hours without exceeding the temperature limit, with cables of the same characteristics as shown in Fig. 8.

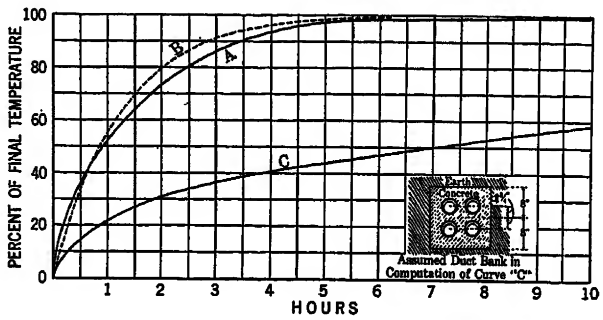

Fig. 7—Temperature Rise Curves; 400,000-Cm. 28-Kv., Shielded Cable

Curve A—Temperature rise; cable suspended in still air
Curve B—Same conditions as in "A", exponential rise assumed
Curve C—Temperature rise; four cables in ducts as shown, with all cables carrying the same load

Application to Standard Belted Cable

The well-known equation for obtaining the steady-state temperature rise of the conductor above the outside surface of the insulation is as follows[8,11]:

$$(T_1 - T_2) = \frac{Q_0 S_i G}{2\pi}. \tag{12}$$

where G is the geometric factor.

For the shielded cable,[12]

$$G = l\,n\,\frac{r_2}{r_1}.$$

Assume that r_2 is the total distance from the center of the conductor to the lead sheath. Obtain G from Simons curves for the cable under consideration[11] and solve for an equivalent S_i. In average cables this equivalent S_i is about double the true value for the insulation under consideration. Use this value of S_i; in equations (8), and (9a).

This approximate solution has been found to give results, within 5 or 10 per cent of the true value as far as the variation in the temperature is concerned. The average temperature corresponding to the zero frequency component of the heat cycle is computed as usual by use of equation (12). Hence for ordinary load cycles the net error of this approximation cannot be over 5 per cent. A rigorous solution for the standard cable would involve much more labor which is useless until the constants of the cable can be obtained with greater accuracy.

Probable Magnitude of Errors

The change in resistance and dielectric losses with change of temperature for the ordinary load cycles met with in practise will not be over 5 or 6 per cent. If it is desired to take this into account a correction may be applied to the watts loss curve corresponding to the estimated temperature at that time.

The value of temperature rise to be assumed in making this correction can be taken as half the ultimate temperature rise produced by the average watts loss over an hourly period. This is based on the fact that in average cables the temperature at the conductor reaches about one-half its ultimate in an hour after a steady load is applied.[1] The error introduced by change in resistance will be less than half of one per cent if this correction is made.

For sector conductors the heat flow will be more concentrated around the regions of smaller radius. This will produce a greater temperature rise in these regions than elsewhere on the conductor. This effect is usually neglected in steady-state solutions, although an approximate solution for sector cables is found in the Bibliography No. 17. The error in neglecting the effect of the sector shape will not be over 1 or 2 per cent.

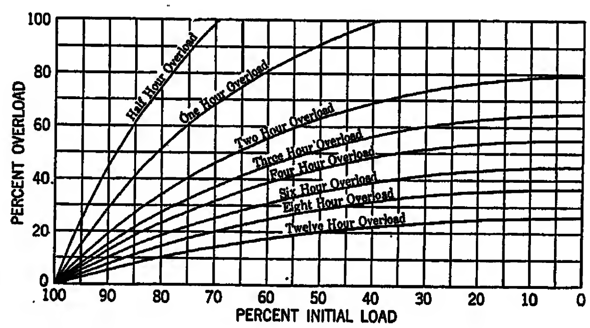

Fig. 8—Overloads, Based on 100 Per Cent Initial Load, which May be Applied without Exceeding the Temperature Rise for 100 Per Cent Initial Continuous Load

Four three-conductor 400,000-cm. 28-kv. shielded cables installed in a four-duct bank; all assumed to be loaded to the same value

Large conductors show considerable proximity effect when three are concentrated in the small area inside the cable sheath. The current tends to crowd to the inside of the cable when three-phase current is flowing. However, under the assumed condition of uniform current distribution the total variation in temperature through the conductor is only one degree for ordinary load cycles. The smallness of this variation is due to the high heat conductivity of the copper. This can be shown by the solution of equation (14) for the temperature at the center of the conductor. Hence if the proper

equivalent a-c. resistance is used for the cable in question, the error in temperature due to unsymmetrical current distribution will not be over 1 per cent. Of course, this assumes all the losses, excluding dielectric losses, are in the conductor. There are undoubtedly some losses in the sheath even in three-conductor cables, especially where large conductors are close to the sheath as they are in shielded cables. Not knowing definitely where all these losses are located, the conservative method would be to assume them all in the conductor.

Neglecting the temperature drop in the copper foil around the conductors in shielded cables introduces about 8 to 9 per cent error in the steady-state solution. A similar error will be introduced in the solution for variable load. This error will be minimized in the total temperature rise if the steady-state component of the temperature cycle is computed taking this drop in the foil into account[12]. This error is partly compensated for by neglecting the heat storage in the lead sheath.

The over-all error in the method is thus not over 5 per cent and is probably much less if the watts loss curve is corrected for temperature. It is doubtful if the value of the insulation heat conductivity is known within 5 per cent, plus or minus, so it is useless to expend the labor necessary in obtaining greater refinement.

Conclusions

Computations based on the theory presented here demonstrate that the temperature curve for any cable and load cycle may be readily calculated without excessive labor with accuracies well within the accuracy of the cable constants. The effects of changes in load cycle from day to day die out in a few hours so the theory can be applied to any day of the week with little effect from the previous day's cycle.

Assumptions are reduced to a minimum, the order of magnitude and direction of the errors are known and hence can be allowed for with any necessary accuracy.

Short-time overload ratings of any duration may be readily applied to correspond to any initial loading with confidence that the calculated temperature rise will not be exceeded.

Acknowledgments

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Appendix A

I—Derivation of Expressions for Heat Flow in the Conductor

The amount of heat entering an annular ring of thickness $d\,r$, and distance r from the center, per unit length per unit time is,

$$-\frac{2\pi}{S_c}\frac{\partial}{\partial r}(T\,r) = -\frac{2\pi}{S_c}\left(r\frac{\partial T}{\partial r} + T\right).$$

The amount of heat leaving this annular ring is,

$$-\frac{2\pi}{S_c}\frac{\partial}{\partial r}\{r\,(T - d\,T)\}\,.$$

where

$$d\,T = -\,d\,r\frac{\partial T}{\partial r}\,.$$

This is equal to,

$$-\frac{2\pi}{S_c}\left(r\frac{\partial T}{\partial r} + T + d\,r\frac{\partial}{\partial r}\left\{r\frac{\partial T}{\partial r}\right\}\right)$$

or

$$-\frac{2\pi}{S_c}\left(r\frac{\partial T}{\partial r} + T + r\,d\,r\frac{\partial^2 T}{\partial r^2} + d\,r\frac{\partial T}{\partial r}\right)\,.$$

Hence the heat stored in the annular ring is,

$$\frac{2\pi}{S_c}\left(r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\right)d\,r + 2\,\pi\,r\,i^2\,p\,d\,r$$

$$= 2\,\pi\,r\,\rho_c\,C_c\frac{\partial T}{\partial t}\,d\,r\,.$$

where

i = Current density in the conductor,
p = Resistivity of the conductor.

Hence the expression for heat flow in the conductor is as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{S_c I^2 R}{\pi r_1^2} = C_c \rho_c S_c \frac{\partial T}{\partial t}. \qquad (1)$$

For sinusoidal variation of heat and temperature,

$$\bar{T} = T_m \epsilon^{j\omega t}.$$

Hence

$$\frac{\partial^2 \bar{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{T}}{\partial r} + \frac{\bar{Q}_0 S_c}{\pi r_1^2} = j\, q_c^2\, \bar{T}. \qquad (13)$$

The solution of equation (13) when

$$\frac{Q_0 S_c}{\pi r_1^2} = 0 \text{ is,}$$

$$\bar{T} = \bar{A}\,\bar{I}_0 (q_c r\sqrt{j}) + \bar{B}\,\bar{K}_0 (q_c r\sqrt{j})^9$$

where

$\bar{I}_0 (q_c r\sqrt{j})$ is a Bessel function of first kind and zero order with semi-imaginary argument.

$\bar{K}_0 (q_c r\sqrt{j})$ is a Bessel function of second kind and zero order with semi-imaginary argument, and

$\bar{A}$ and $\bar{B}$ are arbitrary constants to be determined by the boundary conditions.

When $r = 0$, $\bar{K}_0 (q_c r\sqrt{j}) = \infty$.

Hence $\bar{B}$ must be equal to zero.

The complete solution of equation (13) is then as follows:

$$\bar{T} = \bar{A}\,\bar{I}_0 (q_c r\sqrt{j}) + j\frac{\bar{Q}_0 S_c}{\pi q_c^2 r_1^2}.$$

Applying the boundary conditions and solving for $\bar{A}$,

$$\bar{T} = \left(\bar{T}_1 + j\frac{S_c \bar{Q}_0}{\pi q_c^2 r_1^2}\right)\left(\frac{\bar{I}_0(q_c r\sqrt{j})}{\bar{I}_0(q_c r_1\sqrt{j})}\right) - j\frac{S_c \bar{Q}_0}{\pi q_c^2 r_1^2} \qquad (14)$$

from which

$$\frac{\partial \bar{T}}{\partial r_1} = q_c\sqrt{j}\left(\bar{T}_1 + j\frac{S_c \bar{Q}_0}{\pi q_c^2 r_1^2}\right)\left(\frac{\bar{I}_0'(q_c r_1\sqrt{j})}{\bar{I}_0(q_c r_1\sqrt{j})}\right).$$

or

$$\bar{Q}_1 = -\frac{2\pi q_c r_1 \gamma}{S_c}\left(\bar{T}_1 + j\frac{\bar{Q}_0 S_c}{\pi q_c^2 r_1^2}\right). \qquad (3)$$

where

$$\gamma = \{\sqrt{j}\,\bar{I}_0' (q_c r_1\sqrt{j})\} / \{\bar{I}_0 (q_c r_1\sqrt{j})\}.$$

II—Derivation of Expressions for Heat Flow in the Insulation

By a similar method to that shown in Part I, the Fourier equation for heat flow in a cylinder is obtained for the heat flow in the insulation, as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = C_i \rho_i S_i \frac{\partial T}{\partial t}. \qquad (2)$$

The solution for sinusoidal temperature and heat waves is,

$$\bar{T} = \bar{A}\,\bar{I}_0 (q_i r\sqrt{j}) + \bar{B}\,\bar{K}_0 (q_i r\sqrt{j}).$$

Applying the boundary conditions and solving for $\bar{A}$ and $\bar{B}$,

$$\bar{T} = \frac{(\bar{T}_1 k_2 - \bar{T}_2 k_1)\bar{I}_0 (q_i r\sqrt{j}) + (\bar{T}_2 i_1 - \bar{T}_1 i_2)\bar{K}_0 (q_i r\sqrt{j})}{i_2 k_1 - i_1 k_2}$$

where

$$i_1 = \bar{I}_0 (q_i r_2\sqrt{j}).$$
$$i_2 = \bar{I}_0 (q_i r_2\sqrt{j}).$$
$$k_1 = \bar{K}_0 (q_i r_1\sqrt{j}).$$
$$k_2 = \bar{K}_0 (q_i r_2\sqrt{j}).$$

Hence

$$\frac{\partial \bar{T}}{\partial r} = \frac{(\bar{T}_1 k_2 - \bar{T}_2 k_1)(q_i\sqrt{j})\,\bar{I}_0' (q_i r\sqrt{j}) + (\bar{T}_2 i_1 - \bar{T}_1 i_2)(q_i\sqrt{j})\,\bar{K}_0' (q_i r\sqrt{j})}{i_2 k_1 - i_1 k_2}$$

from which

$$\bar{Q}_1 = \frac{2\pi}{S_i}\left(\frac{q_i r_1 \bar{T}_1(k_2 i_1' - i_2 k_1') + q_i r_2 \bar{T}_2(i_1 k_1' - k_1 i_1')}{i_2 k_1 - i_1 k_2}\right)$$

$$\bar{Q}_2 = \frac{2\pi}{S_i}\left(\frac{q_i r_1 \bar{T}_1(k_2 i_2' - i_2 k_2') + q_i r_2 \bar{T}_2(i_1 k_2' - k_1 i_2')}{i_2 k_1 - i_1 k_2}\right)$$

where

$$i_1' = \sqrt{j}\,\bar{I}_0' (q_i r_1\sqrt{j}),$$
$$i_2' = \sqrt{j}\,\bar{I}_0' (q_i r_2\sqrt{j}),$$
$$k_1' = \sqrt{j}\,\bar{K}_0' (q_i r_1\sqrt{j}),$$
$$k_2' = \sqrt{j}\,\bar{K}_0' (q_i r_2\sqrt{j}).$$

It can be shown that,[8]

$$i_1 k_1' - k_1 i_1' = -\frac{1}{q_i r_1} \text{ and,}$$

$$k_2 i_2' - i_2 k_2' = \frac{1}{q_i r_2}.$$

Hence the following two equations represent completely the heat flow and temperature in the insulation:

$$\bar{Q}_1 = \frac{2\pi}{S_i \sigma}(q_i r_1 \tau \bar{T}_1 - \bar{T}_2), \qquad (5)$$

$$\bar{Q}_2 = \frac{2\pi}{S_i \sigma}(\bar{T}_1 - q_i r_2 \psi \bar{T}_2), \qquad (6)$$

where

$$\sigma = i_2 k_1 - i_1 k_2,$$
$$\psi = i_2' k_1 - i_1 k_2',$$
$$\tau = i_1' k_2 - i_2 k_1'.$$

The following notations are equivalent for the Bessel functions used in the above equations:

$$\bar{I}_0 (q r\sqrt{j}) = Ber\, qr + j\, Bei\, qr.$$
$$\sqrt{j}\,\bar{I}' (q r\sqrt{j}) = Ber'\, qr + j\, Bei'\, qr.$$
$$\bar{K}_0 (q r\sqrt{j}) = Ker\, qr + j\, Kei\, qr.$$
$$\sqrt{j}\,\bar{K}_0' (q r\sqrt{j}) = Ker'\, qr + j\, Kei'\, qr.$$

Appendix B

Derivation of an Expression for Temperature Rise with Suddenly Applied Load

For a single-conductor cable suspended in still air at constant temperature, equations (3), (5), and (9) may be combined and an explicit expression obtained for the temperature rise of the conductor above the sheath surface, as follows:

$$\bar{T}_1 = -j\left(\frac{\bar{Q}_0\,\gamma}{\pi\,q_c\,r_1}\right)\times$$

$$\left(\frac{1}{\dfrac{q_i\,r_1\,\tau}{S_i\,\sigma} - \dfrac{2\,K}{S_i\,\sigma(S_i\,\sigma\,D + 2\,q_i\,r_2\,\psi\,K)} - \dfrac{q_c\,r_1\,\gamma}{S_c}}\right). \quad (15)$$

For the three-conductor cable in air, equations (3), (5), and (9a) are combined to form the expression,

$$\bar{T}_1 = -j\left(\frac{\bar{Q}_0\,\gamma}{\pi\,q\,r_1}\right)\times$$

$$\left(\frac{1}{\dfrac{q_i\,r_1\,\tau}{S_i\,\sigma} - \dfrac{6\,K}{S_i\,\sigma(S_i\,\sigma D + 6\,q_i\,r_2\,\psi\,K)} - \dfrac{q_c\,r_1\,\gamma}{S_c}}\right). \quad (16)$$

For single-conductor cables in ducts,

$$\bar{T}_3 = \frac{N\,\bar{Q}_2\,S_e\,\sigma_e}{\pi\,q_e\,D'\,\tau_e}\,. \quad (10)$$

Substituting for $\bar{T}_3$ in equation (7), the following expression is obtained:

$$\bar{Q}_2 = \frac{\pi\,D\,\bar{T}_2}{(K + D\,\lambda)}\,,$$

where

$$\lambda = \frac{N\,S_e\,\sigma_e}{q_e\,D'\,\tau_e}\,.$$

Hence for single-conductor cables in ducts,

$$\bar{T}_1 = -j\left(\frac{\bar{Q}_0\,\gamma}{\pi\,q_c\,r_1}\right)\times$$

$$\left(\frac{1}{\dfrac{q_i\,r_1\,\tau}{S_i\,\sigma} - \dfrac{2\,(K + D\,\lambda)}{S_i\,\sigma\,\{S_i\,\sigma\,D + 2\,q_i\,r_2\,\psi\,(K + D\,\lambda)\}} - \dfrac{q_c\,r_1\,\gamma}{S_c}}\right) \quad (18)$$

Similarly for three-conductor cables in ducts,

$$\bar{T}_1 = -j\left(\frac{\bar{Q}_0\,\gamma}{\pi\,q_c\,r_1}\right)\times$$

$$\left(\frac{1}{\dfrac{q_i\,r_1\,\tau}{S_i\,\sigma} - \dfrac{6\,(K + D\,\lambda)}{S_i\,\sigma\,\{S_i\,\sigma\,D + 6\,q_i\,r_2\,\psi\,(K + D\,\lambda)\}} - \dfrac{q_c\,r_1\,\gamma}{S_c}}\right) \quad (19)$$

It can be shown that.[13,14]

$$T_0 = \frac{2\,Q_0}{\pi}\int_0^\infty \frac{1}{\omega}\,\delta_r \sin\omega\,t\;d\omega.$$

where δ is the impedance function corresponding to sinusoidal variation of heat flow from any frequency from 0 to ∞. Applying this formula to equations (15) to (18), a solution for temperature can be obtained when a rectangular wave of heat of magnitude Q_0 is suddenly applied to each conductor.

Appendix C

Approximate Expressions for Some Heat-Flow Functions

The following expressions are well within 1 per cent of the exact ones when the limiting conditions specified are fulfilled. They can be used where the functions required are outside the range of the charts given in Figs. 2 to 4.

When $q\,r$ is less than 0.3,

$$\bar{I}_0\,q\,r\sqrt{j} = 1 - \frac{q^4\,r^4}{64} + j\,\frac{q^2\,r^2}{4}\,.$$

$$\sqrt{j}\,\bar{I}_0'\,q\,r\sqrt{j} = -\frac{q^3\,r^3}{16} + j\,\frac{q\,r}{2}$$

$$\bar{K}_0\,q\,r\sqrt{j} = (0.116 - l\,n\,q\,r)\left(1 - \frac{q^4\,r^4}{64}\right) + \frac{\pi\,q^2\,r^2}{16} - \frac{3\,q^4\,r^4}{128} + j\left((0.116 - l\,n\,q\,r)\left(\frac{q^2\,r^2}{4}\right) - \frac{\pi}{4}\left(1 - \frac{q^4\,r^4}{64}\right) + \frac{q^2\,r^2}{4}\right).$$

$$\sqrt{j}\,\bar{K}_0'\,q\,r\sqrt{j} = -(0.116 - l\,n\,q\,r)\left(\frac{q^3\,r^3}{16}\right) - \left(1 - \frac{q^4\,r^4}{64}\right)/q\,r + \frac{\pi\,q\,r}{8} - \frac{3\,q^3\,r^3}{32} + j\left((0.116 - l\,n\,q\,r)\left(\frac{q\,r}{2}\right) - \frac{q\,r}{4} - \frac{\pi\,q^3\,r^3}{64} + \frac{q\,r}{2}\right).$$

$$\sigma = l\,n\,\frac{r_2}{r_1} + j\left(\frac{q^2\,(r_2^2 + r_1^2)}{4}\,l\,n\,\frac{r_2}{r_1} - \frac{q^2(r_2^2 - r_1^2)}{4}\right)$$

$$\tau = \frac{1}{q\,r_1} + j\left(\frac{q\,r_1}{2}\,l\,n\,\frac{r_2}{r_1} + \frac{q^2\,r_2^2}{4\,q\,r_1} - \frac{q\,r_1}{4}\right).$$

$$\psi = \frac{1}{q\,r_2} + j\left(\frac{q\,r_2}{2}\,l\,n\,\frac{r_1}{r_2} + \frac{q^2\,r_1^2}{4\,q\,r_2} - \frac{q\,r_2}{4}\right).$$

$$\gamma = j\,q\,r_1/2$$

When $q\,r$ is greater than 2,

$$\bar{I}_0\, q\, r\sqrt{j} = \frac{\epsilon^{\frac{qr}{\sqrt{2}}}}{2\sqrt{\pi\, q\, r}}\Bigg(\Big(\sqrt{2}+\frac{1}{8\,q\,r}\Big)\cos\Big(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\Big) + \frac{1}{8\,q\,r}\sin\Big(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\Big) + j\Big(-\frac{1}{8\,q\,r}\cos\Big(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\Big) + \Big(\sqrt{2}+\frac{1}{8\,q\,r}\Big)\sin\Big(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\Big)\Big)\Bigg).$$

$$\sqrt{j}\,\bar{I}_0'\, q\, r\sqrt{j} = \frac{\epsilon^{\frac{qr}{\sqrt{2}}}}{2\sqrt{\pi\, q\, r}}\Bigg(\Big(\sqrt{2}-\frac{3}{8\,q\,r}\Big)\cos\Big(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\Big) - \frac{3}{8\,q\,r}\sin\Big(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\Big) + j\Big(\frac{3}{8\,q\,r}\cos\Big(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\Big) + \Big(\sqrt{2}-\frac{3}{8\,q\,r}\Big)\sin\Big(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\Big)\Big)\Bigg).$$

$$\bar{K}_0 q r\sqrt{j} = \frac{1}{2}\sqrt{\frac{\pi}{q\,r}}\,\epsilon^{-\frac{qr}{\sqrt{2}}}\Bigg(\Big(\sqrt{2}-\frac{1}{8\,q\,r}\Big)\cos\Big(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\Big) + \frac{1}{8\,q\,r}\sin\Big(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\Big) + j\Big(\frac{1}{8\,q\,r}\cos\Big(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\Big) - \Big(\sqrt{2}-\frac{1}{8\,q\,r}\Big)\sin\Big(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\Big)\Big)\Bigg).$$

$$\sqrt{j}\,\bar{K}_0'\, q\, r\sqrt{j} = \frac{1}{2}\sqrt{\frac{\pi}{q\,r}}\,\epsilon^{-\frac{qr}{\sqrt{2}}}\Bigg(-\Big(\sqrt{2}+\frac{3}{8\,q\,r}\Big)\cos\Big(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\Big) + \frac{3}{8\,q\,r}\sin\Big(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\Big) + j\Big(\frac{3}{8\,q\,r}\cos\Big(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\Big) + \Big(\sqrt{2}+\frac{3}{8\,q\,r}\Big)\sin\Big(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\Big)\Big)\Bigg).$$

When $\frac{r_2}{r_1}$ is greater than 5,

$$\tau/\sigma = \sqrt{j}\,\bar{K}_0'\, q\, r_1\sqrt{j} \;/\; \bar{K}_0\, q\, r_1\sqrt{j}.$$
$$\psi/\sigma = \sqrt{j}\,\bar{I}_0'\, q\, r_1\sqrt{j} \;/\; \bar{I}_0\, q\, r_1\sqrt{j}.$$

Appendix D

Sample Calculations

I. Calculation of the temperature rise from sheath surface to conductor for a three-conductor 400,000 cir. mils. shielded cable.

The constants of the cable are as follows:

D = 3.1 in. (7.88 cm.).
r_1 = 0.363 in. (0.923 cm.).
r_2 = 0.707 in. (1.797 cm.).
S_c = 0.284 deg. cent. per watt per cm.³
S_i = 650 deg. cent. per watt per cm.³
K = 1,100 deg. cent. per watt per cm.²
ρ_c = 6.9 g. per cm.³ (Equivalent for stranded cable).
ρ_i = 1.16 g. per cm.³
C_c = 0.000107 watt-hr. per g.
C_i = 0.000493 watt-hr. per g.
ω = 0.262 radians per hr. for 24 hr. period.

Hence for the fundamental component (24 hr. period),

$$q_i\, r_1 = 0.923\sqrt{1.16\times0.000493\times650\times0.262} = 0.288.$$

$$q_i(r_2-r_1) = 0.874\sqrt{1.16\times0.000493\times650\times0.262} = 0.272.$$

$$q_c\, r_1 = 0.923\sqrt{6.9\times0.000107\times0.284\times0.262} = 0.00685.$$

$$q_c^2\, r_1^2/S_c = 0.00017.$$

Entering the charts shown in Figs. 2 to 4, the following values are obtained for the heat flow constants:

$$\sigma = 0.668 + j\,0.0085 = 0.668\underline{/0.7^\circ}.$$
$$\tau = 3.47 + j\,0.104 = 3.47\underline{/1.7^\circ}.$$
$$\psi = 1.79 + j\,0.085 = 1.79\underline{/2.7^\circ}.$$

Substituting in equation (8) and letting $\bar{Q}_0$ equal 1.0 watt per conductor per foot of cable,

$$0.01043 - j\,0.00017\,\bar{T}_1 = 0.0046\underline{/-0.7^\circ}(1.00\,\bar{T}_1\underline{/1.7^\circ} - \bar{T}_2).$$

Substituting in equation (9a) and letting $\bar{T}_3 = 0$, since the base temperature is to be considered constant,

$$0.00239\,\bar{T}_2 = 0.0046\underline{/-0.7^\circ}\,(\bar{T}_1 - 1.003\,\bar{T}_2\underline{/2.7^\circ}.$$

Solving these equations for $\bar{T}_1$ it is seen that,

$\bar{T}_1/\bar{Q}_0 = 6.43\underline{/-13.9^\circ}$ deg. cent. per watt per conductor per foot of cable.

For the constant or zero frequency component, the following expression is used.[12]

$$Q_0/(T_1 - T_2) = 61\sqrt{t_1/S_c\, S_i\, r_2}\, G \tanh \phi\sqrt{S_c\, r_2/S_i t_1\, G} + \frac{61\,(\pi-\phi)}{S_i\, G}$$

thermal mhos per foot of cable, where

Q_0 = Watts per conductor per foot of cable.
ϕ = π – Arc of contact of lead sheath and copper shield, expressed in radians, and
t_1 = Thickness of copper shield.

Substituting in this formula,

$(T_1 - T_2)/Q_0$ = 2.45 deg. cent. per watt per conductor per foot.

For the temperature rise in the sheath, under steady load,

$$(T_2 - T_3)/Q_0 = \frac{1100\times3}{3.14\times3.1\times2.54^2\times12} = 4.37 \text{ deg. cent. per watt per conductor per foot.}$$

Hence when $T_3 = 0$,

$T_1/Q_0 = 6.82$ deg. cent. per watt per conductor per foot.

Table I gives the heat flow constants and the value $\overline{T}_1/\overline{Q}_0$ with the time lag of $\overline{T}_1$ behind $\overline{Q}_0$ for all harmonics up to and including the sixth.

TABLE I

Harmonic	σ	τ	ψ	$q^2_c\, r_1^2/S_c$	$\overline{T}_1/\overline{Q}_0$	Time lag, (hours)
0					6.82	0
1	$0.668\underline{/0.7°}$	$3.47\underline{/\ 1.7°}$	$1.79\underline{/\ 2.7°}$	0.00017	6.43	0.93
2	$0.668\underline{/1.4°}$	$2.45\underline{/\ 3.5°}$	$1.27\underline{/\ 5.5°}$	0.00034	5.82	0.87
3	$0.668\underline{/2.1°}$	$2.01\underline{/\ 5.2°}$	$1.04\underline{/\ 8.2°}$	0.00051	5.17	0.80
4	$0.668\underline{/2.9°}$	$1.75\underline{/\ 6.9°}$	$0.91\underline{/11.0°}$	0.00068	4.55	0.74
5	$0.668\underline{/3.6°}$	$1.56\underline{/\ 8.6°}$	$0.81\underline{/13.8°}$	0.00085	4.05	0.66
6	$0.668\underline{/4.4°}$	$1.42\underline{/10.4°}$	$0.75\underline{/16.6°}$	0.00002	3.58	0.60

TABLE II

ω	σ	τ	ψ	γ	δ/π
0.226	$0.668 + j\,0.014$	$2.50 + j\,0.145$	$1.28 + j\,0.120$	$j\,0.0032$	$63.4\ \overline{\backslash 18.5°}$
0.900	$0.667 + j\,0.064$	$1.24 + j\,0.288$	$0.63 + j\,0.234$	$j\,0.0064$	$38.7\ \overline{\backslash 45.7°}$
2.023	$0.660 + j\,0.143$	$0.790 + j\,0.435$	$0.375 + j\,0.342$	$j\,0.0090$	$21.2\ \overline{\backslash 58.9°}$
3.61	$0.640 + j\,0.257$	$0.520 + j\,0.565$	$0.220 + j\,0.452$	$j\,0.0127$	$14.0\ \overline{\backslash 58.4°}$
5.67	$0.595 + j\,0.395$	$0.298 + j\,0.705$	$0.080 + j\,0.558$	$j\,0.0159$	$10.6\ \overline{\backslash 57.6°}$
8.12	$0.520 + j\,0.560$	$0.062 + j\,0.815$	$-0.080 + j\,0.635$	$j\,0.0190$	$8.48\overline{\backslash 57.3°}$
11.03	$0.380 + j\,0.730$	$-0.195 + j\,0.895$	$-0.295 + j\,0.695$	$j\,0.0222$	$6.42\overline{\backslash 45.3°}$

TABLE III

ω	$\sin \omega t$			$\delta_r/\pi\,\omega$	$\frac{Q_0\,\delta_r}{\pi\,\omega}\sin\omega t$		
	$t = 0.4$	$t = 0.8$	$t = 2.0$		$t = 0.4$	$t = 0.8$	$t = 2.0$
0.226	0.0003	0.1800	0.4368	6540	0.788	1.570	3.810
0.900	0.3518	0.6587	0.9744	739	0.346	0.649	0.960
2.023	0.7242	0.9987	−0.7880	133	0.128	0.177	−0.140
3.61	0.9921	0.2453	0.5878	50.1	0.066	0.016	0.039
5.67	0.7660	−0.9848	−0.9397	24.6	0.025	−0.032	−0.031
8.12	−0.1045	0.2079		14.3	−0.002	0.0040	
11.03	−0.9537	−0.9998		10.1	−0.013	0.0077	

Substituting the values given in Table I, for a heat flow represented by the following equation:

$$
\begin{aligned}
Q_0 = 1.47 &+ 0.300 (\sin \omega t - 3.05 \text{ hr.}) \\
&+ 0.092 (\sin 2\, \omega t - 0 \text{ hr.}) \\
&+ 0.148 (\sin 3\, \omega t - 1.39 \text{ hr.}) \\
&+ 0.037 (\sin 4\, \omega t - 0.75 \text{ hr.}) \\
&+ 0.100 (\sin 5\, \omega t - 0.60 \text{ hr.}) \\
&+ 0.060 (\sin 6\, \omega t - 1.50 \text{ hr.})
\end{aligned}
$$

the temperature wave is represented as follows:

$$
\begin{aligned}
T_1 = 10.0 &+ 1.93 (\sin \omega t - 3.98 \text{ hr.}) \\
&+ 0.54 (\sin 2\, \omega t - 0.87 \text{ hr.}) \\
&+ 0.77 (\sin 3\, \omega t - 2.19 \text{ hr.}) \\
&+ 0.17 (\sin 4\, \omega t - 0.01 \text{ hr.}) \\
&+ 0.40 (\sin 5\, \omega t - 1.26 \text{ hr.}) \\
&+ 0.22 (\sin 6\, \omega t + 0.90 \text{ hr.})
\end{aligned}
$$

Curves of the heat and temperature waves represented by the above two equations are shown in Fig. 6.

II. Calculation of the temperature rise from sheath surface to conductor for a three-conductor 400,000 cir. mils shielded cable, suspended in still air, when a steady load is suddenly applied.

The constants of the cable are the same as those in part I of this Appendix except that S_i is 750 deg. cent. per watt per cm.[3] instead of 650.

TABLE IV

T_0/Q_0	$t = 0.2$	$t = 0.4$	$t = 0.8$	$t = 2.0$	$t = 4.0$	$t = \infty$
Deg. cent. per watt	1.44	2.24	3.25	5.15	6.57	7.00
Per cent of final temp.	20.6	32.0	46.5	73.5	93.8	100

The calculation of δ/π is shown in tabular form in Table II.

The calculation of $\dfrac{Q_0\,\delta_r}{\pi\,\omega}\sin\omega t$ for $t = 0.4$ hr., 0.8 hr., and 2.0 hr. is shown in tabular form in Table III.

The calculation of T_0/Q_0 for $t = 0.2$ hr., 0.4 hr., 2.0 hr., and 4.0 hr. is shown in tabular form in Table IV.

The values in Table IV were obtained by integrating $\displaystyle\int_0^\infty \frac{2\,Q_0\,\delta_r}{\pi\,\omega}\sin\omega t\, d\omega$ for the different values of t. A curve of temperature vs. time, plotted from the above Table IV, is shown in Fig. 7, curve A.

Discussion

F. H. Buller: It is interesting to note from Fig. 7 how much the percentage temperature rise in the ducts is slowed down by the increased heat storage capacity. Actually, however, the rate of rise in degrees is faster for the cables in ducts than in air, as indicated by Fig. 19 of the report of the British Electrical and Allied Industries published in the *I. E. E. Journal* for May 1923.

On page 987 of his paper, Mr. Church states that the heat storage capacity of the lead sheath has been neglected. A simple calculation shows that in a large cable the thermal capacity of the sheath is often equal to 80 per cent of that of the conductor and equal to 25 per cent of that of the cable as a whole; and if the cable is furnished with a reinforced sheath or a heavy armor such as used for buried cables or for exposed bus connection cables, the effective thermal capacity of the "sheath" may be an even larger percentage. One cannot help but feel that Mr. Church's equations should be investigated with a view to discovering what the effect of this omission might be of his results; and that if the effect is considerable, the equation should be revised to include the factor which has been omitted.

Mr. Church gives an integral equation (11) for the thermal transient which will occur if a load is suddenly applied to a cable. He points out that in most cases this integral is difficult to evaluate except by mechanical means. He further mentions that when the impedance function is not too complicated, other methods are available for obtaining the value of the transient. In this connection it might be mentioned that at least two methods are known which give an explicit solution in terms of Bessel functions and exponentials for the case of a steady load suddenly applied to a cable suspended in still air. One of these methods involves the use of the Heaviside operational calculus and has been treated by Mr. Shanklin and the present writer in an article which is shortly to be published in the *General Electric Review.* Briefly, this method consists of setting up an operational equation from differential equations similar to Mr. Church's equation (2) for the boundary conditions which obtain on the cable conductor and sheath and of solving this operational equation by the use of the Heaviside expansion theorem. This method gives a solution which is mathematically more explicit and complete than Mr. Church's equation (11).

This, of course, does not mean that Mr. Church's method is without value; for the more complicated case of a cable in ducts and possibly also of a cable buried in earth, the integral equation offers a practical means of solution which will probably be very useful. Mr. Church deserves credit for drawing attention to this method.

The Heaviside operational equation can also be used to give results analogous to those obtained by Mr. Church for cyclical loads by the use of a very simple substitution. The details of this substitution can be found in standard texts on the operational calculus, notably that written by Dr. Berg. Here, again, the present writer does not desire to detract from the value of Mr. Church's work but merely to add an alternative procedure which may be useful in future investigations along these lines.

E. A. Church: In Fig. 7 of the paper, the cable and earth constants were such that the total final temperature difference between the region in the earth at constant temperature and the conductors, and the final temperature difference between the air adjacent to the sheath and the conductors were in the ratio of 2.74 to one. Assuming a total temperature rise of 75 deg. cent. for the cables in ducts, the rise at the end of two hours would be 23 deg. cent. and for a cable suspended in air, *carrying the same load,* the rise would be 20 deg. cent. This approximates the results given in the Fig. 19 mentioned in Mr. Buller's discussion. If, however, the loading on the cables is adjusted to give the same ultimate temperature rise when suspended in air, the temperature will rise much faster than when the cables are in ducts embedded in the earth.

Although the heat storage capacity of the lead sheath may be in some cases 25 per cent of the total storage capacity of the cable, the actual heat storage in the lead sheath rarely exceeds 10 or 15 per cent on account of the relatively lower temperature of the sheath in comparison with the conductor and insulation adjacent to it. The effect of this is small and as stated on page 987 is partly compensated for in three-conductor cables by neglecting the temperature rise in the copper foil around each conductor, the calculation of which would be very difficult for variable loading.

This heat storage may easily be taken into account if it is felt that, as in the case of armored cables mentioned by Mr. Buller, it is of appreciable amounts. Taking heat storage in the sheath into account introduces two more equations of the form of equations (5) and (6) into the set of simultaneous equations necessary for the solution of the temperature of the cable.

To show that the effect of the heat storage in the sheath is quite small in ordinary cases the following calculation was made, for the same size and type of cable as the one described in Appendix D of the paper:

For the lead sheath:

$$(r_2 - r_1) = 0.125 \text{ in. } (0.317 \text{ cm.})$$
$$r_2 = 1.55 \text{ in. } (3.94 \text{ cm.})$$
$$S = 0.298 \text{ deg. cent. per watt per cm.}^3$$
$$\rho = 11.4 \text{ g. per cm.}^3$$
$$C = 0.0000348 \text{ watt-hr. per g.}$$

Hence by the same method as given in Appendix D,

$$q\, r_1 = 0.0201, \text{ and}$$
$$q\, r_2 = 0.0219, \text{ for the fundamental component of the heat-flow cycle.}$$

Referring to the approximate formulas given in Appendix C it can be seen that with $q\,r$ of the order of magnitude obtained above, very little error will be introduced even in the third or fourth harmonics by assuming that:

$$\sigma = l\,n\, r_2/r_1.$$
$$\tau = l/q\, r_1.$$
$$\psi = l/q\, r_2).$$

These values when substituted in equations (5) and (6) give for the sheath:

$$\overline{Q}_1 = \overline{Q}_2 = \frac{2\,\pi}{S\, l\, n\, r_2/r_1}\,(\overline{T}_1 - \overline{T}_2)$$

which is equivalent to saying that the heat storage in the sheath is negligible.

Proximity Effect in Cable Sheaths

BY HERBERT BRISTOL DWIGHT*

Fellow, A. I. E. E.

Synopsis.—The electrical characteristics of the lead sheaths of underground cables can be calculated by the methods used for computing proximity effect. Theoretically, any group of round wires and thin tubes can be calculated. Cases which are taken up in this paper are (1) a thin tube and an enclosed wire, (2) the current density, voltage drop, and power loss in the sheath of a three-conductor cable, (3) two single-conductor cables with the sheaths open-circuited, (4) two single-conductor cables with the sheaths short-circuited, and (5) three single-conductor cables in a plane.

* * * * *

PROXIMITY effect, that is, unequal distribution of alternating current over the cross-section of a conductor caused by current in another conductor, occurs whenever parallel conductors carry alternating current. When the parallel conductors are non-magnetic round wires or infinitely thin round tubes, of any grouping whatever, the effective resistance and reactance can, theoretically, always be calculated. Such a calculation can be carried out for either the loss or the voltage drop. The intensity of the electric field or the magnetic field at any point, due to the irregular current distribution, and the exact amount of shielding, can be computed.

In this paper are given formulas, with numerical examples, for some problems connected with sheaths of underground cables. These are supplementary to the formulas previously published by the writer in the *Electric Journal*.[1]

The problem of the distribution of alternating current in a round wire caused by the proximity of a current in a small filament parallel to the wire, was proposed and solved by Charles Manneback.[2] It seems that this particular problem had not previously been proposed or attacked, but it has proved to be a most useful problem, as it has led to the solution of many others. The result of the above problem is a short algebraical expression for current density, which has the desired characteristic that it gives equal voltage drops at all parts of the conductor section. The current adds up to zero over the section, that is, it is merely a circulating current and does not affect an ammeter connected in series with the conductor. It is not affected by whatever ammeter current there may be and it is not affected by the circulating current caused by another outside filament. These currents are simply added together, as they separately produce equal voltage drops at all parts of the section. The effect of an outside round conductor can be found by integrating the effect of its filaments.

The calculation of the circulating current in an infinitely thin tube caused by a current I in an outside parallel filament has been published by the writer.[3]

The result is

$$i_{(\theta)} = -\frac{I}{\pi c t} \sum_{n=1}^{\infty} \frac{c^n}{s^n} \frac{l^4 + j\, l^2 n}{l^4 + n^2} \cos n\,\theta \tag{1}$$

where

c = mean radius of the tube, in cm.,
t = thickness of the tube, in cm. (considered very small compared with c),
s = distance from the filament to the center of the tube, in cm.,

$$l^2 = \omega \frac{2\pi c t}{\rho_s} \tag{2}$$

$\omega = 2\pi f$
f = frequency in cycles per second,
ρ_s = resistivity of the tube, in abohms per cm. cube.

This is the same as the last part of (14), reference 3, where it is to be noted that the current in the outside filament is $-I$.

A formula for the current density in a very thin tube, caused by a current I in a filament inside the tube, will now be given. If, in reference 3, the filament Y is inside the tube, then instead of equation (4) of that article we must use

$$\operatorname{logn} \frac{D}{c} = -\sum_{n=1}^{\infty} \frac{s^n}{n\, c^n} \cos n\,\theta \tag{3}$$

where c is the radius of the tube, in cm. and θ is the angle indicated in Fig. 1, which is to be used in place of Fig. 1 of reference 3.

The calculation will be the same as the derivation of (14) in reference 3, except that in every case $\frac{a}{s}$ and $\frac{a^n}{s^n}$ become $\frac{s}{c}$ and $\frac{s^n}{c^n}$. The letter c is used in this paper for the mean radius of the tube or sheath. The current density in the tube due to I at Y, Fig. 1, is

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1. "Losses in Grounded Sheaths of Single-Conductor Cables," by H. B. Dwight, *Electric Journal*, February, 1924, p. 62.

2. "An Integral Equation for Skin Effect in Parallel Conductors," *Journal of Mathematics and Physics*, April, 1922, and *Research Bulletin No. 30*, Massachusetts Institute of Technology.

3. *Proximity Effect in Wires and Thin Tubes*, by H. B. Dwight, TRANS. A. I. E. E., 1923, p. 850.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

$$i_{(\theta)} = -\frac{I}{\pi c t}\sum_{n=1}^{\infty}\frac{s^n}{c^n}\,\frac{l^4 + j\,l^2 n}{l^4 + n^2}\cos n\,\theta \qquad (4)$$

If three-phase currents are carried by conductors Y_1, Y_2, and Y_3 as in Fig. 2, and the sheath is open-circuited,

$$i_{(\theta)} = -\frac{I}{\pi c t}\sum_{n=1}^{\infty}\frac{s^n}{c^n}\left(\frac{l^4 + j\,l^2 n}{l^4 + n^2}\right)$$

$$\left[\cos n\,\theta\left(1 - \cos\frac{2\,n\,\pi}{3}\right) + j\,\sqrt{3}\sin n\,\theta\sin\frac{2\,n\,\pi}{3}\right] \qquad (5)$$

The reactive drop in the filament at X due to the element of current $i_{(\theta)}\,c\,t\,d\,\theta$ is

$$j\,\omega\,2\,c\,t\,i_{(\theta)}\,\text{logn}\,\frac{p}{g}\,d\,\theta$$

taking account of flux up to a certain large distance p.

Integrating this expression around the circle and

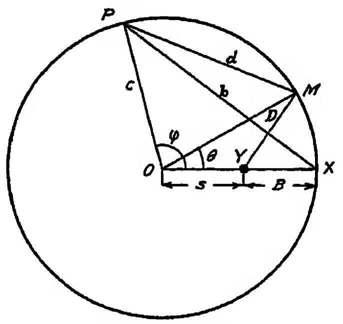

FIG. 1—THIN TUBE AND ENCLOSED FILAMENT

adding the drops due to the three power currents and the resistance drop, the drop in the sheath is found to be

$$j\,\omega\,I\left[\text{logn}\,\frac{\left(c + \frac{s}{2}\right)^2 + \frac{3\,s^2}{4}}{(c-s)^2}\right.$$

$$+\sum_{n=1}^{\infty}\frac{2\,s^n}{c^n}\left(\frac{-n + j\,l^2}{n^2 + l^4}\right)\left(1 - \cos\frac{2\,n\,\pi}{3}\right)$$

$$\left.-\sum_{n=1}^{\infty}\frac{2\,s^n}{n\,c^n}\left(\frac{l^4 + j\,l^2 n}{l^4 + n^2}\right)\left(1 - \cos\frac{2\,n\,\pi}{3}\right)\right]\text{abvolts per cm.} \qquad (6)$$

where I is in abamperes. This formula gives a result equal to zero in a numerical problem. The term logn denotes natural logarithm.

The loss in the open-circuited sheath of a three-phase cable is obtained from (5) and is

$$\frac{\text{Loss in open-circuited lead sheath}}{\text{Loss in 3 copper conductors at 0 frequency}}$$

$$= \frac{2\,a^2}{c\,t}\,\frac{\rho_s}{\rho_c}\sum_{n=1}^{\infty}\frac{s^{2n}}{c^{2n}}\left(\frac{l^4}{l^4 + n^2}\right)\sin^2\left(\frac{n\,\pi}{3}\right) \qquad (7)$$

where t is the actual thickness of the lead sheath in cm., ρ_s is the resistivity of lead and ρ_c that of copper, at the temperatures considered, and where a is the radius in cm. of a solid copper wire of the same resistance as the stranded copper conductor. The current is assumed to be uniformly distributed over the sections of the copper conductors. Note that t has here a different definition from that in reference 1. Expression (7) was published by Dr. F. W. Carter, in the *Proceedings of the Cambridge Philos. Soc.*, Vol. XXIII, 1927, p. 905, equation (19). It may be mentioned that equation (16) of that paper, for the loss in an open-circuited thin tube due to an external filamentary current, is the same result as was published by the writer in paragraph I, Part II, reference 3, in 1923.

Example I. Three-conductor, three-phase cable, 500,000 cir. mils. per conductor, sheath open-circuited, $c = 4.5$ cm., $t = 0.40$ cm., $a = 0.898$ cm., $\rho_s = 25{,}000$, $\rho_c = 2{,}100$ at 75 deg. cent., $f = 60$ cycles, $s = 2.0$ cm., $l^2 = 0.170$.

$$\frac{\text{Loss in open-circuited lead sheath}}{\text{Loss in 3 copper conductors at 0 frequency}} = 0.05 = 5 \text{ per cent.}$$

Example II. Three 1,000,000-cir. mil unsheathed single-conductor cables in a 3/16 in. lead sheath or pipe. $c = 4.55$ cm., $t = 0.48$ cm., $a = 1.27$ cm., $\rho_s = 25{,}000$, $\rho_c = 2{,}100$, $f = 60$, $s = 2.31$ cm., $l^2 = 0.207$.

$$\frac{\text{Loss in open-circuited lead sheath}}{\text{Loss in 3 copper conductors at 0 frequency}} = 0.15 = 15 \text{ per cent.}$$

Note that lead-sheathed, three-conductor cables as large as in this example have been manufactured.

If the pipe were made of iron instead of lead, the loss would be expected to be considerably greater. See Question No. 2913, *Electric Journal*, April, 1929, page 184, concerning three 1,250,000-cir. mil conductors in an iron pipe, in which the loss in the pipe was neglected.

The e. m. f. induced in the open-circuited sheaths in a single-phase circuit of single-conductor underground cables, assuming that the sheaths are infinitely thin, though of the same total resistance as the actual lead sheaths, and assuming that the main current is uniformly distributed over the cross-sections of the round conductors, is given by the following formula.

Let

$$A_n = \frac{2\,c^n}{s^n}\,\frac{(l^4 + j\,l^2 n)}{(l^4 + n^2)}$$

$$B_n = \frac{A_n}{2}\sum_{k=1}^{\infty}\frac{c^k}{s^k}\,A_k\,\frac{(n + k - 1)\,!}{(n-1)\,!\,k\,!}$$

$$C_n = \frac{A_n}{2} \sum_{k=1}^{\infty} \frac{c^k}{s^k} B_k \frac{(n+k-1)\,!}{(n-1)\,!\,k\,!}$$

. . . .

where n and k denote integers.

$$N_n = A_n + B_n + C_n + \ldots$$

Voltage drop in abvolts per cm. of sheath, for a power current I,

$$= j\,\omega\,2\,I\,\text{logn}\,\frac{s}{c} + j\,\omega\,I\,\text{logn}\,\frac{s^2+c^2}{s^2}$$

$$-j\,\omega\,I\left(\frac{N_2}{2} - \frac{N_4}{4} + \frac{N_6}{6} - \ldots\right)$$

$$-j\,\omega\,I\left(\frac{c\,N_1}{H}\cos\alpha + \frac{c^2\,N_2}{2\,H^2}\cos 2\,\alpha + \ldots + \frac{c^n\,N_n}{n\,H^n}\cos n\,\alpha + \ldots\right)$$

$$-\frac{\omega\,I}{l^2}\left(N_2 - N_4 + N_6 - \ldots\right) \qquad (8)$$

where $H^2 = s^2 + c^2$ and $\cos\alpha = \frac{s}{H}$. The term logn denotes natural logarithm. The drop in all parts of the sheath is the same. This expression was calculated for the upper point of the sheath.

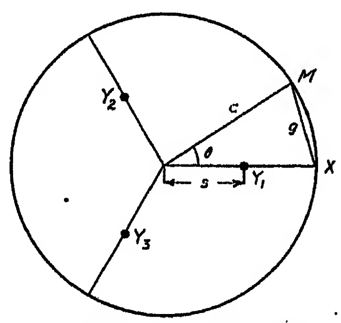

FIG. 2—THREE-PHASE CABLE AND SHEATH

Example III. 2,000,000-cir. mil single-conductor cables, single-phase circuit, 60-cycle, sheaths open-circuited, c = 2.97 cm., t = 0.357 cm. actual thickness of lead sheath, s = 10.66 cm., ρ_s = 25,000. Absolute value of drop by formula (8), is $\omega\,I$ 2.552. If the first term only of (8) is used, the drop is $\omega\,I$ 2.556. The difference is 0.16 per cent which is practically negligible.

To find the expression for the zero-frequency inductance of the sheath, the terms of (8) which contain ω to the first power only, are required. This will involve A_n only. At low frequency, l will be less than 1.

$$\frac{A_n}{j\,l^2} = \frac{2\,c^n}{s^n}\,\frac{1}{n + j\,l^2} = \frac{2\,c^n}{n\,s^n}\left(1 + \frac{j\,l^2}{n}\right)^{-1}$$

The part independent of l is $\frac{2\,c^n}{n\,s^n}$.

The part of the last line of (8), which involves ω to the first power only, is

$$-j\,\omega\,2\,I\left(\frac{c^2}{2\,s^2} - \frac{c^4}{4\,s^4} + \frac{c^6}{6\,s^6} - \ldots\right)$$

$$= -j\,\omega\,I\,\text{logn}\left(1 + \frac{c^2}{s^2}\right)$$

This cancels the second part of (8). The third and fourth parts contain ω to the second and higher powers, and so the zero-frequency inductance of the sheath is given by $2\,\text{logn}\,\frac{s}{c}$, from the first term of (8). The so-called frequency terms, that is, terms involving frequency to higher powers, are seen from Example III to have a negligible effect in a practical case with large conductors. Their effect would be less with smaller conductors.

A series of articles has been published from 1923 to the present time by W. A. Cramp in the *Journal I. E. E.* (England) stating that a term $\text{logn}\,\frac{s-c}{c}$ should be used in formulas for loss and voltage drop in sheaths at low frequency, in place of the term $\text{logn}\,\frac{s}{c}$ used by other writers on the subject.

In case of disagreement between formulas of this type for low frequencies, the proximity effect calculation can be used as a criterion, for it gives convergent series at low frequencies. By computing the terms which are being neglected, it can be determined if they are in fact negligible. This can be done both by taking numerical examples, and by finding the formula for practically zero frequency, as has been done in the single-phase problem just taken up. If different assumptions have been made, their effect can usually be determined by separate proximity effect calculations.

It is shown in the preceding paragraphs that the correct low-frequency term is $\text{logn}\,\frac{s}{c}$. The term $\text{logn}\,\frac{s-c}{c}$ gives results which differ by a considerable percentage, as is shown in Fig. 3.

Carefully made tests are of value in checking calculations. A set of such tests has been published by W. S. Clark and G. B. Shanklin in the A. I. E. E. TRANS., 1919, p. 917. The results of a number of their measurements on open-circuited sheaths are shown by circles in Fig. 3. They are seen to agree closely with equation (8). Fig. 3 is taken from the thesis of W. M. Swingle of Massachusetts Institute of Technology.

When the measurements given in a set of tests do not plot as a smooth curve, or show discrepancies between

themselves, internal evidence is given that the tests are untrustworthy at least to the extent indicated by the discrepancies. The mutual inductance of non-magnetic conductors depends on their size and position, and is a constant with varying current, the frequency and other conditions reported, remaining constant.

A set of tests on open-circuited sheaths which has been referred to quite often by W. A. Cramp is that by P. Dunsheath, *Journal I. E. E.* (England), Vol. 65, 1927, p. 469. One set of test results on non-magnetic cables at 6-in. centers (Table D, page 493 and Fig. 17) shows 30 per cent change in mutual inductance, while the only change reported is that the current changed from 150 to 300 amperes. This can be seen by dividing each

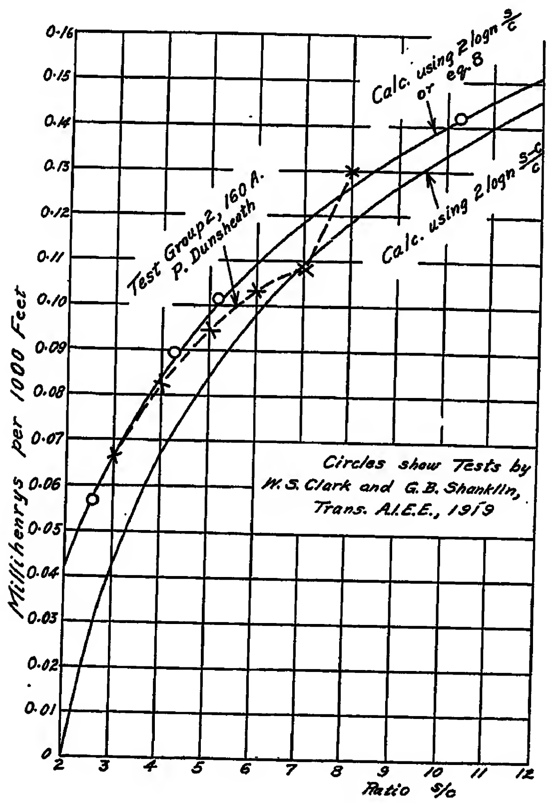

FIG. 3—MUTUAL INDUCTANCE OF CABLES AND SHEATHS

value of measured e. m. f. by the corresponding current, and shows a serious discrepancy in this set of measurements. Another similar set of measurements at different spacings (Table A, p. 492 and Fig. 5) is indicated by crosses and a dotted line in Fig. 3. While the height of the complete curve might require changing, owing to incomplete published data, yet the shape of the curve shows that the measurements are not precise enough to give any evidence that would help toward a choice between the two conflicting formulas illustrated. The statement that the dotted line showed that formula (8) was correct at close spacings and that the other formula was more nearly correct at wide spacings is seen to be without justification. These tests were first published in Fig. 5 by Mr. Dunsheath in the form of a curve, and he showed the same departure of the readings from a smooth curve that is shown in Fig. 3 of this paper.

The consideration of the two formulas of Fig. 3 is a good illustration of the usefulness of proximity effect calculations. The precise values of current and voltage in every part can be computed by the proximity effect method. The exact amount of shielding due to the sheath is calculated. Two different solutions are obtained for the cases of open-circuited sheaths and short-circuited sheaths. The open-circuited voltage between the sheaths suddenly becomes zero when a bond is applied and short-circuit current flows, even though the sheaths are separated by a considerable distance. There is no reason why the voltage between the open-circuited sheaths should gradually approach zero as the sheaths come near the position where they touch each other. The open-circuit voltage is not zero just before they touch.

It is satisfactory that the formulas given in the preceding paragraphs are not long. While some proximity effect formulas contain many terms, that is not a reason for giving up all proximity effect calculations. In cable sheath calculations, the higher frequency proximity effect terms are usually of small effect, except for the largest sizes of cables. The initial terms can often be adjusted to be the same as the most convenient and accurate low-frequency formula. It is advisable to have the higher frequency terms always available, so that one will estimate, almost automatically, whether they are needed or not. If they are not needed, they will cause little or no extra work. If they are needed, nothing else but proximity effect terms can take their place.

A number of proximity effect formulas for cable sheaths is given in a technical report of the British Electrical and Allied Industries Research Association by E. B. Wedmore, P. D. Morgan and S. Whitehead.[4] It may be noted that formulas **(118)** and **(119)** of that report are the same as the first terms of the proximity effect formulas published in Section V of reference 3.

Formula **(28)** of reference **1** for the loss in short-circuited, non-magnetic sheaths in a single-phase circuit can be improved by making it more rapidly convergent for wide spacings between the cables. This is done by a slight change in the calculation, making the initial terms the same as in the usual impedance formula. The two formulas give the same numerical results for cases for which they are both convergent.

Using the same notation as described under equation (1),

$$P = \frac{-j\,\omega \operatorname{logn} \dfrac{s}{c}}{j\,\omega\, 2 \operatorname{logn} \dfrac{s}{c} + \dfrac{\rho_s}{2\,\pi\, c\, t}} \tag{9}$$

4. *Journal I. E. E.* (England), Vol. 67, March, 1929, p. 359.

$$F_n = j\,l^2\,(1 + 2\,P)\,\frac{c^n}{n\,s^n} \tag{10}$$

$$B = j\,l^2 \Big[-2\,A \operatorname{logn} \frac{s}{c} + \sum_{k=1}^{\infty} \frac{c^k}{k\,s^k}\,F_k \Big] \tag{11}$$

$$G_n = j\,l^2 \Big[2\,A\,\frac{c^n}{n\,s^n} - \frac{F_n}{n} + \frac{c^n}{n\,s^n} \sum_{k=1}^{\infty} \frac{(n+k-1)\,!}{(n-1)\,!\,k\,!}\,\frac{c^k}{s^k}\,F_k \Big] \tag{12}$$

In the equations for B and G_n, $A = 0$. For C and H_n use the formulas for B and G_n respectively, except change A to B and F to G. Similarly, for D and I_n, change A to C and F to H, and so on.

$$L = P + B + C + \ldots \tag{13}$$

$$M_n = F_n + G_n + H_n + \ldots \tag{14}$$

$$\frac{\text{Loss in sheath}}{\text{Loss in conductor at zero frequency}} = \frac{a^2}{c\,t}\,\frac{\rho_s}{\rho_c} \Big[2\,|\,L\,|^2 + \sum_{n=1}^{\infty} |\,M_n\,|^2 \Big] \tag{15}$$

where a is the radius of a solid copper wire of the same resistance as the conductor, and where ρ_c is the resistivity of copper, in the same units as ρ_s. $|\,L\,|^2$ is the square of the absolute value of L.

For an approximate formula, use

$$\frac{2\,a^2}{c\,t}\,\frac{\rho_s}{\rho_c}\,|\,P\,|^2 \tag{16}$$

By using equation (25), reference 1, to calculate the drop in the conductor, the effective impedance of the conductor with short-circuited sheath is found to be

$$R_c + j\,X_c + j\,\omega\,4\,L \operatorname{logn} \frac{s}{c} - j\,\omega\,2 \sum_{n=1}^{\infty} \frac{c^n}{n\,s^n}\,M_n \tag{17}$$

where R_c and X_c are the resistance and reactance of the conductor without sheath. L and M_n are complex quantities. If the numerical value of (17) is found, the real part is the effective resistance, and the unreal part is the effective reactance, of the cable with short-circuited sheath, in a single-phase circuit. The effective resistance will be in agreement with the loss in the conductor plus the loss in the sheath as given by (15).

By putting $L = P$ and $M_n = 0$, the following well-known approximate formulas are obtained:

$$\text{Effective resistance} = R_c + \frac{R_s\,X_m^2}{R_s^2 + X_m^2} \tag{18}$$

$$\text{Effective reactance} = X_c - \frac{X_m^3}{R_s^2 + X_m^2} \tag{19}$$

where X_m is the mutual reactance of the sheath and the conductor, and is given by

$$X_m = 2\,\omega \operatorname{logn} \frac{s}{c} \text{ in absolute units.} \tag{20}$$

$$= 2\,\pi\,f\,\frac{140.4}{10^6} \log_{10} \frac{s}{c} \text{ ohms per 1,000 ft. of cable.} \tag{21}$$

See equations (6), (7), (10) and (11) of the article by Donald M. Simmons in the *Electric Journal*, August 1925, p. 366. In numerical calculations, resistance and reactance should be in the same units. The second part of (18) is equivalent to (16). Equation (19) is equivalent to (187) of reference 4.

Example IV. 2,000,000-cir. mil, single-conductor cables, single-phase circuit, 60 cycle, c = 2.97 cm., t = 0.357 cm. actual thickness of lead sheath, s = 6.30 cm. (lead sheaths touching) a = 1.796 cm., ρ_s = 25,000, ρ_c = 2,100, f = 60, l^2 = 0.1005.

$$\frac{\text{Loss in sheath}}{\text{Loss in conductor at zero frequency}} = 49 \text{ per cent, by equation (15).}$$

The result by the approximate formula (16) or (18) is 41 per cent. These results agree with those in Example II, reference 1. The rapidity of convergence of (15) of this article is about the same as that of (28), reference 1, for this example in which the cables are very close together. If the cables are on about 6-in. centers, as they would be if in ducts, the convergence of (15) is more rapid than that of (28), reference 1, and its first term is a better approximate formula.

It has recently been pointed out in Holland, England, and the United States that when three single-conductor cables with short-circuited lead sheaths lie in a plane and carry balanced three-phase current, the losses in the two outer sheaths are not the same. This difference is apparent in the zero-frequency expression. To take care of this, a formula for the loss in the second outside sheath should be added to (32) and (34) of reference 1. Using the notation of that article, let

$$P_o = A_{co} + B_{co} + C_{co} + \ldots$$

$$P_n = A_{dn} + B_{dn} + C_{dn} + \ldots \quad n \neq 0$$

$$\frac{\text{Loss in sheath of second outside cable}}{\text{Loss in conductor at zero frequency}} = \frac{a^2}{c\,t} \Big[2\,|P_o|^2 + |P_1|^2 + |P_2|^2 + \ldots + |P_n|^2 + \ldots \Big] \tag{22}$$

This gives, for Example V of the above article, a result of 1.80 or 180 per cent to compare with 235 per cent in the sheath of the first outside cable.

The writer wishes to acknowledge valuable suggestions from Mr. R. W. Atkinson, chief electrical engineer of the Standard Underground Cable Company, and Mr. K. W. Miller of the Commonwealth Edison Company, regarding this paper.

In conclusion, various problems in proximity effect and shielding in wires and thin tubes, can be solved, as required. In some cases, it is desired to compute the results to show that certain effects are practically negligible. In other cases, the effects are of engineering importance, and the design of conductors will depend upon the computed values.

Discussion

R. W. Atkinson: I wish to discuss Prof. Dwight's formula (7) relating to sheath losses in three-conductor cables. With the large sizes of cables now frequently used, the sheath losses in three-conductor cables are not negligible and, therefore, the formula developed by Prof. Dwight will be very useful indeed. The formula given is readily available for use and is actually much less imposing than might be assumed from its appearance by the average engineer. In order, however, to improve the convenience of its application, we have transformed it to the following form:

$$P = \frac{24\,\pi^4 f^2 s^2 t\, a^2}{c\,\rho_c\,\rho_s}\left[\frac{1}{1+l^4} + \frac{s^2}{4_c^2}\left(\frac{1}{1+\frac{1}{4}\,l^4}\right) + \ldots\right] \tag{7'}$$

P denotes the ratio of sheath loss to zero frequency copper loss and the rest of the notation is the same as in Prof. Dwight's paper.

The series in the bracket converges so rapidly in all practical cases at commercial frequencies that only the first two terms in the bracket need be taken into account. Moreover, at 60 cycles the term l^4 is nearly equal to $\frac{s^2}{4_c^2}$, the variation being of the order of 2 per cent of the value of P. We may therefore, obtain a fair degree of accuracy by neglecting the bracket entirely at 60 cycles.

By making a simplifying assumption which neglects the effect of the redistribution of flux due to the sheath currents, Mr. Louis Meyerhoff of our company has independently derived a formula for sheath loss. This formula is

$$P = \frac{24\,\pi^4 f^2 s^2 t\, a^2}{c\,\rho_c\,\rho_s}\left[1 + \frac{s^2}{4_c^2} + \ldots\right] \tag{VII}$$

It will be noted that formulas (VII) and (7′) are identical except for the term l^4 which appears in the bracket of the latter formula. The value of l^4 in practical cables at 60 cycles varies between 0.02 and 0.05, so that the value of sheath loss as determined by (VII) is from 2 to 5 per cent higher than the value as determined by (7). This indicates the degree of error which may be expected if losses of this type are calculated without taking into account the effect of the induced currents on the flux distribution. The calculations are greatly simplified without causing serious error.

If we take out of formula (7) the factor $\pi\, a^2$ and replace it with a symbol representing the cross-sectional area of the conductor, the formula will apply to stranded as well as to solid conductors. We have also found experimentally that it is applicable with reasonable accuracy to sector-shaped conductors as well, provided the current is assumed to be concentrated at the center of the small diameter of the sector.

In order to permit of simpler application of formula (7) all dimensions may be expressed in inches and the conductor area in circular mils. Substituting for f the value of 60, and for ρ_c and ρ_s their values at 20 deg. cent. (1,720 and 22,000, respectively) and neglecting all terms in the bracket, the formula becomes

$$\text{Per cent } P_{60} = \frac{2.3\, s^2\, t\, A}{10^4\, c} \tag{7''}$$

where

Per cent P_{60} is the sheath loss at 60 cycles expressed in per cent of the copper loss at zero frequency.
s is the distance in inches between the center of each conductor and the center of the cable.
t is the thickness of the lead sheath in inches.
A is the area of each conductor in circular mils.
and c is the mean sheath radius, in inches.

To obtain the loss at any temperature other than 20 deg. cent. the value of per cent P_{60} as obtained by (7″) should be multiplied by the factor

$$\frac{1}{1 + 0.004\ (T_c + T_s - 40)},$$

in which T_c and T_s are the respective conductor and sheath temperatures in degrees centigrade.

Ordinarily, the values of s and c are not readily available, but they can be computed from the following:

For round conductors, the values are accurately:

$$c = 1.078\,d + 2.155\,a + b + \frac{t}{2}$$

$$s = 0.578\,d + 1.155\,a$$

For stranded sector conductors, the values are approximately:

$$c = 0.878\,d' + 2.155\,a + b + \frac{t}{2}$$

$$s = 0.471\,d' + 1.155\,a$$

where

d is the diameter of the conductor in inches,
d' is the diameter of a stranded circular conductor of the same area as the sector conductor under consideration,
a is the conductor insulation thickness in inches,
b is the belt insulation thickness in inches ($b = o$ for type H cables) and c, s and t have the same significance as for formula (7″).

It may be pointed out that the above discussion applies only to three-conductor cables which do not have a magnetic binder.

Another source of losses in three-conductor cables is the skin effect and proximity effect of the conductors. We have measured in our laboratory the losses in stranded sector cable of various sizes from 350,000 cir. mils to 900,000 cir. mils and have found that on the cables tested the value of combined skin effect and proximity effect is approximately equal to 90 per cent of the skin effect calculated by the standard formula for a round single conductor of the same size. By adding this value of skin effect and proximity effect to the sheath loss we obtain the total increase of a-c. loss over d-c. loss. Theoretically, the combined loss will differ somewhat from the simple sum of the two component losses, but the order of difference is smaller than the degree of accuracy with which we are concerned.

The apparent a-c. resistance of a cable will also be affected by the proximity of other cables, the effect depending upon the number, proximity, and physical dimensions of the contiguous cables. Each case must be considered as a special problem. It seems that a fair approximation of this effect can be obtained by making numerical computations on the assumption that the induced currents do not affect the flux distribution. This effect is being studied.

H. B. Dwight: The approximate sheath loss formula given by Mr. Atkinson is a very appropriate one for this problem. It can be used with confidence, as it gives the sheath loss of three-conductor cables up to 1,000,000 cir. mils and more, within a very few per cent.

The test results which he gives are of considerable value. It is to be hoped that further measurements of effective resistance and reactance will be published, for in the cases of stranded conductors and laminated busbars, measurements must be used as well as calculations.

Losses in Transformers for Use With Mercury Arc Rectifiers

BY E. V. DeBLIEUX[1]
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Synopsis.—This paper developes the mathematical formulas and discusses the assumption involved in the methods for determining the losses of transformers for use with mercury arc rectifiers. The methods discussed have been proposed for the A. I. E. E. Standards.

From test readings of current taken under service conditions, a rigid calculation of copper losses is made. From this calculation it is shown that the proposed methods give values that are accurate within satisfactory limits.

The paper also presents the advantages of the proposed methods for determining the losses of transformers for use with mercury arc rectifiers.

* * * * *

Part I

A NUMBER of papers and books has been published in the last few years which includes tables and calculations giving the salient features of the various transformer connections which are suitable for use with mercury arc rectifiers.

The calculations set forth in these publications are based on the assumption that the secondary current waves are rectangular in shape.

Similarly, calculations of effective values of current and of losses are based on this rectangular wave shape and on the further assumption that the effective resistance of the winding is the same at all frequencies. Since the reactance of the circuit alters the current wave shape from the assumed form and as the effective resistance is different for the various harmonics which go to make up the current waves, calculations based on the stated assumptions may be very approximate.

It is the purpose of this paper to review these theoretical calculations, to calculate particularly the copper losses of a typical transformer designed for use with power rectifiers and, based on the conclusions obtained from the calculations, to set forth commercial methods of making tests of losses of the commonly used connections.

Current Harmonics

If a rectifier transformer with p secondary phases supplies current to a rectifier and the circuit is assumed to be free of reactance, the current per phase will be as shown for phases 1 and 2 in Fig. 1a. Since there are p phases each phase will carry current for the $\frac{2\pi}{p}$ portion of a cycle, or with the axis as shown, from $-\frac{\pi}{p}$ to $+\frac{\pi}{p}$. The magnitude of the current is designated by the letter J.

With this arrangement, the wave may be analyzed into a series of cosine terms because of its symmetry about the coordinate axis. The amplitude of the nth harmonic is then—

$$a_n = \frac{1}{\pi} \int_{-\frac{\pi}{p}}^{+\frac{\pi}{p}} J \cos n\,\theta\, d\,\theta = \frac{2\,J}{n\,\pi} \sin \frac{n\,\pi}{p} \qquad (1)$$

Solving for the harmonics in the secondary current wave of a six-phase and a three-phase rectifier transformer, the values will be as given in Table I.

Fig. 2c shows one of the commonly used six-phase

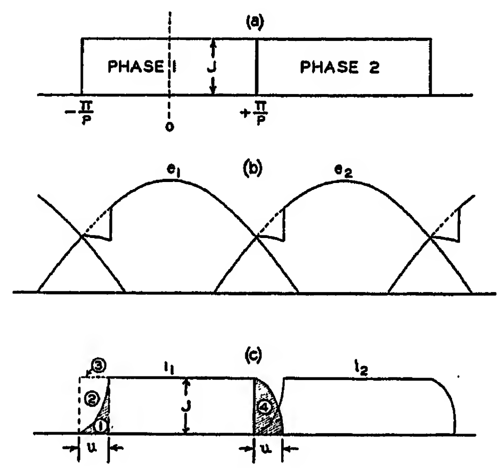

FIG. 1

rectifier connections. Since current flows in each of the (b) portions of the winding for $\frac{2\pi}{p}$ or 1/6 of a cycle the values in the $p = 6$ column of Table I will apply to this portion of the winding. The (c) portion of the winding carries the current of two successive b coils and therefore for ⅓ of a cycle. It is therefore equivalent to a three-phase rectifier and the values in the $p = 3$ column apply to this section of the winding. The current which flows in the primary coils is the resultant of the currents which flow in the b and c secondary coils which are on the same leg. The even harmonics of the b and c coils neu-

1. General Trans. Engg. Dept., General Electric Co., Pittsfield, Mass.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

tralize each other because they are, 180 deg. out of phase, of equal magnitude and flow through coils having equal turns. It is therefore the resultant of the odd harmonics that flow in the primary coils.

Effect of Reactance

Due to the reactance of the transformer windings and the supply lines, it requires a definite time for the secondary current to rise to its full value at the start of each period of conduction and again to decrease to zero at the end of the period of conduction.

As a result the current wave is altered from the shape as shown in Fig. 1a and assumes the shape shown by the solid lines of Fig. 1c.

Figs. 1b and 1c show the voltage and current waves of anodes 1 and 2 of the connection shown in Fig. 2c. Anode 1 carries the full d-c. current until the point in the cycle is reached where the voltage of anodes 1 and 2 are equal, at which point anode 2 begins burning.

Beginning at this point two anodes are connected

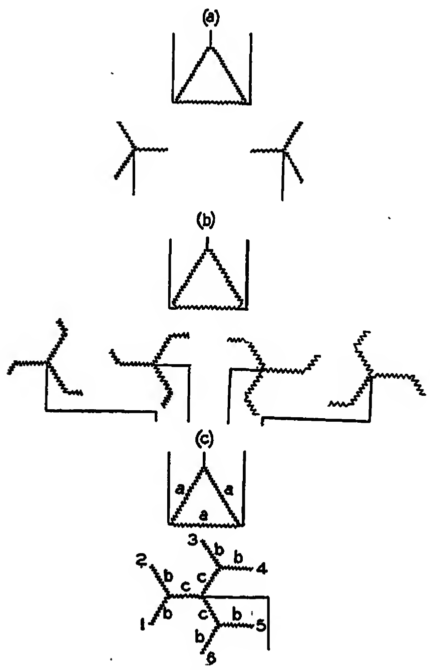

Fig. 2

simultaneously to the cathode and constitute an electrical connection between the connected transformer phases, short-circuiting the difference in voltage between the two phases. This difference in voltage circulates a current through the loop thus formed, which current flows in an opposite direction to the d-c. current carried by phase 1.

This condition continues until the instantaneous value of the circulating current equals the d-c. current carried by phase 1 at which time the resultant current in phase 1 is zero and its arc goes out thus terminating the short circuit and leaving phase 2 carrying the d-c. current.

By this means, commutation of current is effected.

TABLE I

	$p = 6$	$p = 3$
Fundamental	100 %—0.318 I_{DC}	100 %—0.552 I_{DC}
Second harmonic	86.8%—0.276 I_{DC}	50 %—0.276 I_{DC}
Third	66.6%—0.212 I_{DC}	0.00
Fourth	43.4%—0.138 I_{DC}	25 %—0.138 I_{DC}
Fifth	20 %—0.064 I_{DC}	20 %—0.110 I_{DC}
Sixth	0.00	0.00
Seventh	14.3%—0.045 I_{DC}	14.3%—0.079 I_{DC}
Average = d-c. comp.	$= \frac{I_{DC}}{p} = 0.167\ I_{DC}$	$0.333\ I_{DC}$
R. m. s. = a-c. and d-c.	$= \frac{I_{DC}}{\sqrt{p}} = 0.408\ I_{DC}$	$0.577\ I_{DC}$
R. m. s. a-c. comp. $= I_{DC} \sqrt{\frac{1}{p} - \frac{1}{p^2}}$	$= 0.373\ I_{DC}$	$0.471\ I_{DC}$

This period of commutation, *i. e.*, of overlap of current of two phases is denoted by u. (See Fig. 1c.)

Using the instant when anode 2 starts burning as the origin and applying Kirchoff's second law to the closed loop through phases 1 and 2—

$$e_1 = L\frac{d\,i_1}{d\,t} - L\frac{d\,i_2}{d\,t} + e_2 + a_1 - a_2 \tag{2}$$

in which

e_1 and e_2 are the instantaneous voltages of phase 1 and phase 2 respectively,

a_1 and a_2 are the arc drops of phase 1 and phase 2 respectively,

i_1 and i_2 are the instantaneous currents of phase 1 and phase 2 respectively.

L = the inductance of the circuit between anode and neutral.

Assuming the arc drop of the two burning anodes to be equal the terms a_1 and a_2 cancel out.

Also from Fig. 1c,

$$i_1 + i_2 = J$$

$$e_1 = E\sqrt{2}\cos\left(u\,t + \frac{\pi}{p}\right)$$

$$e_2 = E\sqrt{2}\cos\left(u\,t - \frac{\pi}{p}\right)$$

in which

E = Effective value of transformer secondary line-to-neutral voltage

t = time

$u = 2\,\pi\,f$

Substituting the above values of e_1 and e_2 in (1) and solving simultaneously

$$i_1 = J - \frac{E\sqrt{2}\sin\pi/p}{X}(1 - \cos u\,t) \tag{2a}$$

in which $X = \omega L$

Equating equation (2) to zero at time $t = u$, at which time i_1 = zero, then,

$$\cos u = 1 - \frac{J X}{E \sqrt{2} \sin \pi/p} \qquad (3)$$

It should be noted from Fig. 1c that the effect of reactance is to increase the period during which a phase carries current by the angle u without altering the maximum or average value of the current.

As a result it is to be expected that the effective value of current is decreased.

The change in mean square value due to this increase in the period of conduction may be represented by the following equation:

$$\Delta i_1 = \frac{1}{2\pi}\int_0^u i^2 (1-\cos x)^2\, dx + \frac{1}{2\pi}\int_0^u (J - i\,(1-\cos x)^2\, dx - \frac{1}{2\pi}\int_0^u J^2\, dx \qquad (4)$$

Referring to Fig. 1c, the first term represents the area (1) the second term represents the area (2) which is the same as (4) and the third term represents the area (3) which is enclosed in the dotted lines and is the sum of (1) and (2).

Integrating for i_1

$$\Delta i_1 = \frac{1}{2\pi}\left[2 i^2 \left(\frac{3u}{2} - 2 \sin u + \frac{1}{4} \sin 2u \right) - 2 J i\,(u - \sin u) \right] \qquad (5)$$

From the figure $i = \dfrac{J}{1 - \cos u}$ at $x = u$

Substituting this value for i in (5) and collecting terms:

$$\Delta i_1 = - \frac{J^2 (2 + \cos u) \sin u - (1 + 2 \cos u)\, u}{2 \pi (1 - \cos u)^2}$$

The minus sign indicates that the change in effective value of the current is a decrease as was anticipated.

The effective or root mean square value of I for flat topped waves is $\dfrac{J}{\sqrt{p}}$ and the mean square value is therefore $\dfrac{J^2}{p}$. Due to commutation, the mean square value therefore becomes

$$I_{eff}^2 = \frac{J}{p^2} - \frac{J^2(2 + \cos u)\sin u - (1 + 2\cos u)\, u}{2\pi(1 - \cos u)^2} \qquad (6)$$

which expression is equivalent to and may be reduced to the forms given by various authors.[2]

2. "Mercury Arc Rectifiers and Their Circuits," Prince and Vogdes, p. 115,

Archiv. fur Elektrotechnik, Daellenbach and Gerecke, Vol. 14, No. 2, Jan. 15, 1925, pp. 17-246.

"Mercury Arc Power Rectifiers," Marti and Winograd, p. 114.

Equation (6) is based on the assumption that the effective transformer resistance is the same for all the harmonics which make up the current wave for which the expression was derived and that therefore in determining the effective value of current the harmonics may be added in the usual manner to determine the resultant r. m. s. value.

Columns 2, 3, and 4 of Table II show in per cents of the fundamental component the magnitude of each of the harmonics in the various coil currents. The values were obtained when a d-c. current of value J was delivered by a six-phase rectifier, of the connection shown in Fig. 2c, fed from a system whose capacity is large compared with that of the rectifier. The harmonics in the primary and in the anode (b coil) currents were measured. The harmonics in the current of the c coils were determined by proportion from the measured reductions of the corresponding harmonics in the current of the a and b coils.

It will be noted that the actual per cent harmonics in the current are not so large as the theoretical values in Table I and that the ratio between theoretical harmonic current and the test value is greater the higher the order of the harmonic.

This is due to the impedance of the circuit which increases with the order of the harmonic.

Effective Resistance

Based on formulas developed over a period of years and tests of a large number of transformers a formula for eddy loss in transformer windings has been developed, in which the eddy current loss varies approximately in accordance with the following equation:

$$\text{Watts (eddy loss)} = 0.86\, Y\, X^{1.875} \qquad (7)$$

in which

X = frequency and Y represents the sum of the other factors which determine the eddy loss.

It should be noted that in this expression the eddy loss varies in accordance with slightly less than the square of the frequency instead of with the square as given in various other similar formulas, and therefore, calculations based on it will give lower losses than the conventional formulas.[3] For a given winding all of the factors that enter in the Y term of equation (7) are constant, therefore the proportionate resistance to the various harmonics will be as determined from equation (7) and as given in columns 5 and 6 of Table II.

In connection with the loss values determined later in the paper, it should be noted from columns 5 and 6 of Table II that the effective resistance at fundamental

3. "Magnetic Leakage in Transformers," J. M. Weed, *G. E. Review*, Vol. 15, p. 759, Dec. 1912.

"*Reduction of Eddy Current Losses*, J. M. Lyons, A. I. E. E. Journal, July, 1913.

"Eddy Currents in Large Slot Wound Conductors," A. B. Field, *Proc. A. I. E. E.*, 1905, p. 659.

"Stray Losses in A-C. Armature Windings," Dr. I. Rusch, *Elektrotechnik und Maschinenbau*, Nos. 4 and 5, Jan. 1910.

TABLE II

1 Order of harmonic	2 Test amplitude (peak values) *a* coil	3 *b* coil	4 *c* coil	5 Effective coil resistance *a*	6 *b* and *c*	7 Effective values of *I* (see equation (8) *a* coil	8 *b* coil	9 *c* coil
D. C.		52.5	60.3	1	1		51.1	58.7
1	100	100	100	1.05	1.12	102.5	105.8	105.8
2		73	42.3		1.44		88.5	50.7
3	22	41.5	0	1.18	1.96	24	58.0	
4		23.5	13.55		2.62		38.0	21.9
5	7.7	7.7	7.7	2.03	3.46	11.0	14.3	14.33
6								
7	5.7	5.7	5.7	2.2	5.6	8.44	13.5	13.5
8		8.0	4.61		6.9		21.0	12.1
9	3.8	7.5		4.07	8.36	7.68	21.4	
10		5.3	3.07		9.99		16.75	9.7
11	2.5	2.5	2.5	5.51	11.8	5.88	8.6	8.6
12								
13	2.2	2.2	2.2	7.18	15.8	5.9	8.75	8.75
14		3.3	1.81		17.6		13.82	7.58
15	1.5	3.0		9.02	18.65	4.5	12.95	
16		2.0	1.17		22.85		9.57	5.6
17	0.9	0.9	0.9	11.21	25.6	3.02	4.55	4.55
18								
19	0.95	0.95	0.95	13.44	30.9	3.46	5.29	5.29
20		1.4	0.78		34		8.16	4.55
21	0.7	1.3		16.08	37.2	2.87	7.93	
22		0.8	0.48		40.75		5.11	3.06
23	0.34	0.34	0.34	18.95	44.2	1.48	2.26	2.26
24								
25	0.43	0.43	0.43	21.86	51.1	2.01	3.08	3.075
R. m. s. a-c. component in per cent of fundamental						75.6	113.5	87.1
Calculated r. m. s. a-c. component (from Table I)						0.816	0.373	0.471
Test r. m. s. a-c. component						0.835	0.361	0.481
D-c. component						0	0.1625	0.324
Calc. r. m. s. (a-c. + d-c.) (From Table I)						0.816	0.408	0.577
Test r. m. s. (a-c. + d-c)						0.835	0.396	0.584
Test r. m. s. in per cent of calc. r. m. s.						102.2	97.5	101.2

frequency for the *a* coils is 5 per cent, and for the *b* and *c* coils is 12 per cent, greater than the corresponding d-c. resistance, indicating an eddy and stray loss of 5 per cent for the *a* coils and 12 per cent for the *b* and *c* coils, which are conservative values.

COPPER LOSS CALCULATION

From Table II the loss produced by each harmonic may be separately determined.

In order to make the application general, the effective coil resistance to a given harmonic, has been expressed in terms of the resistance to the fundamental as developed from the following equations:

$$\text{Watts loss} = I_h^2 R_h = I_h^2 K R_f = (I_h \sqrt{K})^2 R f = I^2 R_f \quad (8)$$

in which

I_h = Effective value of harmonic current

R_h = Effective coil resistance at harmonic frequency

R_f = Effective coil resistance at fundamental frequency

$$K = \frac{R_h}{R_f}$$

$$I = I_h \sqrt{K}$$

Comparing the percentage values of I' given in columns 8 and 9 of Table II with the corresponding values of Table I expressed on a percentage basis, shows that they are approximately the same, indicating that for each harmonic the effect with respect to the copper losses of the reduction of effective current value due to the reactance of the circuit is approximately balanced by the effect of the increase in effective resistance with the order of the harmonic.

The percentage values of I' will therefore be as shown in columns 7, 8, and 9 of Table II for windings *a, b, c* respectively.

With this change all harmonics have been expressed on the basis of the same resistance and the r. m. s. value of the series of harmonics which flow in each winding may be determined from the square root of the sum of their squares. This determination gives for the *a, b,* and *c* coils respectively, the values shown at the bottom of columns 7, 8 and 9 of Table II.

For the particular transformer used for the test, which is of average design, the losses in the various windings were distributed approximately as shown below when based on the theoretical r. m. s. values of current given in Table I.

Winding	(*a*)	(*b*)	(*c*)
Copper loss	42%	33%	25%

Correcting these percentages for the difference between the theoretical r. m. s. values on which they are based and the test values of Table II, the following percentages are obtained.

Winding	(*a*)	(*b*)	(*c*)
Copper loss	43	32.2	25.3

The sum of the losses of the three windings is 100.5 per cent indicating that the actual losses are 0.5 per

cent greater than the value calculated from the theoretical r. m. s. values of current. Making a similar calculation using the values of Table I gives 133.6 per cent losses.

There are several points in regard to this calculation which should be pointed out. The supply voltage wave contained a number of harmonics of small magnitude which slightly increased the magnitude of the current harmonics and for which no correction was made. The magnitude of these harmonics is given in Table III.

It should be noted from Table II that due to the harmonics in the supply voltage, the 19th and 25th harmonics in the high-voltage winding are larger than

TABLE III

Harmonic	Per cent volts a-c. supply rectifier shut down
Fundamental	100
5	0.50
7	0.44
11	0.11
13	0.06
17	0.32
19	0.06
23	0.015
25	0.020

the 17th and 23rd harmonics respectively, whereas from theoretical calculations, they should be smaller.

Had the tests been made on a connection such as is shown in Fig. 2a or 2b the harmonic currents in the secondary windings would be those for a three-phase circuit, since these connections have multiple three-phase secondaries. The magnitude of the harmonics would therefore approximate those of the *c* coils of the transformer tested. Since the primary harmonic currents are proportional to the secondary currents, the accuracy of the primary winding copper loss determination would approximate that of the test.

The tested connection which has been discussed in detail is therefore representative of the results which would be obtained for any one of the three commonly used connections illustrated.

Conclusion

The transformer copper losses are dependent on the magnitude of the harmonics in the current waves which harmonics vary in magnitude depending on the reactance of the load and of the supply, both of which are variable, as well as on the reactance of the rectifier transformer.

A calculation of losses based on the r. m. s. values of current for flat-topped waves without overlapping (Table I), gives results of the correct magnitude.

Therefore, the proposed A. I. E. E. Standards for Mercury Arc Rectifiers which recommends that the calculation of losses of rectifier transformers be based on flat-topped waves without overlapping, will give values which are accurate within satisfactory limits. Calculation by this method has a number of advantages in addition to eliminating uncertain reactance calculations. These advantages are listed below:

1. It eliminates involved mathematical calculations.
2. It eliminates factors which are design features and known only to the manufacturer.
3. It enables purchaser to check the manufacturer's calculations.
4. It gives safe values of current from which the purchaser may determine the sizes of cables to use in making connections.
5. It reduces cost and shortens the time of delivery since it obviates the necessity of having the manufacturer set up the complete apparatus, duplicating the supply and load reactances and measuring losses by the input-output method. As is well known, the input-output method is subject to considerable inaccuracy due to the fact that it involves the determination of the difference between two large quantities.
6. As will be shown in the third part of the paper, it permits separate and direct measurement of the losses of the transformer by the usual methods employed in testing power transformers.

Part II

It is the purpose of this section of the paper to develop the mathematical basis of the methods of determining the copper losses of power transformers for use with rectifiers which are proposed for the A. I. E. E. Standards for Mercury Arc Rectifiers. Proof will also be developed that the proposed methods give the losses which would be obtained under rectifier operation with flat top secondary current waves without overlapping, it being the conclusion from Part I that the loss thus obtained is equivalent, within satisfactory limits of accuracy, to that obtained with the actual wave forms obtained under service conditions.

The three most commonly used transformer connections are shown in Figs. 2a, 2b, and 2c.

It should be noticed that two of the circuits *a* and *b* have multiple secondaries and that the third *c* has a single interconnected secondary. As a result different tests are proposed for the two types of circuits to obtain the desired results. The various other transformer connections used at times, fall into one or the other of these classes.

Under rectifier operation each anode and the associated transformer winding carries one block of current per cycle as shown in Fig. 1. In order to avoid the saturation of the transformer cores by the direct current carried by the secondaries, an even number of anodes and associated transformer secondary windings, divided into pairs displaced 180 degrees electrically are provided.

Under this condition the primaries carry blocks of current similar to those in the secondaries but only 180 degrees apart, the alternate blocks being of opposite sign.

The heating effect of such a current will be twice that of a current consisting of pulses in one direction only and the r. m. s. value of the primary current will therefore be $\sqrt{2}$ times that of the secondary current when the transformation ratio is unity.

Testing of Transformers with Multiple Secondaries

Considering the connection shown in Fig. 2a:

If I_p is the primary current, I_s the secondary current, and the transformation ratio is unity, then as has been shown in the preceding paragraph $I_p = \sqrt{2}\, I_s$.

If R_p = resistance of one primary coil

and R_s = resistance of one secondary coil

then total copper loss $W_{cu} = 3\, I_p^2\, R_p + 6\, I_s^2\, R_s$

$$= 3\, I_p^2\, (R_p + R_s) \qquad (9)$$

If only one secondary winding is short-circuited for the copper loss test, then I_p will equal I_s instead of $\sqrt{2}\, I_s$ and the resistance will be R_s.

The total copper loss for this test will therefore be

$$W_{cu}' = 3\, I_p^2\, R_p + 3\, I_p^2\, R_s = 3\, I_p^2\, (R_p + R_s) \qquad (10)$$

which is identical with equation (9) and therefore gives the correct loss.

Considering the transformer connection shown in Fig. 2b:

For this connection the transformer copper losses under rectifier operation will be

$$W_{cu} = 3\, I_p^2\, R_p + 12\, I_s^2\, R_s \qquad (11)$$

The various symbols have the same meaning as explained in the preceding paragraph for the connection of Fig. 2a.

It can be shown that for the connection of Fig. 2b.

$$I_s = 0.364\, I_p$$

Substituting this value for I_s in equation (11)

$$W_{cu} = 3\, I_p^2\, R_p + 1.59\, I_p^2\, R_s \qquad (12)$$

Two tests will be required to determine the copper losses of this connection. For the first test two of the secondary wyes are short-circuited and the losses measured with rated sinusoidal current held in the primary. Assuming a 1:1 ratio of transformation the total copper losses will be:

$$L_1 = 3\, I_p^2\, R_p + 1.5\, I_p^2\, R_s \qquad (13)$$

For the second test all four secondary wyes are short-circuited and losses measured as in the first test. For the second test the losses will be:

$$L_2 = 3\, I_p^2\, R_p + 0.75\, I_p^2\, R_s \qquad (14)$$

Solving equations (13) and (14) simultaneously for R_p and R_s

$$R_p = \frac{2\, L_2 - L_1}{3\, I_p^2} \qquad (15)$$

$$R_s = \frac{L_1 - L_2}{0.75\, I_p^2} \qquad (16)$$

Substituting these values of R_p and R_s in equation (12)

$$W_{cu} = 1.12\, L_1 - 0.12\, L_2 \qquad (17)$$

which is an expression for the transformer copper lost in terms of the loss measurements of the two tests connections.

It is essential to successful rectifier operation that the multiple secondary circuits be so placed with respect to the primary that each can operate independently as the full secondary. For this reason short-circuiting only half of the secondary coils does not alter materially the magnitude or distribution of the stray losses. Furthermore, the slight alteration that does occur tends to raise the reactance and therefore the losses. The increase is slight, however, as tests of a number of transformers give values of impedance with one-half of the secondary short-circuited that average only 5 per cent higher than the value with the full secondary shorted-circuited. The maximum increase in reactance obtained in the extreme case was less than 10 per cent.

Testing of Transformers with a Single Set of Secondary Windings

Under rectifier operation assuming flat-topped waves without over-lapping each of the outer b coils (Fig. 2c)

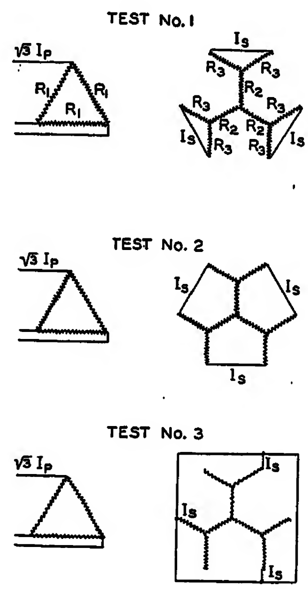

Fig. 3

of the secondary carry the full d-c. for one-sixth of a cycle. Its effective value therefore, is

$$I_b = \frac{I_{DC}}{\sqrt{6}} \qquad (18)$$

Similarly the inner c coils carry the total d-c. for one-third of a cycle and the effective value is

$$I_c = \frac{I_{DC}}{\sqrt{3}} \qquad (19)$$

Assuming a 1:1 ratio of primary to one secondary coil

$$I_a = \frac{2\, I_{DC}}{\sqrt{6}} \qquad (20)$$

The copper losses are therefore

$$W_{cu} = 6\, I_b^2\, R_b + 3\, I_c^2\, R_c + 3\, I_a^2\, R_a \qquad (21)$$

Substituting in (21) for I_a, I_b, and I_c, their values from equations (18), (19), and (20)

$$W_{cu} = (2\, R_a + R_b + R_c)\, I_{DC}^2 \qquad (22)$$

From (20)

$$I_{DC} = \frac{\sqrt{6}\, I_a}{2}$$

Substituting this value for I_{DC} in (22)

$$W_{cu} = (3\, R_a + 1.5\, R_b + 1.5\, R_c)\, I_a^2 \qquad (23)$$

In order to make an independent test of the power transformer and obtain the losses for rectifier operation as given in equation (23) three copper loss tests will be made to obtain three equations and permit solving for the three unknowns—the effective a-c. resistance of the three sections of the windings.

The three test connections are shown in Fig. 3.

Denoting the loss which will be measured in tests (1), (2), and (3) by L_1, L_2, and L_3 respectively, the loss equations will be as follows:

$$L_1 = 3\, I_p^2\, R_1 + 6\, I_s^2\, R_3 = I_p^2(3\, R_1 + 2\, R_3) \qquad (24)$$

$$L_2 = 3\, I_p^2\, R_1 + 3\, (\sqrt{3}\, I_s)^2\, R_2 + 6\, I_s^2\, R_3 = I_p^2\, (3\, R_1 + 3\, R_2 + 2\, R_3) \qquad (25)$$

$$L_3 = 3\, I_p^2\, R_1 + 3\, I_s^2\, R_2 + 3\, I_s^2\, R_3 = I_p^2\, (3\, R_1 + R_2 + R_3) \qquad (26)$$

Solving equations (24), (25), and (26) simultaneously for R_1, R_2, and R_3

$$L_2 - L_1 = 3\, I_p^2\, R_2 \quad \text{therefore} \quad R_2 = \frac{L_2 - L_1}{3\, I_p^2} \qquad (27)$$

$L_1 - L_3 = I_p^2\, (R_3 - R_2)$ therefore

$$R_3 = \frac{L_1 - L_3}{I_p^2} + R_2 = \frac{2\, L_1 + L_2 - 3\, L_3}{3\, I_p^2} \qquad (28)$$

$$R_1 = \frac{L_1}{3\, I_p^2} - \frac{2\, R_3}{3} = \frac{6\, L_3 - 2\, L_2 - L_1}{9\, I_p^2} \qquad (29)$$

Substituting in (23) the values of R_1, R_2 and R_3, from equations (27), (28) and (29) and collecting terms

$$W_{cu} = \frac{L_1 + 2\, L_2 + 3\, L_3}{6} \qquad (30)$$

which is an expression for the copper loss in terms of the measured copper loss of the three test connections.

Part III

Iron Loss

While the current waves are distorted due to the harmonics discussed in the previous sections of this paper, the voltage wave impressed on the primary, when the rectifier is fed from a system whose capacity is reasonably large, approximates a sine wave.

The core loss tests should therefore be made in the same manner as for ordinary power transformers.

Conclusions

It is therefore possible to test the main transformers of the various rectifier connections independently of the rectifiers, making the tests by the simple methods used for ordinary power transformers. From the readings thus obtained the losses which would be obtained under rectifier operation with flat-topped waves and no overlapping may be determined. This loss as shown in the first part of the paper, approximates within satisfactory limits the loss obtained under actual operating conditions.

Discussion

W. M. Goodhue: Referring to equation (2), the symbol L is used as the inductance of the circuit between anode and neutral and this implies that the inductance of the two anode circuits are equal to each other and are constant at all times. If transformers having only one (two-terminal) secondary coil and one primary coil per "window" of the transformer could be used, the leakage flux paths would have a definite shape and a single value of leakage inductance could be used, provided the exciting current required for the iron is neglected. But with rectifiers, however, this arrangement is impracticable, since the d-c. component in the secondary current cannot be reflected in the primary and the iron would therefore be saturated in a single direction and to an extreme degree. Hence with rectifiers at least two secondary coils (or a three-terminal coil) per window must be used, so that the d-c. components of their m. m. fs. may oppose each other. In Fig. 2c, either coils b_1 and b_3, or coils b_2, b_3 and c_{56} etc. must all occupy one window. Thus the two coils corresponding to the two anodes which are firing simultaneously must, at least in many parts of the cycle, occupy the same window. Under these conditions the leakage flux paths must be changing *shape* with time as the proportion of current between the two anode currents changes with time, the sum of the two currents at all times during the commutation period being nearly constant. This requires that at least three different leakage flux components (or self and mutual inductances) must be considered instead of one single inductance as in equation (2). A rigorous solution of these conditions would lead to differential equations of at least the third order if all unknowns but one are to be eliminated. Under these conditions it may be shown that the use of several constant self and mutual inductances is sound with the changing shape of the flux path, and with undetermined and peculiar wave forms.

The use of such constant inductances presumes that the leakage flux can be divided into components, each leakage flux being proportional to the m. m. f. producing it, and of constant shape, each m. m. f. being taken *singly*. This holds true in spite of the fact that one m. m. f. when acting alone would saturate the iron. To show that this presumption is correct requires an analysis of the boundary condition of the *unsaturated iron* when *one* m. m. f. *alone* is acting; this analysis may be rigorously made by the use of vector methods. Let A be the *vector* potential of the magnetic field intensity H, that is Curl $A = H$; A, however, may be restricted by making Div $A = O$; Curl H = Curl Curl $A = -\Delta^2 A$, where Δ^2 is the Laplacian operator, so that A is of potential function form with $\frac{1}{4\pi}$ Curl H replacing the density of charge, ρ, in electrostatics; that is,

$$-\Delta^2 A = 4\pi \left[\frac{1}{4\pi} \text{Curl } H \right] = 4\pi \rho.$$

$$A_p = \int\int\int \frac{\rho}{r_p}\, d\,v = \int\int\int \frac{\frac{1}{4\,\pi}\,\text{Curl}\,H}{r_p}\, d\,v$$

Where

A_p = vector potential of point p.
$d\,v$ = element of volume.
r_p = length of line drawn from P to $d\,v$.
$\frac{1}{4\,\pi}$ Curl H = current density (of winding).

To take account of the boundary condition $H = O$ at an iron surface with one m. m. f. acting *alone*, set $A = O$ at the iron surface. This gives a *vector* surface charge, σ_1, induced on the iron surface (analogous to electrostatic induced charge). Then the total A due to one m. m. f. acting alone is, with its σ, denoted by A_{p1}

$$A_{p1} = \int\int\int \frac{\frac{1}{4\,\pi}\,\text{Curl}\,H_1}{r_p}\, d\,v + \int\int \frac{\sigma_1}{r_p}\, d\,S$$

Curl H_1 is 0.4 π times the current density (amperes per sq. cm.) in the coil containing m. m. f., F_1 and σ_1 is an induced vector charge due to this same m. m. f., so that A_1 is completely determined. Thus Curl A_1 is determined, giving H_1 the leakage field density due to one m. m. f. *alone* with the boundary condition of unsaturated iron. As σ depends only on the geometry and the current density, H_1 is proportional to the m. m. f., F_2. For similar reasons any one of the three constant self and three mutual leakage inductances may likewise be used, all of them being needed in the differential equations. Propagation and distributed capacitance effects have, however, been neglected; proximity effect on the leakage inductances with certain connections (as conductors in parallel) may occur also, although with conductors in series, it is chiefly the equivalent resistance which is affected.

Referring again to equation (2) the resistance drops have been neglected. *These resistance drops are not negligible during commutation,* because of the small magnitude of the e. m. fs. involved. Also there appear to be errors made in the signs and time origins of e_1 and e_2, for e_1 and e_2 essentially oppose each other at the moment of commutation, considering the closed loop, through phases 1 and 2. Hence e_1 and e_2 should have opposite signs if on the same side of the equation rather than the same sign as given in equation (2).

Furthermore, on page 1005 it is stated that the core loss test is to be made in a manner similar to that used in ordinary power transformers. It should be remembered, however, that the exciting current of ordinary transformers is exceedingly small. Even if a circuit is used which, with proper placing of the coils in the windows of the transformer or transformers avoids extreme saturations due to the full effect of the working load m. m. f., a very slight unbalance due to mechanical and electrical discrepancies in the accuracy of construction and operation of the entire system would produce very considerable saturation. Some method of testing can probably be devised whereby this condition may be similated as for example by circulating a direct current of the correct magnitude in addition to the working alternating exciting current through the windings. The result could then be plotted in some such form as per cent increase in core loss as a function of the per cent unbalance under several operating conditions. To obtain the direct-current component, a small direct-current e. m. f. might be inserted during the core loss test in series between the supply alternator and the transformer, using a direct-current ammeter to indicate the amount of unbalance.

E. V. DeBlieux: It must be recognized that a rigorous solution of the conditions during commutation requires, as pointed out by Mr. Goodhue, that the mutual inductances and the resistance drops be considered. In addition, the fact that the arc drops are not equal at each instant and that there is a ripple in the d-c. current must also be considered.

In order to simplify the calculations incidental to commercial design, these refinements have been eliminated and the simple formula given in equation (2) obtained. This simplification was justified both by test results and the following considerations:

During the interval when current is commutating between two anodes the current in the other anodes which are active, is

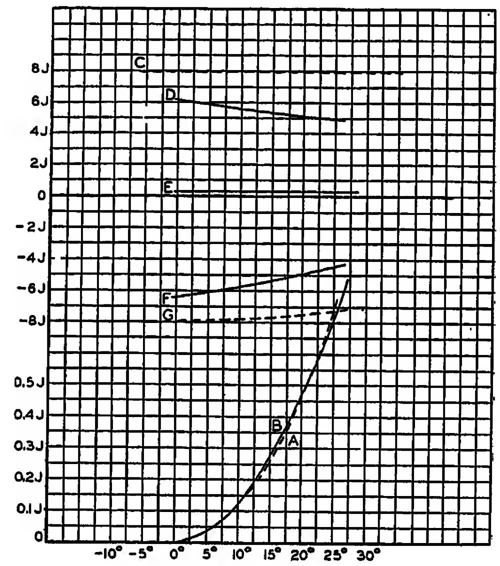

FIG. 1

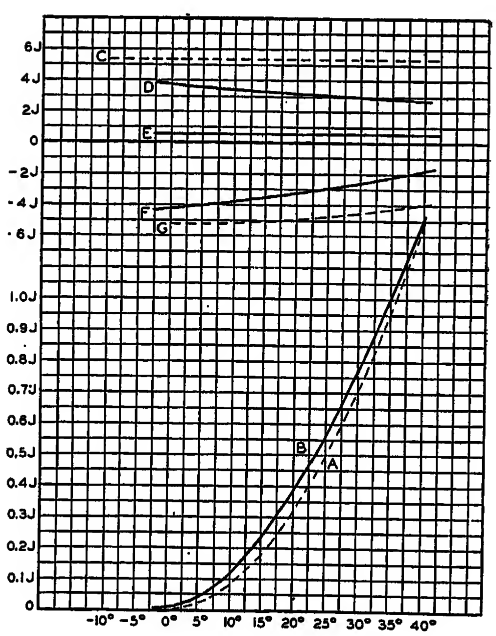

FIG. 2

fairly constant. As a result the mutual effects between the coils active in the commutation and other coils are small. For some connections such as Fig. 2A when current is commutating between two anodes all other secondary coils of the same legs with the two involved in the commutation are idle. Therefore, there are no mutual effects in this case. The same is true of Fig. 2C with the exception that for commutations such as between anodes 2 and 3 there is an inductive effect between coils B_2 and B_3, but as the

primary instead of flowing in the opposite direction as stated above.

The equivalent circuit with negative impedance coupling may be used for making calculations but cannot be produced by using simply resistances, reactance coils, and condensers because oft he negative resistance component in the coupling impedance. However a

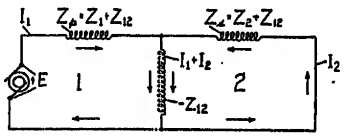

FIG. 2—EQUIVALENT CIRCUIT OF TWO-CIRCUIT TRANSFORMER EMPLOYING NEGATIVE IMPEDANCE COUPLING

negative impedance device is stated to have been used in a form of Wheatstone bridge for measuring the capacities in cables.[2]

There appears to be no advantage to be gained by using the equivalent circuit in Fig. 2 instead of that in Fig. 1. The circuit in Fig. 2 has been given to show how the relative directions of the currents in two coupled circuits can be reversed by the use of negative coupling because this principle will be employed later in some of the equivalent circuits of the three-circuit transformer.

THREE-CIRCUIT TRANSFORMER

Case. I. In the three-circuit transformer the first case to be considered is where the magnetic circuits are

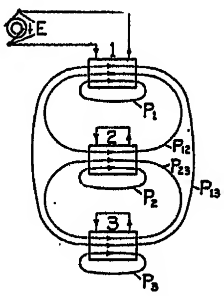

FIG. 3—DIAGRAM DESCRIBING CONDITIONS IN TRANSFORMER IN CASE I (*a*)

so arranged that it is impossible for mutual fluxes to exist which interlink with the conductors in each of the three circuits. The diagram in Fig. 3 shows a circuit arrangement in which the conductors in each circuit are represented by coils which are placed so that their axes are parallel to each other. The current in the coils is assumed to be flowing in the directions indicated by the arrows and the coils are wound so that the fluxes due to the ampere-turns in each coil flow through the coils from left to right. There are three stray fluxes, that interlink with conductors in only one circuit, which flow in the paths P_1, P_2 and P_3 and six mutual fluxes that flow in paths P_{12}, P_{13} and P_{23} and interlink with the conductors in circuits 1 and 2, 1 and 3 and 2 and 3. Because of the directions of the currents and the arrangement of the circuit the mutual fluxes in each path flow in opposition to each other. There are three distinct transformer relations due to the magnetic coupling between the circuits which are shown schematically in the diagram in Fig. 4.

In circuit 1, in Fig. 3, there is a voltage E from the generator, an induced voltage $Z_{12} I_2$ due to current flowing in circuit 2 and an induced voltage $Z_{13} I_3$ due to current in circuit 3. In circuit 2, there is an induced

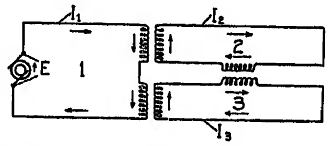

FIG. 4—SCHEMATIC DIAGRAM DESCRIBING TRANSFORMER RELATIONS BETWEEN THREE COUPLED CIRCUITS

voltage $Z_{12} I_1$ due to current in circuit 1 and an induced voltage $Z_{23} I_3$ due to current flowing in circuit 3 and in circuit 3 there are induced voltages $Z_{13} I_1$ and $Z_{23} I_2$ due to current flowing in circuits 1 and 2. The voltage relations in the three circuits may be expressed in the following equations.

$$E + Z_{12} I_2 + Z_{13} I_3 = Z_1 I_1 \qquad (6)$$

$$Z_{12} I_1 + Z_{23} I_3 = Z_2 I_2 \qquad (7)$$

$$Z_{13} I_1 + Z_{23} I_2 = Z_3 I_3 \qquad (8)$$

in which Z_{12}, Z_{13} and Z_{23} represent the mutual impedance due to the flux interlinking with circuits 1 and 2, 1 and 3 and 2 and 3; Z_1, Z_2 and Z_3 are the total impedances in each circuit and I_1, I_2 and I_3 represent respectively the current in each circuit.

Equations (6) (7) and (8) may also be written,

$$E = (Z_1 - Z_{12} - Z_{13}) I_1 + Z_{12} (I_1 - I_2) + Z_{13} (I_1 - I_3) \qquad (9)$$

$$O = (Z_2 - Z_{12} - Z_{23}) I_2 + Z_{12} (I_2 - I_1) + Z_{23} (I_2 - I_3) \qquad (10)$$

$$O = (Z_3 - Z_{13} - Z_{23}) I_3 + Z_{13} (I_3 - I_1) + Z_{23} (I_3 - I_2) \qquad (11)$$

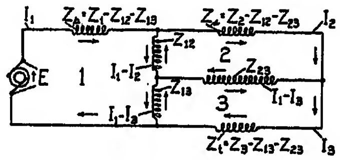

FIG. 5—EQUIVALENT CIRCUIT, CASE I (*a*)

The equivalent circuit of the three-circuit transformer, in the case under consideration, may be drawn from the voltage relations expressed by the equations as shown in Fig. 5.

In the equations given above and in the equivalent circuit external loads on the transformer have not been considered. Any external loads, or impedances not directly coupled with the adjacent circuits, would be included in the total impedances Z_1, Z_2 or Z_3 in the

2. "Direct Capacity Measurements," by George A. Campbell, *Bell System Technical Journal*, July 1922, pp. 24 and 29.

equations and included in the diagram in the series impedances $Z_p = Z_1 - Z_{12} - Z_{13}$, $Z_s = Z_2 - Z_{12} - Z_{23}$ or $Z_t = Z_3 - Z_{13} - Z_{23}$ in the corresponding circuit.

Case II. When the coils and the magnetic circuit in a three-circuit transformer are so arranged that there is a path for magnetic flux which interlink with all three circuits the condition may be described schematically as shown in the diagram in Fig. 6 in which the coils are represented by heavy black lines and the directions of the current flowing in the coils and the directions of the fluxes are as indicated by the arrows. There are stray fluxes flowing in the paths P_1, P_2, and P_3 and mutual fluxes which interlink with only two circuits in paths

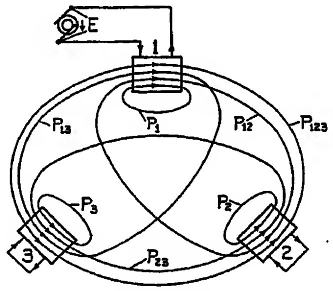

Fig. 6—Diagram Describing Conditions in Transformer in Case II (*h*)

P_{12}, P_{13}, and P_{23} and mutual fluxes that interlink with all three circuits flow in the path P_{123}. The magnetic coupling between circuits 1 and 2 is caused by the fluxes that flow in the paths P_{12} and P_{123} that are due to the ampere-turns in coils 1 and 2. The mutual impedance Z_{12}, in this case, represents the impedance due to all flux interlinkages between circuits 1 and 2. In a similar manner the mutual fluxes which interlink with conductors in circuits 1 and 3 may be seen to flow in the paths P_{13} and P_{123} and the mutual impedance is represented by Z_{13}. The mutual fluxes interlinking with circuits 2 and 3 flow in the paths P_{23} and P_{123} and the mutual impedance due to these fluxes is represented by Z_{23}. When a mutual flux flowing through any coil is in the same direction as the stray flux the induced voltage due to the mutual flux is in the same direction as the induced voltage due to self induction and when the mutual flux is in the opposite direction it induces a voltage in an opposite direction to that due to self induction. In each circuit there is, in addition to the voltage due to self induction, an induced voltage due to interlinkage of flux caused by ampere-turns in each of the other coils.

The voltage equations which include these induced voltages may now be written from an inspection of the diagram in Fig. 6 to describe the relations in the transformer as follows:

$$E + Z_{12} I_2 + Z_{13} I_3 = Z_1 I_1 \qquad (12)$$

$$Z_{12} I_1 - Z_{23} I_3 = Z_2 I_2 \qquad (13)$$

$$Z_{13} I_1 - Z_{23} I_2 = Z_3 I_3 \qquad (14)$$

By comparing these equations with equations (6), (7) and (8) it will be seen that they are of the same form except for the negative signs before the terms $- Z_{23} I_2$ and $- Z_{23} I_3$. If the negative signs are assumed to indicate a negative impedance the equations may be rewritten as in the following equations (15), (16) and (17) which are in the same form as equations (9), (10) and (11).

$$E = (Z_1 - Z_{12} - Z_{13}) I_1 + Z_{12} (I_1 - I_2) + Z_{13} (I_1 - I_3) \qquad (15)$$

$$O = (Z_2 - Z_{12} + Z_{23}) I_2 + Z_{12} (I_2 - I_1) + (-Z_{23})(I_2 - I_3) \qquad (16)$$

$$O = (Z_3 - Z_{13} + Z_{23}) I_3 + Z_{13} (I_3 - I_1) + (-Z_{23})(I_3 - I_2) \qquad (17)$$

The equivalent circuit of the transformer in the present case may now be drawn to express the voltage relations described by the equations as shown in the diagram in Fig. 7.

In deriving the equivalent circuit, circuit 1 has been assumed to be the primary and circuits 2 and 3 two secondary circuits, but circuit 2 might have been selected as the primary, and circuits 1 and 3 the secondary circuits. By following the same line of reasoning the equivalent circuit would then have the same form as that in Fig. 7 but the mutual impedance between circuits 1 and 3 would have the negative sign instead of the impedance between circuits 2 and 3. The mutual impedances would have been Z_{12}, $- Z_{13}$ and Z_{23} and the series impedances $Z_p = Z_1 - Z_{12} + Z_{13}$, $Z_s = Z_2 - Z_{12} - Z_{23}$ and $Z_t = Z_3 + Z_{13} - Z_{23}$. In a similar manner circuit 3 might have been taken as the primary and circuits 1 and 2 as secondary circuits in which case the mutual impedances would be $- Z_{12}$, Z_{13} and Z_{23} and the series impedances $Z_p = Z_1 + Z_{12} - Z_{13}$, $Z_s = Z_2 + Z_{12} - Z_{23}$ and $Z_t = Z_3 - Z_{13} - Z_{23}$. It will be seen in each case that the series impedances are such that the total open circuit impedance in each circuit is Z_1, Z_2 or Z_3. If the three equivalent circuits that have been described are each equivalent circuits of the transformer they must be equivalent to each other and any one may be substituted

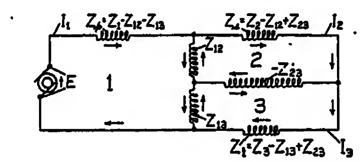

Fig. 7—Equivalent Circuit, Case II (*h*)

in place of another. For instance when a potential is applied to circuit 1 the three equivalent circuits are as shown in the diagrams in Figs. 7, 8, and 9.

The circuit in Fig. 7 represents the condition in the transformer when the zero phase direction of the current in circuit 1 is assumed to be in one direction and the currents in circuits 2 and 3 are in the opposite direction. Fig. 8 represents the condition when the currents in circuits 1 and 3 are assumed to be in the same direction and the current in circuit 2 is in the opposite. The diagram in Fig. 9 indicates that the currents in circuits 1 and 2 are in the same direction and that in circuit 3 is in the opposite direction. There is still another equivalent circuit in which all three of the mutual impedances are

negative impedances which represent the condition when the currents in all three windings are assumed to flow in the same direction.

In the equivalent circuit in Fig. 5, which represents the case where there is no mutual flux which threads all three windings, it will be seen that the mutual impedances are all positive. The signs of any two of the mutual impedances may be changed from positive to negative and the circuit will still be theoretically the

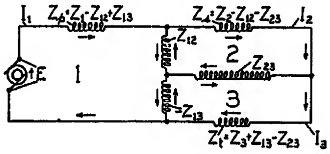

FIG. 8—EQUIVALENT CIRCUIT, CASE II (g)

equivalent of the transformer and equivalent to the circuit in the diagram in Fig. 5, provided corresponding changes are made in regard to the assumption as to the direction of the flow of current in the windings of the transformer.

It can be seen that the mutual impedances in all of the equivalent circuits are connected together in the form of a Y. These impedances may be replaced in any of the circuits by an equivalent Δ combination of impedances which will give a form of circuit from which calculations can be more easily made. The present circuits have a Y combination of impedances within a delta combination and when the proposed substitution is made the arrangement will be a Δ within a Δ combination with the adjacent sides of the two deltas connected in parallel as shown in Fig 14.

TWO-CIRCUIT TRANSFORMER THEORY APPLIED TO THE THREE-CIRCUIT TRANSFORMER

In either of the two cases of the three-circuit transformer it may be that the impedance in one of the

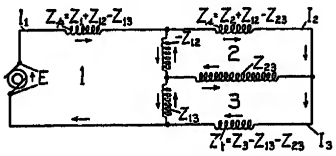

FIG. 9—EQUIVALENT CIRCUIT, CASE II (f)

secondary circuits remains unchanged in which case the other two circuits may be considered as a two-circuit transformer. To describe this condition consider the equivalent circuit in Fig. 5 and assume that the impedance in circuit 3 does not change. The impedances Z_{13}, Z_{23} and $Z_t = Z_3 - Z_{13} - Z_{23}$ are then to be considered as constant quantities. By referring to the diagram it can be seen that these impedances are connected together in the form of a Δ. The Δ arrangement can be replaced in the usual manner by an equivalent Y combination. When this change is made the circuit becomes as shown in the diagram in Fig. 10 in which the impedance $Z_A = Z_{13}\, Z_{23}/Z_3$, $Z_B = Z_{23} - (Z_{13}\, Z_{23} + Z_{23}^2)/Z_3$, $Z_C = Z_{13} - (Z_{13}\, Z_{23} + Z_{13}^2)/Z_3$.

The circuit is clearly an equivalent circuit of a two-circuit transformer in which $Z_{12} + Z_A$ is the mutual impedance and $Z_1 - Z_{12} - Z_{13} + Z_C$ is the series impedance in circuit 1 and $Z_2 - Z_{12} - Z_{23} + Z_B$ is the series impedance in circuit 2. If Z_{12}' represents the mutual impedance and Z_p' and Z_s' the series impedance in circuits 1 and 2 respectively and Z_1' the total impedance in the primary and Z_2' the total impedance in the secondary we have

$$Z_p' = Z_p + Z_C = Z_1 - Z_{12} - \frac{Z_{13}^2 + Z_{13}\, Z_{23}}{Z_3} \tag{18}$$

$$Z_s' = Z_s + Z_B = Z_2 - Z_{12} - \frac{Z_{13}\, Z_{23} + Z_{23}^2}{Z_3} \tag{19}$$

$$Z_{12}' = Z_{12} + Z_A = Z_{12} + \frac{Z_{13}\, Z_{23}}{Z_3} \tag{20}$$

$$Z_1' = Z_1 - \frac{Z_{13}^2}{Z_3} \tag{21}$$

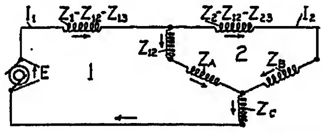

FIG. 10—EQUIVALENT CIRCUIT OF A THREE-CIRCUIT TRANSFORMER REDUCED TO A TWO-CIRCUIT TRANSFORMER

$$Z_2' = Z_2 - \frac{Z_{23}^2}{Z_3} \tag{22}$$

The last three equations are the most important because they fully describe the impedance characteristics of the two-circuit transformer.

In any of the equivalent circuits that have been discussed no matter what the signs are before the mutual impedances when reduced to a two-circuit transformer the total impedance of the primary Z_1' and of the secondary circuit Z_2' are the same as expressed in equations (21) and (22). In a Case I problem, when there is no mutual flux through all three windings, the mutual impedance will be as given by equation (20) or by equation (23).

$$Z_{12}' = -Z_{12} - \frac{Z_{13}\, Z_{23}}{Z_3} \tag{23}$$

and in a Case II problem the mutual impedances will be as given in equations (24) or (25)

$$Z_{12}' = Z_{12} - \frac{Z_{13}\, Z_{23}}{Z_3} \tag{24}$$

$$Z_{12}' = -Z_{12} + \frac{Z_{13}\, Z_{23}}{Z_3} \tag{25}$$

In the following table the circuits a, b, c and d are equivalent to each other and represent the conditions described in the problem in Case I and circuits e, f, g and h are equivalent circuits representing conditions in the problem in Case II. The circuits in either case differ from each other because the equations from which they are derived are based on a different assumption as to the directions in which the currents in the transformer are flowing. The directions in which the currents in each circuit are assumed to flow are indicated in the diagram in Fig. 11. In the table the characteristic impedances of the two-circuit transformer are given in terms of the characteristics of the three-circuit transformer. A set of three fundamental equations may be written in a form which will correspond with any one of the equivalent circuits described in the table. These equations are expressed in terms of the three-circuit characteristic impedances and the currents I_1, I_2, and I_3. In any set of equations the current I_3 may be eliminated algebraically with the result that two equations are obtained which describe the conditions in circuits 1 and 2 and also furnish a proof that the corresponding equivalent circuit shown in Fig. 10 truly represents the conditions in the transformer.

IMPEDANCES IN EQUIVALENT CIRCUITS

Equivalent circuit	Problem	Mutual impedances in three-circuit diagram	Two-circuit diagram Z_1'	Z_2'	Z_{12}'	Equivalent circuit shown
a	Case I	$Z_{12},\ Z_{13},\ Z_{23}$	$Z_1 - \frac{Z_{13}^2}{Z_3}$	$Z_2 - \frac{Z_{23}^2}{Z_3}$	$Z_{12} + \frac{Z_{13}\,Z_{23}}{Z_3}$	Figs. 5 and 10
b	"	$Z_{12},\ -Z_{13},\ -Z_{23}$	"	"	"	
c	"	$-Z_{12},\ Z_{13},\ -Z_{23}$	"	"	$-Z_{12} - \frac{Z_{13}\,Z_{23}}{Z_3}$	
d	"	$-Z_{12},\ -Z_{13},\ Z_{23}$	"	"	"	
e	Case II	$-Z_{12},\ -Z_{13},\ -Z_{23}$	"	"	$-Z_{12} + \frac{Z_{13}\,Z_{23}}{Z_3}$	
f	"	$-Z_{12},\ Z_{13},\ Z_{23}$	"	"	"	Fig. 9
g	"	$Z_{12},\ -Z_{13},\ Z_{23}$	"	"	$Z_{12} - \frac{Z_{13}\,Z_{23}}{Z_3}$	Fig. 8
h	"	$Z_{12},\ Z_{13},\ -Z_{23}$		"	"	Fig. 7

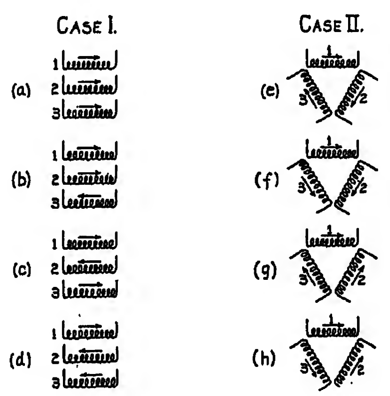

FIG. 11—SHOWING RELATIVE DIRECTIONS OF THE CURRENTS IN THE TRANSFORMER

The two-circuit transformer theory has been applied to describe conditions in circuits 1 and 2 when taken in the presence of circuit 3. In a similar manner equivalent circuits can be drawn that are equivalent to the transformer as regards circuits 1 and 3 in the presence of circuit 2 and there are circuits which are equivalent to circuits 2 and 3 when taken in the presence of circuit 1.

RATIO OF TURNS IN THE WINDINGS IN THE TRANSFORMER

Nothing has been said in the present discussion in regard to the ratio of the number of turns in the windings of the transformer and it is not ordinarily necessary to consider the number of turns in making circuit calculations but the equivalent circuit is sometimes used to study conditions that exist within the transformer and for this purpose the equivalent circuit may be reduced to a circuit which represents a transformer that has a 1:1:1 ratio of the number of turns in the windings. If a_2 represents the ratio of turns between the primary and secondary and a_3 the ratio of turns between the primary and tertiary windings, the impedances in the transformer may be reduced to primary terms by multiplying the total impedance Z_2 of the secondary by a_2^2 and the total impedance Z_3 of the tertiary by a_3^2, the mutual impedances Z_{12} and Z_{13} by a_2 and a_3 respectively and the mutual impedance Z_{23} by $a_2\,a_3$; furthermore, the voltages in circuits 2 and 3 would vary directly in proportion to the ratio of the number of turns in the windings and the currents would vary inversely as the ratio of the number of turns.

CONDITIONS IN THE MAGNETIC CIRCUIT

In the transformer described in Case I, the conditions in the magnetic circuit can be determined from the currents, voltages, and impedances in the electrical circuits. The ampere-turns are determined from the currents flowing and the number of turns in the windings. The permeances of the various paths in the magnetic circuits may be determined from the inductances

in the electrical circuits, because it is well known that the inductances may be written directly in terms of permeances.[3] The flux which interlinks with the conductors in a coil may be determined from the induced voltage they produce, provided it is sufficiently accurate to assume that all lines of force interlink with all convolutions in the coil.

In the particular problem given above in Case I it is possible to determine the permeances of the various paths described in the magnetic circuit shown in Fig. 3. But in more complicated problems in Case I and those in Case II as shown in Fig. 6 it does not appear to be possible to separate the permeance of the path P_{123} from the permeances of the paths P_{12}, P_{13} and P_{23} and it appears to be impossible to separate the permeances of the paths for the stray flux from the permeances of the paths in which there are mutual fluxes.

Discussion of the Equivalent Circuit of the Three-Circuit Transformer

Two very simple problems of the three-circuit transformer have been considered in each of which there is only one coil in each circuit and one path for the mutual flux which interlink with each pair of coils. In practise there may be a number of coils in each circuit and a number of paths for the mutual flux. The fluxes in each path between two circuits, including the path which interlinks with the conductors in all three circuits, cause a component of the total mutual impedance between the circuits. Each component may be represented in the equivalent circuit by a positive or negative component of coupling impedance, depending upon the direction of the fluxes flowing in the mutual path, and the sum of the components determines the sign of the resultant coupling impedance.

It has been shown that there are eight possible combinations of positive or negative coupling impedances in the equivalent circuit, four of which represent the conditions described in the problem in Case I and the other four conditions in the problem in Case II. A proof of this statement can be given in either case of the three-circuit transformer for any condition that may exist as follows:

Consider the equivalent circuits h and g shown in the diagrams in Figs. 7 and 8. The circuit in Fig. 7 represents conditions in the transformer as expressed by the following equations,

$$E_1 + Z_{12} I_2 + Z_{13} I_3 = Z_1 I_1 \tag{26}$$

$$E_2 + Z_{12} I_1 - Z_{23} I_3 = Z_2 I_2 \tag{27}$$

$$E_3 + Z_{13} I_1 - Z_{23} I_2 = Z_3 I_3 \tag{28}$$

and the circuit in Fig. 8 expresses the conditions in the transformer as described by the equations,

$$E_1 + Z_{12} I_2 - Z_{13} I_3 = Z_1 I_1 \tag{29}$$

$$E_2 + Z_{12} I_1 + Z_{23} I_3 = Z_2 I_2 \tag{30}$$

$$E_3 - Z_{13} I_1 + Z_{23} I_2 = Z_3 I_3 \tag{31}$$

In these equations it has been assumed that there may be an a-c. potential applied in each of the three circuits.

By examining the equations it will be seen that the expressions become identical if the sign of the current I_3 and the applied potential E_3 are reversed in either set of equations. Both sets of equations as they are written give the same result in that they accurately describe the magnitude and phase of the currents flowing in the transformer. The difference in the equations is due to the fact that they are based on a different assumption as to the zero phase direction of the current I_3 and the potential E_3 which can be seen by referring to the diagrams in Fig. 11. In the same manner the other two equivalent circuits e and f in Case II can be shown to be equivalent to the circuits in Figs. 7 and 8 and it can also be proven that the four equivalent circuits a, b, c and d in Case I are equivalent to each other and equivalent to the transformer.

A three-circuit transformer when separated from its external loads may have one side of each of the three circuits connected to a common terminal in which case it can be treated as a network having four external terminals. It is well known that such a network cannot be represented theoretically by an equivalent circuit having less than six impedances. The equivalent circuits that have been given cannot be reduced to a circuit having a smaller number of impedances unless it is found to be permissible in some cases to make assumptions that will simplify the problem.

There are equivalent circuits used in the design of power transformers and autotransformers which have been derived by an entirely different method of reasoning by Boyajian[4] and MacLeod[5] on the assumption that the magnetizing currents are negligible or that the mutual impedances are infinite as compared with other impedances in the circuit. Some of these equivalent circuits contain only three impedances and are claimed to give very satisfactory results in solving three-circuit transformer problems that arise in power work.

The present investigation of the theory of the three-circuit transformer was undertaken in connection with a study of the a-c. intermittent train control system of the type installed on the Chicago, Indianapolis and Louisville Railway.

Numerical Example

Consider three coils each mounted on a laminated iron core and arranged as shown in the sketch in Fig. 12 and assume an a-c. potential of 16 volts at a frequency of 360 cycles per second applied to the primary circuit. An ammeter (I_1) and a condenser C_1 are connected in series with the generator in the primary, a load consist-

3. "The Magnetic Circuit," by V. Karapetoff, Fourth Edition, published by McGraw-Hill Book Co., page 9 and Article 58.

4. *Theory of Three-Circuit Transformers*, by A. Boyajian, A. I. E. E. Trans., 1924, p. 508.

"New Theory of Transformers and Autotransformer Circuits," by A. Boyajian, *General Electric Review*, February 1929, p. 110.

5. "New Equivalent Circuits for Autotransformers and Transformers with Tapped Secondaries," by D. R. MacLeod, *General Electric Review*, February 1929, p. 120.

ing of an ammeter (I_2) and condenser C_2 in the secondary and a load connected in the tertiary consisting of the ammeter (I_3). It is required to compute the impedances in the equivalent circuit and the current flowing in each of the coils. The arrangement of the circuit is the same as shown schematically in Fig. 3 and the currents are assumed to flow in the directions indicated by the arrows. The phase relations of the currents in the equivalent circuit have been computed with reference to the phase of the voltage of the generator as standard phase. The phase angles of the currents I_2 and I_3 as determined by calculation are interpreted to mean their phase relative to the zero phase directions shown in Fig. 12 at the instant the voltage of the generator is a maximum in the direction indicated. The condition described is a problem in Case I in which the mutual impedances in the equivalent circuit are all positive impedances and are arranged as shown in the diagram in Fig. 13.

Impedance of Coils.

Primary. $z_1 = 309^{\omega}$ $r_1 = 13.02^{\omega}$ $x_1 = 308.7^{\omega}$ $L_1 = 136.5$ mh. phase angle 87 deg. 35 min., power factor 0.0421, d-c. resistance 9.15^{ω}, iron loss resistance 3.87^{ω}.

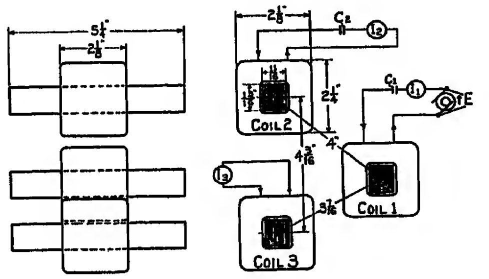

Fig. 12—Arrangement of Three Coupled Circuits Explained in the Numerical Example

Winding. 900 turns, No. 23 A. W. G., s. c. c. copper wire.

Secondary. $z_2 = 233^{\omega}$ $r_2 = 10.96^{\omega}$ $x_2 = 232.8^{\omega}$ $L_2 = 103.0$ mh. phase angle 87 deg. 18 min., power factor 0.0470, d-c. resistance 6.32^{ω}, iron loss resistance 4.64^{ω}.

Winding. 800 turns, No. 22 A. W. G., s. c. c. copper wire.

Tertiary. $z_3 = 182^{\omega}$ $r_3 = 10.8^{\omega}$ $x_3 = 181.6^{\omega}$ $L_3 = 80.3$ mh. phase angle 86 deg. 36 min., power factor 0.0594, d-c. resistance 5.2^{ω}, iron loss resistance 5.6^{ω}.

Winding. 700 turns, No. 22 A. W. G., s. c. c. copper wire.

Impedance of Loads.

Primary. Ammeter resistance 1.18^{ω}, condenser $C_1 = 1.633$ μf., $X_{C1} = 270^{\omega}$.

Secondary. Ammeter resistance 2.37^{ω}, condenser $C_2 = 2.00$ μf., $X_{C2} = 221^{\omega}$.

Tertiary. Ammeter resistance 9.6^{ω}.

Total Impedances.

The total impedance in each circuit consists of the sum of the impedance of the coil and the external load in the circuit. The values are calculated to be as follows,

$$Z_1 = 14.2 + j\,38.7 = 41.233 \angle 69° 51' \text{ ohms}$$
$$Z_2 = 13.33 + j\,11.8 = 17.809 \angle 41° 32' \text{ ohms}$$
$$Z_3 = 20.4 + j\,181.6 = 182.742 \angle 83° 35' \text{ ohms}$$

Mutual Reactances.

The mutual reactances between the circuits have been

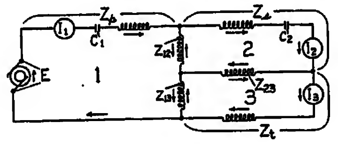

Fig. 13—Equivalent Circuit Described in Numerical Example

determined from tests that were made on the coils when arranged as shown in the sketch in Fig. 12. The results of the tests are:

$$X_{12} = 2\,\pi\,f\,M_{12} = 15.74 \text{ ohms}, \quad M_{12} = 6.96 \text{ mh.}$$
$$X_{13} = 2\,\pi\,f\,M_{13} = 24.6 \text{ ohms}, \quad M_{13} = 10.88 \text{ mh.}$$
$$X_{23} = 2\,\pi\,f\,M_{23} = 7.02 \text{ ohms}, \quad M_{23} = 3.11 \text{ mh.}$$

Mutual Resistances.

The resistive components of the mutual impedances, designated R_{12}, R_{13} and R_{23}, have been separated from the iron loss resistance in the coils in each circuit by assuming that the resistances vary in proportion to the corresponding reactances. The resistance $R_{12} + R_{13}$ has been separated from the iron loss resistance in circuit 1, the resistance $R_{12} + R_{23}$ from the iron loss resistance in circuit 2 and the resistance $R_{13} + R_{23}$ from the iron loss resistance in circuit 3. From the values of these re-

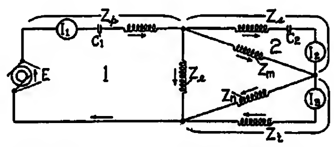

Fig. 14—A Form of Circuit which is Equivalent to the Circuit Shown in Fig. 13

sistances the resistance $R_{12} + R_{13} + R_{23}$ has been obtained. The latter resistance is then separated into the resistances R_{12}, R_{13} and R_{23}, the values of which have been determined to be as follows:

$$R_{12} = 0.322^{\omega} \qquad R_{13} = 0.502^{\omega} \qquad R_{23} = 0.143^{\omega}$$

Mutual Impedances.

The resistive and reactive components of the mutual impedances have been determined and from these values the mutual impedances may be given:

$$Z_{12} = 0.322 + j\,15.74 = 15.744 \angle 88° 50' \text{ ohms}$$
$$Z_{13} = 0.502 + j\,24.6 = 24.605 \angle 88° 50' \text{ ohms}$$
$$Z_{23} = 0.143 + j\,7.02 = 7.021 \angle 88° 50' \text{ ohms}$$

Series Impedances.

The series impedances Z_p, Z_s and Z_t have been determined from the equations:

$$Z_p = Z_1 - Z_{12} - Z_{13}$$
$$Z_s = Z_2 - Z_{12} - Z_{23}$$
$$Z_t = Z_3 - Z_{13} - Z_{23}$$

Numerical values computed from the equations are:

$Z_p = 13.376 - j\ 1.64 = 13.476 \measuredangle\ 69° 59'$ ohms

$Z_s = 12.865 - j\ 10.96 = 16.901 \measuredangle\ 40° 26'$ ohms

$Z_t = 19.755 + j\ 149.98 = 151.375 \angle\ 82° 30'$ ohms

Δ *Impedances.*

The mutual impedances shown in Fig. 13 are connected in the form of a Y and may be replaced by the Δ combination consisting of impedances Z_l, Z_m and Z_n arranged as shown in the circuit in Fig. 14 by employing the well-known equations,

$$Z_l = Z_{12} + Z_{13} + \frac{Z_{12}\, Z_{13}}{Z_{23}}$$

$$Z_m = Z_{12} + Z_{23} + \frac{Z_{12}\, Z_{23}}{Z_{13}}$$

$$Z_n = Z_{13} + Z_{23} + \frac{Z_{13}\, Z_{23}}{Z_{12}}$$

Numerical values computed from the equations are:

$Z_l = 1.943 + j\ 95.509 = 95.539 \angle\ 88° 50'$ ohms

$Z_m = 0.556 + j\ 27.252 = 27.258 \angle\ 88° 50'$ ohms

$Z_n = 0.869 + j\ 42.591 = 42.601 \angle\ 88° 50'$ ohms

The equivalent circuit in Fig. 14 has been reduced to a simpler form of circuit consisting of a series and multiple arrangement of impedances from which the currents in the various branches of the circuit can be easily computed. The required values of current I_1, I_2 and I_3 flowing in the three coils in the circuit have been computed and found to be:

$$I_1 = 0\ 445 \measuredangle\ 43° 29' \text{ ampere}$$
$$I_2 = 0.421 \angle\ 4° 50' \text{ ampere}$$
$$I_3 = 0\ 0718 \measuredangle\ 28° 32' \text{ ampere}$$

The values of the currents I_1, I_2 and I_3 when substituted in equations (6), (7) and (8), together with numerical values of the applied voltage and impedances in the transformer, will be found to satisfy the equations. This proves that the results of calculations made from the equivalent circuit are the same as might have been obtained by a solution of the original equations.

Equivalent Circuits of Imperfect Condensers

BY C. L. DAWES* and W. M. GOODHUE†
Member, A. I. E. E. Non-member

Introduction

THE plates of commercial condensers must necessarily be held in mechanical separation by some substance which is a dielectric. Since such substances cannot be perfect dielectrics, the condensers when connected across alternating voltage must absorb power. It is customary to simulate the electrical performance of such condensers, either by an equivalent series circuit consisting of a perfect condenser C_s and a resistance R_s, Fig. 1; or by an equivalent parallel circuit consisting of a perfect condenser C_p in parallel with a resistance R_p, Fig. 2. In the study of dielectric phenomena both methods are employed. Sometimes with the equivalent parallel circuit, a conductance $G_p = 1/R_p$ is used rather than the parallel resistance R_p, since such conductance frequently gives results which are more convenient in form than those given by resistance.

It frequently happens that equivalent series and parallel capacitances and equivalent series and parallel resistances are used indiscriminately without designation as to whether the equivalent series or equivalent parallel circuit is intended. With usual types of condensers this ordinarily does not cause confusion, since the power factor of most condensers is comparatively low. Under these conditions, the equivalent series capacitance C_s is almost equal in magnitude to the equivalent parallel capacitance C_p, so that it is not necessary to make designation; the equivalent parallel resistance R_p is so large in magnitude as compared with the equivalent series resistance R_s that there is little doubt as to which is intended. However, as the study of dielectric and ionization phenomena progresses, it is becoming

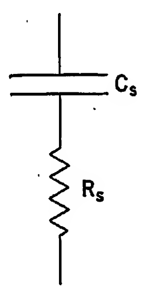

Fig. 1—Equivalent Series Circuit

more common to find that the equivalent series resistance and the equivalent parallel resistance (see Fig. 9) may be of nearly the same order of magnitude; also the equivalent series capacitance may be quite different in magnitude from the equivalent parallel capacitance. (See Fig. 11.) These conditions occur particularly in ionized gases whose power factor is ordinarily high. Therefore, it becomes important not only to discriminate between equivalent series and parallel circuits, but

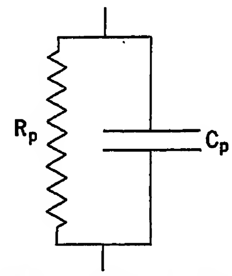

Fig. 2—Equivalent Parallel Circuit

also to investigate the relations existing among their various parameters.

Relationships Among Equivalent-Circuit Parameters

The relationships existing among the parameters of equivalent series and parallel circuits are well known and are as follows:

$$C_p = \frac{C_s}{1 + R_s^2 C_s^2 \omega^2} \tag{1}$$

$$R_p = R_s \frac{1 + R_s^2 C_s^2 \omega^2}{R_s^2 C_s^2 \omega^2} \tag{2}$$

where ω is $2 \pi f$, f being the frequency. The other quantities are given in Figs. 1 and 2.

The power factor of the condenser must be the same whether an equivalent series or an equivalent parallel circuit is considered. In terms of equivalent series-circuit parameters

$$(\text{Power factor}) = \frac{R_s}{\sqrt{R_s^2 + \dfrac{1}{C_s^2 \omega^2}}} = \frac{R_s C_s \omega}{\sqrt{R_s^2 C_s^2 \omega^2 + 1}} \tag{3}$$

From (1), (2) and (3)

$$R_p = R_s \frac{1}{(\text{P. F.})^2} \tag{4}$$

$$C_p = \frac{C_s (\text{P. F.})^2}{R_s^2 C_s^2 \omega^2} \tag{5}$$

If $R_s C_s \omega = \eta$

Then $$R_p = R_s \frac{1 + \eta^2}{\eta^2} \tag{6}$$

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$$C_p = \frac{C_s}{1 + \eta^2} = \frac{C_s\,(\text{P. F.})^2}{\eta^2} \qquad (7)$$

The power factor in terms of parallel parameters

$$\text{Power factor} = \frac{1}{\sqrt{1 + R_p^2\, C_p^2\, \omega^2}} = \frac{1}{\sqrt{1 + \delta^2}} \qquad (8)$$

where $\delta = R_p\, C_p\, \omega = \tan\theta = \cot\psi$. θ is the power-factor angle of the condenser; ψ is the angle of defect.

Consider first an impregnated-paper, high-voltage, underground cable having good dielectric characteristics. Fig. 3 shows the power-factor, the power and the capacitance curves taken at 20 deg. cent. of a typical cable of this character. (300,000 cir. mils, 6/32 in. (0.476) cm. wall.) It is to be noted that there is no evidence of gaseous ionization in this cable, since the power-factor and the capacitance characteristics are both linear and horizontal. The equivalent series resistance per foot R_s of this cable is found to be constant

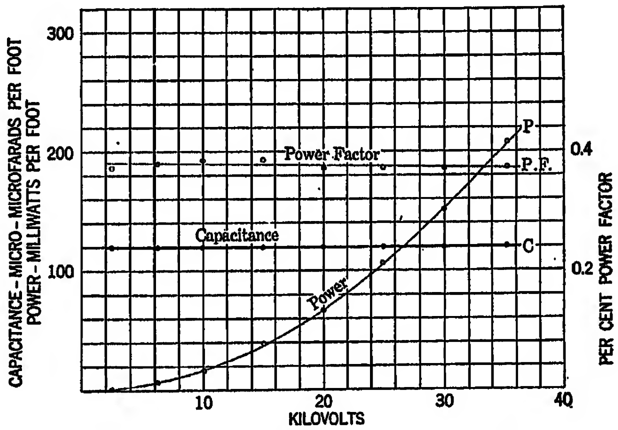

FIG. 3—PAPER CABLE CHARACTERISTICS WITHOUT IONIZATION

300,000 cir. mils, 6/32 in. wall, impregnated paper

at all voltages, up to 40 kv. at least, and is equal to 8.25×10^4 ohms. Likewise, the equivalent series capacitance per foot C_s is found to be practically constant and equal to 119.8 $\mu\mu$f. From equation (2) the equivalent parallel resistance per foot is found to be

$$R_p = 8.25 \times 10^4 \left(\frac{1 + 1.388 \times 10^{-5}}{1.388 \times 10^{-5}} \right)$$

$$= 5.96 \times 10^9 \text{ ohms.}$$

It is to be noted that with this dielectric whose power factor is low, η^2 (equation (6)) is small compared with unity and

$$R_p = \frac{1}{R_s\, C_s^2\, \omega^2} \quad \text{approximately.} \qquad (6a)$$

The equivalent parallel capacitance per foot

$$C_p = \frac{119.8 \times 10^{-12}}{1 + 1.388 \times 10^{-5}} = 119.8\ \mu\mu\text{f.}$$

which is substantially equal to the equivalent series capacitance per foot.

Thus with dielectrics of very low power factor the equivalent parallel resistance is practically equal to the equivalent series resistance multipled by $1/\eta^2$, where $\eta = R_s\, C_s\, \omega$; the equivalent series and parallel capacitances are practically equal. Hence, with dielectrics of very low power factor, the capacitance as a rule may be given without designating whether equivalent series or equivalent parallel capacitance is meant.

After this same cable (Fig. 3) had undergone 150 hours of accelerated life test, at an average voltage gradient of 225 volts per mil, and under weekly cycles of both voltage-gradient variation (from zero to 225 volts per mil) and temperature variation (from 20 deg. to 65 deg. cent.) both its power factor and capacitance increased.

At 30 kilovolts and 63 deg. cent., the power factor is now 0.1209 and the measured equivalent series capacitance is 147 $\mu\mu$f. per foot. The equivalent series resistance per foot of this cable is now found to be 2.20×10^6 ohms.

The equivalent parallel resistance per foot from equation (2) is now

$$R_p = 2.20 \times 10^6 \left(\frac{1 + 0.01483}{0.01483} \right) = 1.506 \times 10^8$$

ohms, as compared with 5.96×10^9 ohms, its former value. It is to be noted in the first instance, when the power factor of this cable is 0.00373, that η^2 is entirely negligible in comparison with unity; but with the power factor equal to 0.1209, the value of η^2 is just becoming appreciable in comparison with unity, its value being approximately 0.015.

The equivalent parallel capacitance now becomes, from equation (1),

$$C_p = \frac{147.0}{1 + 0.01483} = 145.0\ \mu\mu\text{f.}$$

Likewise the equivalent parallel capacitance now differs from the equivalent series capacitance by 1.5 per cent. The power factor 0.1209 is large for dielectrics, but even so the difference between equivalent series and parallel capacitance is almost negligible.

Hence, with most of the usual dielectrics, the difference between the equivalent series capacitance and the equivalent parallel capacitance is small and generally may be neglected.

COMPOSITE SOLID AND GASEOUS DIELECTRICS

When a dielectric circuit consists of a solid dielectric in series with a film of gas, as a rule it behaves like a simple invariable dielectric up to the ionization voltage of the gas, the only loss being that in the solid dielectric. When the gas becomes ionized, however, its losses become relatively large and its electrical parameters such as capacitance, permittivity, equivalent series and parallel resistance vary with the current density, the functions usually being non-linear. The actual mechanism of the ionization and its physical interpretation

are not simple and need not be considered at this time. Moreover, the authors know of no combination of circuit parameters which can simulate the electrical behavior of such films under all conditions. However, at any single value of current density, the electrical behavior, so far as the fundamental components of the voltage and current waves are concerned, can be simulated either by suitable resistance and capacitance in series, or by suitable resistance and capacitance in parallel. This method is used in this paper.

It has been found experimentally that the power loss

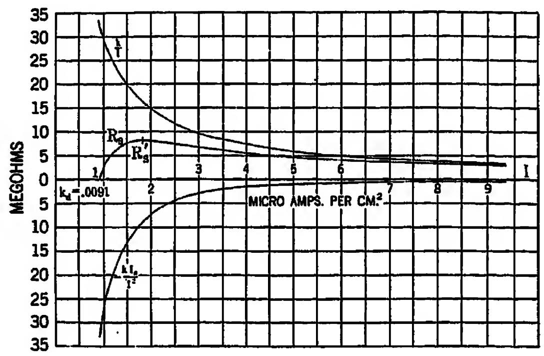

Fig. 4—Equivalent Series Resistance of Glass-Ionized Air Film Model, with 2.17 Mm. Air Gap, at 60 Cycles, 23 Deg. Cent., and 764 Mm. Pressure

in ionized gas films is a simple linear function of the current density.

For example, let the value of current at which ionization begins be I_o. Since the power P increases as a linear function of the current density after ionization begins, its equation becomes

$$P_i = k\,(I - I_o) \qquad (9)$$

where k is a constant.

Neglecting harmonics, at room temperature the power in the solid dielectric varies as the current squared and the total power in the composite dielectric becomes

$$P = k_d\,I^2 + k\,(I - I_o) \qquad (10)$$

Since no ionization loss occurs until $I > I_o$, the second term of equation (10) vanishes for all values of $I \leq I_o$. Hence it follows that the first derivative of the function is discontinuous at the point, $I = I_o$.

If R_s is the equivalent series resistance of the composite dielectric,

$$R_s = \frac{P}{I^2} = k_d + \frac{k}{I} - \frac{k\,I_o}{I^2} \qquad (11)$$

Equation (11) is composed of three simple functions, a linear function whose graph is parallel to the current axis, a rectangular hyperbola in the first quadrant and an "inverse square" function in the fourth quadrant. Fig. 4 shows this R_s-function and its components for 2.17-mm. film of dry air at 23 deg. cent. a barometer pressure of 76.2 cm. and a frequency of 60 cycles per second. The value of k_d is so small, being only 0.0091, that it cannot be shown with the scale used in Fig. 4.

It is to be noted that R_s rises rapidly to a maximum and then decreases. By differentiating (11) with respect to the current and equating the first differential coefficient to zero, it is found that the maximum value R_s' of R_s occurs when

$$I = 2\,I_o \qquad (12)$$

and

$$R_s' = k_d + \frac{k}{4\,I_o} \qquad (13)$$

Due to occluded gases, the dielectric of high-voltage cables is a composite one of solid and gaseous dielectric, the gaseous dielectric becoming ionized after the voltage becomes sufficiently large.

The characteristics of high-voltage cables are now generally given by power-factor, power and capacitance curves as in Fig. 3. These three characteristics for a cable having negligible ionization are shown in Fig. 3. When considerable ionization exists in a cable, however, the power-factor, power and capacitance characteristics have the general appearance of the curves given in Fig. 5, which are the characteristics of a 300,000 cir.-mil 6/32-in. (0.476-cm.) wall, impregnated-paper cable. Both the power-factor and capacitance curves have a discontinuity in their slopes at the ionization voltage of the cables, and then rise rapidly with further increase in voltage. If the voltage is carried to a sufficiently high value, the power-factor curve will rise to a maximum and then decrease; the capacitance curve, however, increases indefinitely.

These three characteristic cable curves of power factor, power and capacitance are obtained either

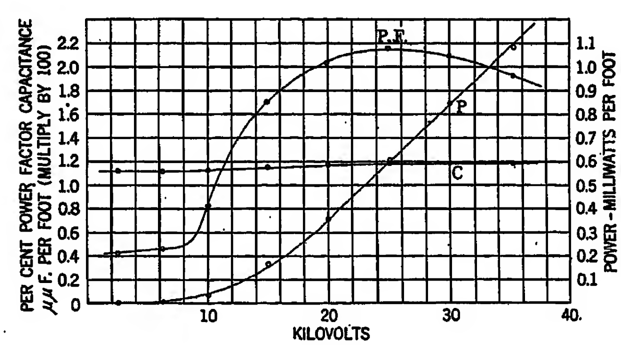

Fig. 5—Paper Cable Characteristics with Ionization
300,000 cir. mils, 6/32 in. wall, impregnated paper

directly or are determined with little computation from data obtained from the methods of bridge measurement now in common use. The bridges which are now commonly used to measure the characteristics of dielectrics at high voltage are the Heaviside type used by the authors and the Schering type.[1] These give directly the equivalent series capacitance. In fact, most types of capacitance bridges give directly the equivalent series capacitance.[2]

Clark and Shanklin,[4] who were among the pioneers in

1. For references see Bibliography.

the study of ionization in high-voltage cables, measured the power taken by cables with a compensated electrodynamometer and in their publications used equivalent parallel resistance as a distinctive cable characteristic. At room temperature, the equivalent parallel resistance of cables is found to remain substantially constant until after ionization begins; then a discontinuity occurs in the slope of the curve, after which the equivalent parallel resistance decreases, at first rapidly and then at a decreasing rate until it reaches a minimum. Fig. 6 shows a typical curve of this character

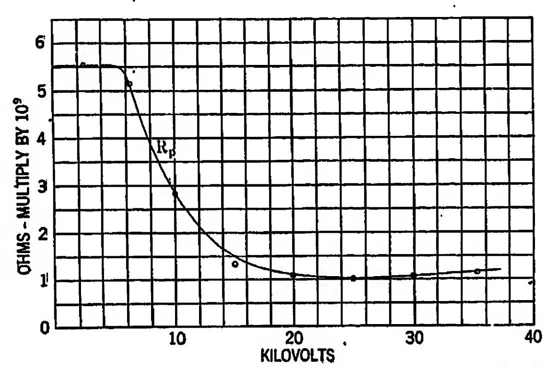

FIG. 6—EQUIVALENT PARALLEL RESISTANCE OF 300,000 CIR. MILS, 6/32 IN. WALL, IMPREGNATED PAPER CABLE

for the cable whose characteristics are given in Fig. 5. This equivalent parallel resistance curve is in distinction to the power-factor curve, which also has a discontinuity in its slope at the ionization voltage, but thereafter shows increasing rather than decreasing values and reaches a maximum, rather than a minimum.

With our present knowledge of ionization phenomena, it is now possible to derive an analytical expression for this equivalent parallel resistance characteristic. The power due to the combined solid-dielectric and ionization power loss is given by

$$P = K E^2 + K_1 (E - E_o) \qquad (14)$$

where K is the solid-dielectric power coefficient, K_1 is the ionization-power coefficient and E_o is the intercept of the ionization-power curve with the E-axis. The second term of equation (14) is zero when $E = E_o$. Moreover, in a cable the gas films do not all ionize simultaneously as does a single uniform flat gas film, (Fig. 4). They ionize successively outwards as the voltage increases.[5] Equation (14) therefore, does not apply during this interval of successive ionization, that is between the values $E = E_o$ and $E = E_1$ where E_o is the voltage at which ionization just begins in a cable and E_1 is the voltage at which the ionization power curve becomes a straight line.

The equivalent parallel resistance

$$R_p = \frac{E^2}{P} = \frac{E^2}{K E^2 + K_1 (E - E_o)} \qquad (15)$$

Although both the numerator and denominator of (15) are simple algebraic expressions, the equation as a whole does not represent any simple curve or combination of simple curves. This is evident from a study of Fig. 6. However, if the equivalent parallel conductance G_p be used, an equation is obtained which may be analyzed into three terms representing three simple graphic functions. That is,

$$G_p = \frac{P}{E^2} = K + \frac{K_1}{E} - \frac{K_1 E_o}{E^2} \qquad (16)$$

The first term K of (16) depends on the properties of the solid dielectric and is a straight line parallel to the voltage axis; the second term is a rectangular hyperbola lying in the first quadrant and the third term is an inverse square function lying in the fourth quadrant. Hence, this function is very similar to (11). Each term of (16) is plotted separately in Fig. 7 for the cable whose characteristics are given in Fig. 6; also the resultant characteristic G_p is plotted, being obtained by combining these three component curves.

In the region of successive ionization, where (16) does not apply, the computed curve is shown dotted. Except in the region of successive ionization, the actual G_p-curve obtained experimentally, and shown solid, coincides with the curve obtained either by combining the component curves or by computation from (16).

It is shown in the Bibliography 5, equation (12) that

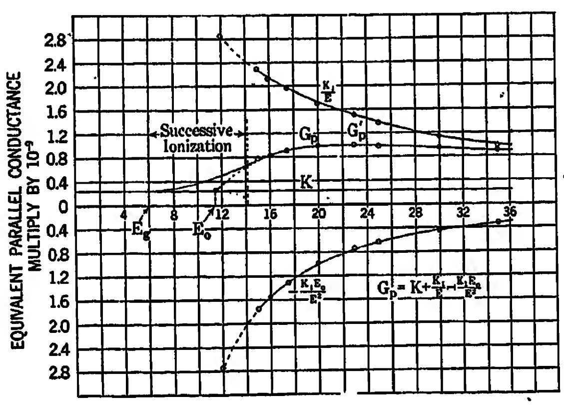

FIG. 7—EQUIVALENT PARALLEL CONDUCTANCE OF 300,000 CIR. MILS, 6/32 IN. WALL, SINGLE-CONDUCTOR CABLE

the power factor of a composite dielectric consisting of a solid dielectric and a gaseous medium is approximately given by $\frac{1}{C\,\omega}\left(K + \frac{K_1}{E} - \frac{K_1 E_o}{E^2}\right)$ when the power factor is not large. C is either the equivalent series or parallel capacitance, since the two are nearly equal when the power factor is not large; ω is $2\,\pi$ times the frequency.

Hence equation (16) is identical in form with the equation for power factor, differing only by the coefficient $\frac{1}{C\,\omega}$.

By finding the first differential coefficient of either (15) or (16) with respect to E and equating to zero, the minimum value of R_p and the maximum value of G_p occurs when

$$E = 2\,E_o \tag{17}$$

From, (17) and (16) the maximum value of G_p

$$G_p' = K + \frac{K_1}{4\,E_o} \tag{18}$$

(See Bibliography 6, equation (5) .)

This relationship is illustrated in Fig. 7, where E_o equals 11.5 kv. and the maximum value of G_p, G_p', occurs at 23 kv.

For cables having ionization (or other dielectrics having ionization) it also becomes possible to express analytically the equivalent series resistance as a function of voltage. The power

$$P = I^2\,R_s = K\,E^2 + K_1\,(E - E_o)$$

Hence

$$R_s = \frac{\{\,K\,E^2 + K_1\,(E - E_o)\,\}\ \ (R_s^2\,C_s^2\,\omega^2 + 1)}{E^2\,C^2\,\omega^2}$$

If the power factor of the cable or similar dielectric is

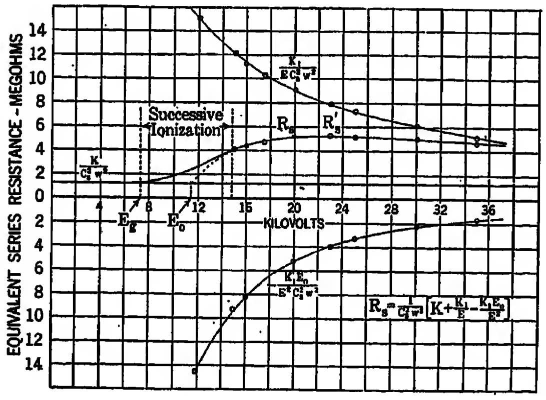

FIG. 8—EQUIVALENT SERIES RESISTANCE OF 300,000 CIR. MILS, 6/32 IN. WALL, SINGLE-CONDUCTOR CABLE

not excessive, $R_s^2\,C_s^2\,\omega^2$ is negligible compared with unity and

$$R_s = \frac{K}{C_s^2\,\omega^2} + \frac{K_1}{E\,C_s^2\,\omega^2} - \frac{K_1\,E_o}{E^2\,C_s^2\,\omega^2} \text{ approximately} \tag{19}$$

If the variation of capacitance with voltage, which in cables is usually small, (see Fig. 5) be neglected, the equivalent-series-resistance characteristic consists of a linear function whose graph is parallel to the E-axis, a rectangular hyperbola in the first quadrant and an inverse square function lying in the fourth quadrant. The equivalent-series-resistance function and its three component curves are shown graphically in Fig. 8 for the same cable whose equivalent parallel resistance and conductance are given in Figs. 6 and 7. This function is similar in character to the equivalent parallel conductance function (equation (16) and Fig. 7) and the power-factor function.[5,6] The maximum value of R_s occurs when $E = 2\,E_o$ and its value is

$$R_s' = \frac{1}{C^2\,\omega^2}\left(K + \frac{K_1}{4\,E_o}\right) \tag{20}$$

It appears, therefore, that many functions relating to ionization phenomena are functions of the character of (11) and (14).

There also follows another interesting relationship when the power factor is not large and the change in capacitance is small.

From (11) and (16)

$$R_s = \frac{P}{I^2} = \frac{P}{E^2\,C^2\,\omega^2} \text{ (approximately) and } G_p = \frac{P}{E^2}$$

Hence

$$R_s = \frac{G_p}{C^2\,\omega^2} \text{ (approximately)}$$

Thus, at constant frequency, if the change in capacitance is small, the R_s-function differs from the G_p-function by a constant quantity only. That this is true may be seen by a study of Fig. 7 which shows the R_s function for the same cable.

EQUIVALENT CIRCUITS OF IONIZED GASES

At any single value of current density and at the fundamental frequency, ionized gaseous media, when considered alone, behave like very leaky condensers. They may therefore be represented by equivalent series and parallel parameters. But it is found that these parameters are quite divergent from those obtained with the usual dielectrics. Furthermore, they are much more difficult to evaluate from measurement data than the dielectrics which have just been considered. For example, when gaseous films become ionized, a relatively large energy loss results and this loss increases as a linear function of the current density, as has already been stated. Also, because of the effects of space charge, the measured capacitance increases with increasing current density and may reach values which are several times greater than the non-ionized value. Hence, the apparent dielectric constant of the ionized gas film will ordinarily have values which are several times the non-ionized value of unity.

These effects cause the power factor to reach values which are much higher than the values ordinarily met in composite solid and ionized dielectrics such as cables, (see Fig. 11). Hence, the approximations which are frequently made with dielectrics, such as assuming that the equivalent series and the equivalent parallel capacitances are substantially equal, are no longer valid.

In Fig. 9 are shown the equivalent series resistance and the equivalent parallel resistance of a 1.53-mm. air film as a function of current density, the frequency being 60 cycles and the pressure 764 mm. The equivalent series resistance obviously is zero until ionization begins. It then rises rapidly to a maximum after which

it decreases, its graph approaching a horizontal line as an asymptote. The analytical function giving the variation of equivalent series resistance R_s with current density is readily derived from (9)

$$I^2 R_s = P = k\,(I - I_o) \tag{22}$$

$$R_s = \frac{k\,(I - I_o)}{I^2} = \frac{k}{I} - \frac{k\,I_o}{I^2} \tag{23}$$

The first term in (23) is the equation of the well-known rectangular hyperbola and the second term is an inverse square function lying in the fourth quadrant. Each component function for the 1.53-mm. air film of Fig. 9 is shown in Fig. 10. The resultant curve R_s is the equivalent series resistance, which has a maximum value and is similar to the R_s-curve, Fig. 9.

The maximum value is readily found by differen-

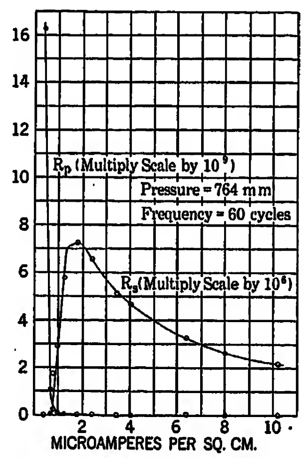

FIG. 9—EQUIVALENT SERIES AND PARALLEL RESISTANCE OF 1.53 MM. IONIZED AIR FILM

tiating (23) with respect to I and equating the first differential coefficient to zero. For the maximum value R_s' of R_s,

$$I = 2\,I_o \tag{24}$$

From (23) and (24)

$$R_s' = \frac{k}{4\,I_o} \tag{25}$$

(See Fig. 23, Bibliography 5 and equation (4) Bibliography 6.)

These relationships are very similar to those for the power-factor, the equivalent parallel conductance, and the equivalent series resistance characteristics of a cable in which ionization exists. (Figs. 5, 7, and 8.)

Fig. 11 shows graphically the equivalent series capacitance, and the equivalent parallel capacitance per cm. cube (or permittivity) of a 1.59-mm. air film as functions of current density. Also the power-factor function is given. It will be noted that the power factor of such a film increases up to nearly 90 per cent, which is a very much higher value than occurs even with composite solid and ionized dielectrics, such as impregnated-paper cables. (See Fig. 5.) Unlike the power-factor curve for cables having ionization (Fig. 5), the power-factor curves which we have so far obtained experimentally for gas films have not shown a maximum, but continue to increase with the current density.

The equivalent series permittivity at the maximum current density of 11.5 microamperes per sq. cm. obtained and shown in Fig. 11 has a value of 24, which illustrates the enormous changes in capacitance which

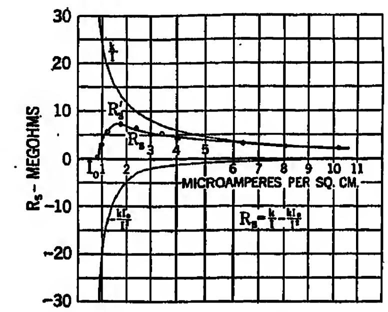

FIG. 10—ANALYSIS OF EQUIVALENT SERIES RESISTANCE FUNCTION OF IONIZED AIR FILM

gas films undergo when ionized. Furthermore, instead of being approximately equal to the equivalent parallel capacitance, the equivalent series capacitance reaches a value at the maximum current density shown in Fig. 11 which is approximately six times the value of the equivalent parallel capacitance. Hence, when analyzing capacitance relations in ionized gas films, it becomes important to specify whether the equivalent series or equivalent parallel capacitance is intended.

It does not appear possible at this time to derive from

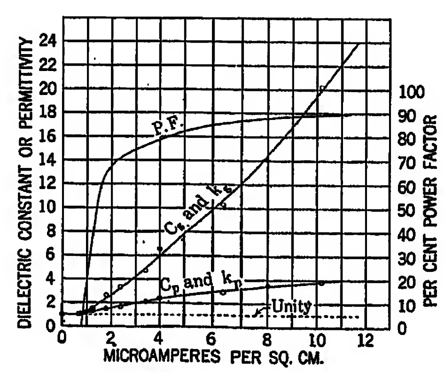

FIG. 11—EQUIVALENT SERIES AND PARALLEL CAPACITANCE OF IONIZED AIR FILM

fundamental considerations any simple equations which give the equivalent series capacitance, and the equivalent parallel capacitance characteristics shown in Fig. 11.

For example, with the equivalent series circuit

$$I = \frac{E}{\sqrt{R_s^2 + \frac{1}{(C_s\,\omega)^2}}} = \frac{E\,C_s\,\omega}{\sqrt{R_s^2 C_s^2\ \omega^2 + 1}} \tag{26}$$

$$C_s = \frac{1}{\omega}\ \frac{I}{\sqrt{E^2 - (I\,R_s)^2}} \tag{27}$$

From (22), $P = I^2 R_s = k (I - I_o)$; substituting for R_s in (27)

$$C_s = \frac{1}{\omega} \frac{I}{\sqrt{E^2 - \frac{k^2}{I^2} (I - I_o)^2}}$$

$$= \frac{I^2}{\omega \sqrt{(E\,I)^2 - k^2 (I - I_o)^2}} \qquad (28)$$

This is not a simple function and it cannot be readily separated into simple component curves. Moreover, no approximate algebraic relationship even between E and I has yet been found. We do know that E increases to a maximum, and then decreases (see Fig. 10, Bibliography 6). However, when equation (28) is plotted as in Fig. 11 (which gives permittivity rather than total capacitance) it is found that C_s increases at a slightly greater rate than the current density. A graphical analysis shows the equation of this curve to be

$$k_s = A + B (I - I_o)^{1.19} \qquad (29)$$

where k_s is the equivalent series permittivity and A and B are constants. Specifically for Fig. 11.

$$k_s = 1 + 1.28 (I - 0.8)^{1.19} \qquad (30)$$

Also, the equivalent parallel permittivity k_p is plotted as a function of current density in Fig. 11. Again the explicit function for this curve is far from being simple and it does not appear that the curve can be analyzed into simple fundamental curves. Also the equivalent series permittivity (or capacitance) characteristic concaves upwards: the equivalent parallel permittivity (or capacitance) characteristic concaves downwards.

From the foregoing relationships, it appears that as a rule equivalent series resistance gives the simple relationships when the current is the independent variable; equivalent parallel conductance and resistance give the simple relationships when the voltage is the independent variable.

Harmonics

It is well known that because of the variable character of the parameters of ionized gas films, a simple harmonic voltage wave cannot produce a simple harmonic current wave, and a simple harmonic current wave in such gas films cannot produce a simple harmonic voltage wave across the gas films. In practise, flat ionized gas films as a rule can exist only under the conditions of "restricted ionization." That is, the volt-ampere characteristics of the ionized gas film alone are highly unstable, and without a stabilizing impedance in series the cumulative reactions would cause a power arc to develop almost instantly. Solid dielectric in series with the gas film acts to stabilize the current and voltage across the film in the same manner as ballast stabilizes a power arc. If a simple harmonic voltage wave is impressed across the ionized gas film and solid dielectric in series, the current wave cannot be a simple harmonic wave because it is determined in part by the variable impedance of the ionized gas film. However, if the impedance of the solid dielectric is large in comparison with that of the ionized gas film, the current wave under these conditions may be nearly a sine wave. Under these conditions, the voltage across the ionized gas film cannot be a sine wave because of the variable character of the gas-film impedance.

Oscillograms show that with short air-gaps and comparatively low voltage, the current wave contains harmonics ranging from the lowest harmonics, well up through the audio-frequency range; at the higher voltages, and also with air films of greater thickness, the current wave contains harmonics from the lowest to probably well beyond the audio-frequency range. When such waves are studied with the usual Duddell string-type oscillograph vibrator, it is found that the harmonics near the resonant frequency of the vibrator cause it to vibrate with large amplitude at its own resonant frequency, thus accentuating on the oscillogram the harmonics of this particular frequency. Moreover, such vibrators cannot respond to the high frequencies

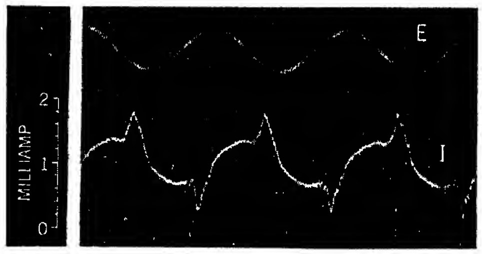

Fig. 12—Oscillogram of Ionization Current for Air Gap 1.59 Mm., Voltage 5,500, Frequency 60 Cycles

that some waves must contain. Hence, such oscillograms may not always give a true representation of the wave; they do, however, give useful information, in that they show changes in the wave shapes and in the lower frequency harmonics that ionization produces in such current waves under different conditions. As an example, the oscillogram* in Fig. 12 shows the voltage and current waves for a 1.59-mm. film of dry air, at 5,500 volts r. m. s. and at a frequency of 60 cycles per second. The voltage wave is not that across the gas film itself, but across the gas film and a slab of glass in series. However, the glass has a very high dielectric constant and very low power loss so that its impedance is small compared with that of the gas film. The voltage at which this oscillogram was taken is known to be well above the ionization voltage. It will be noted that the current increases almost sinusoidally with voltage until the voltage is sufficiently high to start ionization. At the instant of initial ionization, high-frequency harmonics occur in the current wave. These are im-

*A vacuum-tube amplifier of the type developed by S. K. Waldorf was used in obtaining this oscillogram. See Bibliography 7.

mediately followed by a sudden increase or large peak in the current wave.

This oscillogram shows that the harmonics of lower frequencies have the greatest effect on the wave distortion; also the wave has a considerable number of harmonics of high order. For more accurate reproduction of such waves, a cathode-ray type of oscillograph should be used. However, it is difficult to obtain sufficient current sensitivity with this type of oscillograph. Owing to the difficulty of making suitable potential connection across the gas film itself, oscillograms of this potential, so far as is known, have not as yet been obtained. (A complete series of oscillograms for different conditions of ionization will be published later.)

Definitions of Dielectrics with Ionization

The presence of the foregoing harmonics makes ambiguous many of the terms and definitions which are applicable to dielectrics that are practically invariable or in which the ionization is so small that it has but little influence on the wave shape. In view of the rapidly increasing study of gaseous ionization, it has become almost necessary to agree on some definitions of the parameters of ionized gas both when alone and when in combination with solid dielectrics.

One method is to base definitions on r. m. s. values. For example, if the voltage E and current I are measured or computed on a r. m. s. basis the equivalent series resistance $R_s = P/I^2$, where P is the power in watts; and the capacitance $C_s = \frac{1}{\omega} \frac{I}{\sqrt{E^2 - (I R_s)^2}}$.

These parameters might then be called the r. m. s. equivalent series resistance; the r. m. s. equivalent series capacitance; etc.

On the other hand, if the characteristics of the gas film are measured in a bridge having invariable parameters in three of its arms such as a standard air condenser and resistances, and a harmonic voltage is applied across the bridge, the bridge can be balanced with a detector, which responds only to the fundamental frequency. A vibration galvanometer is an example of such a detector. The parameters determined under such conditions might well be termed fundamental equivalent series resistance; fundamental equivalent series capacitance; etc.

It is conceivable that definitions depending on parameters other than the foregoing may also be evolved. The authors believe, however, in view of the magnitude and large number of harmonics which are present in the current wave, particularly, and the complications which these harmonics introduce, that for the present at least the more simple definitions of the parameters of ionized gas films based on fundamental frequencies should be used.

Conclusions

1. The electrical characteristics of imperfect condensers at any one frequency may be represented by resistance and capacitance in series, and by resistance and capacitance in parallel; definite algebraic relationships exist among the parameters of such equivalent series and parallel circuit.

2. Even with dielectrics having relatively high power factor, the equivalent series and parallel capacitances are substantially equal.

3. With composite dielectrics consisting of an invariable dielectric in series with ionized gas films, the equivalent series resistance characteristic and the equivalent parallel conductance characteristic are very similar to the power-factor characteristic provided the total change in capacitance is not large.

4. Ionized gas films have such high losses and high power factor that approximations which are applicable to ordinary dielectrics are not applicable to them.

5. With increasing knowledge of the laws of ionization, it is becoming possible to express series and parallel parameters of ionized gas films as explicit functions of known quantities.

6. The usual dielectric definitions are not applicable to ionized gas films, and to dielectrics containing such films; some appropriate definitions should be formulated; and it is recommended that these definitions be based on the first harmonic components of the voltage and current waves.

The authors are indebted to Professor H. E. Clifford, Dean of the Harvard Engineering School, for his helpful suggestions in the preparation of this paper.

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Discussion

M. G. Malti: Prof. Dawes and Mr. Goodhue have pointed out some very valuable and pertinent facts relative to the representation of imperfect condensers by equivalent circuits. Their treatment is complete and straightforward when, and if, a sine voltage, impressed on an imperfect condenser, produces a sine current. But the variation of the characteristics of an

imperfect condenser with impressed voltage is non-linear. Hence with a sine impressed voltage a non-sine current (Fig. 12 of the paper) is obtained. Moreover the distortion suffered by the current is a function of the nature of the dielectric used, its temperature, its humidity, the magnitude and wave form of the impressed e. m. f. and other factors. Under these conditions it becomes difficult to conceive of a series or a parallel circuit, consisting of a variable resistance and capacitance, which would simulate the behavior of an imperfect condenser. Thus such a circuit can not possibly give the e. m. f. and current oscillograms shown in Fig. 12 of the paper.

The writer is of the opinion that equivalent circuits should be altogether abandoned and the facts studied directly from the oscillograms. Thus Fig. 12 gives the e. m. f. and current curves of an imperfect condenser. The instantaneous power supplied to the condenser may be easily obtained by multiplying corresponding ordinates of e and i and plotting the products to a convenient scale. The average power dissipated in the condenser is the average ordinate of the instantaneous power curve. Here we have a means of determining the e. m. f. current, and power relations of an imperfect condenser. No mention is made of an equivalent series or parallel circuit because, physically, there is no such a thing. The imperfect condenser is an entity which behaves uniquely and cannot be imitated by an equivalent circuit.

C. L. Dawes: The statement of Prof. Malti that the variation of the characteristics of imperfect condensers is non-linear, in a strict sense, is undoubtedly true. That is, the absorption loss does not vary as the voltage squared and the conductance may vary with the voltage gradient. There are, however, many imperfect condensers in which the total loss as measured by precision methods is found to vary sensibly as the voltage squared. The dielectrics of such condensers must then have characteristics which are practically linear and they will not therefore produce any appreciable distortion in the current wave when a sine wave of voltage is impressed across them. For all practical purposes then it becomes possible to simulate such condensers either by an equivalent series circuit or by an equivalent parallel circuit. For example, we have found that cable paper when impregnated with the higher grades of cable-impregnating compounds appears to have characteristics at room temperature which are practically linear, provided all occluded gases are substantially removed.

It is true, however, as Prof. Malti points out, that it would be difficult or perhaps impossible to devise either an equivalent series circuit or an equivalent parallel circuit that would simulate under all conditions dielectrics having non-linear characteristics. The authors were well aware of this fact as is shown in the statement at the top of page 1032 inwhich they state that they "know of no combination of circuit parameters which can simulate the electrical behavior of such (gas) films under all conditions." However, in spite of the fact that such equivalent circuits cannot under all conditions simulate dielectrics having non-linear characteristics they frequently are very useful for purposes of analysis and I cannot agree with Prof. Malti that they should be abandoned entirely. For example a knowledge of the power loss in such dielectrics often is very important as with high-voltage cables. If the impressed voltage wave is sinusoidal the power loss is given by $E^2 G_p$ where E is the r. m. s. value of the impressed voltage and G_p is the equivalent parallel conductance determined for the fundamental component of current. G_p will vary with E as is shown for example in Fig. 7, but the loss at any particular voltage may be readily determined by using the value of G_p corresponding to that voltage. Likewise if the current wave is sinusoidal, the power loss is given by $I^2 R_s$ where I is the r. m. s. value of the current and R_s is the equivalent series resistance to the fundamental.

To depend entirely on oscillograms for the determination of the electrical properties of imperfect condensers, as suggested by Prof. Malti, would, I believe, be not only inconvenient but would frequently be unsatisfactory as regards accuracy. For example, before the oscillogram shown in Fig. 12 could be obtained it was necessary to construct and adjust a 7-tube amplifier, as well as to devise and test out an elaborate shielding system. The taking, developing, printing, and calibrating of the oscillogram was in itself a relatively laborious procedure. To obtain the power by multiplying together the instantaneous values of E and I in itself is also laborious, and is also subject to errors in calibration and in the fact that the width of line is relatively large as compared with the amplitude. Hence the value of power obtained by this method would undoubtedly be less precise than the value obtained by the use of an equivalent circuit.

There are, however, special problems such for example as occur when both voltage and current waves are non-sinusoidal where the use of such oscillograms are necessary if accurate analyses are to be made. We are confronted with several of this type of problem at the present time.

As a matter of fact from an analytical point of view, non-linear dielectric characteristics are not different from non-linear magnetic characteristics. The analysis of dielectric phenomena is however more recent and is therefore less understood. For convenience in solving practical problems we have become accustomed to represent the circuit linked by magnetic material, even though it has non-linear characteristics, by equivalent series and parallel circuits. A common example is the equivalent parallel circuit so frequently used to represent the no-load conditions of transformers and induction motors. Rigorous analysis would require the use of oscillograms of voltage and current, as suggested by Prof. Malti for the dielectric circuit. As a matter of practical convenience, however, we actually use either an equivalent series or parallel circuit under all conditions of operation, although such do not, strictly speaking, simulate the actual circuit under all conditions. This same practical necessity, I believe, will necessitate in many instances the use of equivalent circuits to simulate the behavior of dielectric circuits, even although such circuits do not simulate rigorously the actual circuit.

Measurement of Noise in Electrical Machinery

BY BENJ. F. BAILEY[1]

Fellow, A. I. E. E.

WHEN a new and useful machine is first introduced, nobody thinks much about possible refinements. Thus when electric motors were first used, reliability was the essential quality. Soon afterward price became of importance and still later efficiency was considered. Now at the present time attention is being turned to noise and in the not too distant future, appearance will become of distinct importance.

When the public has had some means of expressing its opinion quietness has always been demanded. In the case of the automobile this demand has led to the development of a machine that is unbelievably quiet especially to those who drove cars twenty years ago. On the other hand, it is hard to see why the public has not protested long ago against the needless noise of street cars and trains. Little progress has been made in this direction in thirty years and it is only now that the competition of quieter vehicles is forcing those responsible to take some action.

Where an electric motor is used to drive a washing machine or a mangle, noise is not so important, but if a motor is used to drive an electric refrigerator the case is quite different. The motor is usually located in close proximity to the living quarters and is in operation much of the time and at any time of night or day. Obviously such a motor must be quiet.

Need of Methods of Measurement

The point has now been reached where the need of adequate means for measuring noise is felt. At present the intensity of a noise is stated about as accurately as the Irishman's reply who when asked how large a certain object was said it was about as large as a lump of coal. Two distinct types of measuring devices are needed. One should, if possible, give with a single reading an indication of the intensity of a sound; much as an ammeter gives at once the current in a circuit. The other should make possible the analysis of a sound into its components in much the same manner that the components of a current can be analyzed by means of an oscillograph. The first type will be useful to the manufacturers and users of machinery in determining quickly and easily whether or not a machine comes within certain prescribed limits of quietness. The second type will be useful to the designer of machinery to enable him to determine and eliminate the sources of noise.

The inspection of parts by ear for undue noise is old in industry. Thus in the manufacture of gears, an inspector runs the different gears and accepts or rejects them in accordance with his judgment. An average gear may be provided which the inspector may use as a standard of comparison. Such an inspection while better than nothing is very imperfect. Probably no two people hear things alike and the sensitivity of the ear differs from day to day.

One gear manufacturer to test the accuracy of sound inspection submitted 100 gears to one of his inspectors. Of these 95 were passed and 5 rejected. Another inspector who knew nothing of the first inspection passed 93 and rejected 7. Only one or two of the gears were rejected by both inspectors. A few days later the same gears were again submitted to the first inspector who did not know that he had previously inspected them. This time he rejected 11. Of the 11, only one had been included in the first five. The other ten he had previously passed.

The accuracy of a direct comparison method may be greatly improved if a number of observers is used. A prominent laboratory was recently called upon to rate a considerable number of motors in regard to noise. These motors were all of the same horsepower and speed. For comparison, five motors were selected and arbitrarily given ratings from 1 to 5, No. 1 being an exceedingly quiet motor, No. 5 a very noisy one while No. 3 was supposed to represent about the average. Six observers compared the motors with these standards. Occasionally all the ratings would agree, particularly in the case of very quiet motors, but in other instances the same motor would be rated all the way from 1 to 3 by different observers. The average of the six ratings was probably fairly reliable.

In somewhat similar tests conducted by a prominent manufacturer a standard of noise was provided as well as the motor under test. The observer placed himself between the two motors and varied his position until they sounded equally noisy. This method is similar to that used in the photometry of lamps.

Difficulties of the Problem

When an attempt is made to develop an instrument which will give an objective indication of the degree of noise from a given source serious difficulties are encountered. If it were merely a question of measuring the physical intensity of the sound the difficulties would not be so great. Unfortunately the effect upon the human ear is not directly proportional to the physical intensity of the sound. A sound of very high pitch, for example, cannot be heard at all and the same

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is true of sounds of very low pitch. Moreover the range which the ear can hear varies with different persons and at different times. The problem is further complicated by the fact that what is really desired is to find out how disagreeable a certain noise is. Some sounds, such as the scratching of a slate pencil upon a slate, while not very loud are exceedingly disagreeable. It is hard to see how an instrument can be made which will take all these things into account.

Reflections

Another difficulty arises from the fact that in an ordinary room we hear not only the sound itself but many reflections of the sound. A sound indoors at a certain distance may easily produce ten times the effect of the same sound at the same distance in open air. Moreover in any ordinary room there will be certain points at which the sound will be focused and will consequently be much louder than at other parts of the room or perhaps at the object emitting the sound. When trying to measure sound by using a microphone as a pick-up it has frequently been observed that the sound three feet from the motor under test may be much louder than at a distance of one foot from the motor. Of course one obvious remedy is to make the test in a sound proof room. This while possible in the laboratory would introduce serious difficulties if an attempt was made to make sound measurements in the course of routine production.

Another method that has been used with some success is to provide a very large rotating vane. This tends to break up reflections and prevent them concentrating at any given point.

Measurement of Vibration

One interesting suggestion is that the noise itself should not measured but the vibration which produces it. If a transformer, for example, produces noise it is because certain parts of the transformer are vibrating. If it is possible to measure the vibration it will give a reasonably accurate indication of the intensity of the sound. This method has actually been applied to the routine testing of roller bearings. In this case the problem is much simplified in that the objects to be tested differ if at all only in size. It would not be so easy to apply this to the case of electric motors, for example, where much of the sound comes from the rotating parts.

Analysis into Components

As previously indicated we can analyze the sound into its various components and determine the intensity of each of these components. One method consists in using a microphone to pick up the sounds and generate a feeble electric current. This is amplified by vacuum tubes and then passed through a variable filter, so that only one component at a time passes beyond the filter. This component can then be measured by a galvanometer or similar indicating device. This method, while obviously slow, gives complete information in regard to the sound produced. In order to determine its effect upon the ear it would be necessary to assign to each frequency an arbitrary importance depending upon the effect of the particular frequency upon the average human ear. It would not appear that such a method could be made sufficiently quick and simple so that it could be used in routine production. It is, however, obviously of great value to the designer, in helping him to reduce noise.

Organizations Interested

Several manufacturers of various types of machinery have become greatly interested in methods for measuring accurately and quickly the noises produced by their product. In the past five years approximately $250,000 has been spent upon such projects at the University of Michigan alone and methods have been devised for measuring the noise produced in ball and roller bearings, gears, electric carpet sweepers, and similar devices. Instruments have been constructed and are used in routine production in several factories.

Certain national societies, among them the National Electrical Manufacturers Association and the American Society of Mechanical Engineers, have become interested in the problem of noise and have appointed committees to consider the subject. The National Electrical Manufacturers Association's committee was instructed, first, to find or devise a method of measuring the noise produced by electrical machinery, secondly, when such a device had been found to formulate standards of allowable noise in electrical machinery. Some of the difficulties in the first part of this problem have already been indicated. When we consider the enormous variation in horsepower, speed, frequency, number of phases, etc. of electric motors alone it will be apparent that the difficulties of the second part of the assignment are no less.

Conclusion

It is very evident that the American people as well as the manufacturers of electrical machinery are becoming "noise conscious." Great progress has been made in the past few years in devising means of measuring noise as well as improving machinery so that it will be less noisy. Although the difficulties of the problem are great there is reason to hope that some device will in the near future be perfected to give an immediate indication of the effect of any given noise upon the human ear. After all what is desired is to eliminate noise. But with such an instrument available progress will be much more rapid. A definite measure of noise in place of the present very imperfect estimate based entirely upon personal judgment will then have been achieved.

Discussion

For discussion of this paper see page 1071.

Indicating Meter for Measurement and Analysis of Noise

BY T. G. CASTNER,* Non-member E. DIETZE,† Non-member G. T. STANTON,‡ Associate, A. I. E. E. and R. S. TUCKER† Associate, A. I. E. E.

Synopsis.—This paper describes a visual indicating meter for the measurement of noise and other sounds. Its design is based on the known characteristics of sound and hearing, which are summarized. Particular attention has been paid to the response of the meter to sounds of short duration The aim has been to make the meter both simple in operation and portable. An attachment for the frequency analysis of noise is under development. Several fields of use of the meter and analyzer are indicated.

FOR the measurement of acoustic noises of various kinds, the advantages of a visual indicating meter are generally recognized. This paper describes such a meter which has recently been developed in the Bell System. As this meter measures noise and sounds in general, it has been called a "sound meter." In designing this meter the aim has been to have its indications in close accord with the present information as to the response of the ear, in so far as is practicable in a simple portable device. In this regard, particular attention has been paid to the response of the meter to sounds of short duration which frequently are an important part of noise. There is under development also an attachment for this sound meter to permit the analysis of sound both on a single-frequency and a band-frequency basis.

To indicate the requirements for a meter of this type this paper discusses the characteristics of noise and hearing, and the attributes which it is desirable that a meter for noise measurement should have in order that its indications be correlatable with the effects of noise.

WHAT A METER FOR NOISE SHOULD MEASURE

Every noise problem arises from the fact that noise is objectionable, *i. e.*, produces certain undesirable effects upon people (such as interference with hearing, annoyance, effect on working efficiency, effect on health). It might appear desirable, therefore, to construct a meter which would measure these effects directly. Most of them, however, are dependent upon the psychology of the individual and require for their evaluation a large amount of further investigation. Furthermore, a meter for measuring one effect probably would have different characteristics from a meter for measuring some other effect, leading to a diversity of meters whose readings would not be comparable.

On the other hand, a meter giving a purely objective description of the noise, *i. e.*, measuring the physical characteristics of acoustic waves, would also be undesirable, as the measurements would bear an unknown relation to the noise as people would hear it, and, therefore, to the effects produced by the noise.

There remains as a practical solution of the problem of noise measurement, a meter having as far as practicable the attributes of hearing. This is a logical solution, since when measuring noise with an indicating meter, the ear is replaced by the meter. Since reactions to noise are dependent upon hearing, the readings of a meter based upon hearing should be correlatable with noise effects.

The sound meter described in this paper is of this type. It aims to fill a need now felt in many diverse kinds of noise problems for a rather simple meter which can be made generally available and can be used with some facility to obtain a large variety of noise data. With preliminary models of such a meter, much valuable information has already been obtained, some of which is summarized herein. Such data should be of assistance in laboratory studies of the various reactions of people to noise.[1] As laboratory data on the effects of given noises on human beings become available, approximate relations between sound meter readings and the amounts of effect of various types can be obtained.

As the basis for the design of the sound meter, loudness has been employed, since loudness is the most elemental and universally appreciated attribute of noise. Since the loudness of a sound depends on its intensity (sound power passing through unit area), frequency, duration, and complexity of wave shape, the effects of each of these upon loudness must be considered in determining the requirements for such a meter.

ESSENTIALS OF THE SOUND METER

The sound meter discussed in this paper consists of a sound pick-up for converting acoustic into electrical energy; a five-stage vacuum tube amplifier; a calibrated gain control; an electrical network for weighting energies at different frequencies in a manner similar to that in which they are weighted for loudness by the ear; a full-wave rectifier and a visual indicating meter having a time of response similar to that of the ear; and suitable means for making an over-all acoustic calibration of the

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1. For references see Bibliography.

instrument. The calibrated gain control makes it possible to measure sound levels over a range of 90 db. (corresponding to a range of 1,000,000,000:1 in input powers) on a d-c. meter having an effective range of only 15 db. The sound meter proper comprises two units: the amplifier unit, and a battery box which also contains the calibrating equipment and has space for carrying the sound pick-up. Fig. 1 shows a schematic diagram of the complete measuring set while Figs. 2 and 3 show the external appearance of an experimental model of the meter and battery box.

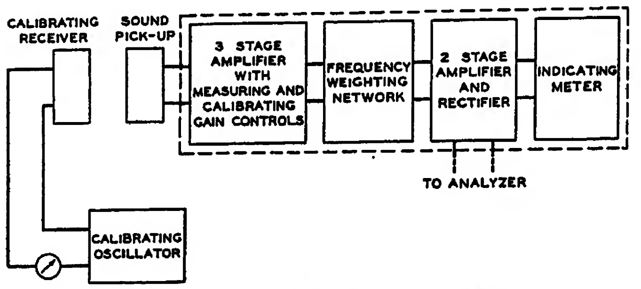

FIG. 1—SCHEMATIC DIAGRAM OF SOUND METER CIRCUIT

The sound meter is calibrated in terms of a 1,000-cycle pressure of 0.001 dyne per sq. cm. in a free progressive wave. This value is called "reference sound level" and has been chosen because it is a convenient reference point and for a 1,000-cycle tone is near the threshold of audibility for an average observer in a completely quiet place.* The level of a sound is taken as the level of a 1,000-cycle tone which gives a meter reading equal to that given by the sound in question, and is expressed in decibels above this reference point. The advantage of this reference point is that it is definite and reproducible, and does not depend on personal equation. It can be related to the threshold of audibility, as accurately as the latter is known.

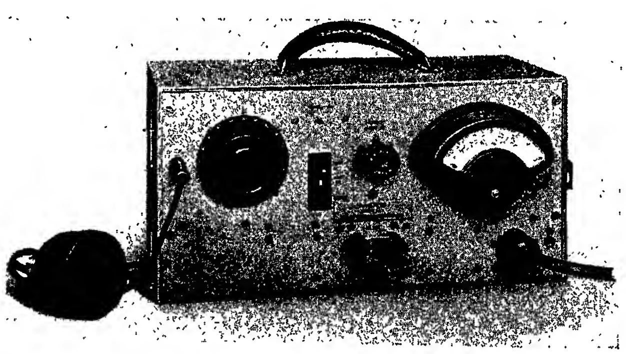

FIG. 2—MODEL OF SOUND METER

CHARACTERISTICS OF NOISE

The characteristics of noise must be studied since they determine the range over which the characteristics of hearing must be considered and impose very severe requirements on measuring apparatus. Noises vary widely in intensity, frequency, duration, and wave shape.

Noise covers the entire audible range of intensities. It varies from the barely audible ticking of a watch to the screaming of a steam whistle or the detonation of heavy artillery which may actually damage the ear of a nearby observer. Fig. 4** illustrates the levels of some commonly encountered noises.[2,3] The available data are not detailed enough to determine accurately the range of noise levels to be expected in specific types of location where particular kinds of machinery may be installed.

As to the frequency range covered by noise, building vibrations sometimes are of such low frequency that the listener is in doubt whether he can hear a sound or only feel vibration; on the other hand, some noises involve very high frequencies, as, for instance, the scraping of a knife on glass or the jingling of keys. Most noises are composed of a large number of frequencies.

FIG. 3—MODEL OF SOUND METER BATTERY BOX

The duration characteristics of noises vary widely. A noise may be steady, like the hum of rotating machinery; it may vary in definite cycles, as the noise of an oscillating fan; it may increase to a maximum and from there on decrease till it disappears, as the roar of a passing train or an airplane. A great deal of noise consists of one or more isolated peaks, like the banging of a door, hammer blows, etc.; or of a series of peaks with or without steady noise. Fig. 5 gives a diagram of noise measured in a restaurant. This figure indicates the magnitude and frequency of occurrence of the clearly

*Reference sound level is between about 5 and 10 db. above the 1,000-cycle threshold of audibility. A more accurate relation is being determined.

**The "zero noise level" of this figure was determined as the threshold of audibility for certain common types of noise.

discernible peaks which stood out above the general noise level, and the causes of these peaks. In noise produced by a great many sources at different distances, as for instance, street noise, generally the nearby noise peaks stand out, whereas those farther away tend to merge with the general noise.

Some noises have very peaked wave shapes. For common noises such as ordinary room or street noise, the ratio of the maximum instantaneous pressure to the r. m. s. pressure over an interval of a few tenths of a second probably varies from 5 to 10. For noises consisting of hammer blows, the banging of doors, etc., this ratio may be as high as 30 or 40.

OUTDOOR NOISES IN NEW YORK CITY		db	OTHER NOISES
DIST. FROM SOURCE IN FEET	SOURCE OR DESCRIPTION OF NOISE	ABOVE ZERO NOISE LEVEL	SOURCE OR DESCRIPTION OF NOISE
35	RIVETER	100	SUBWAY — LOCAL STATION WITH EXPRESS PASSING (1)
15-20	ELEVATED ELECTRIC TRAIN ON OPEN STRUCTURE	90	
15-75	VERY HEAVY STREET TRAFFIC WITH ELEVATED LINE	80	SUBWAY CAR AT FREE-RUNNING SPEED, INTERIOR OF CAR (2)
15-50	MOTOR TRUCK		
		70	
15-75	BUSY STREET TRAFFIC		
15-50	PASSENGER AUTOMOBILE		AVERAGE OF 18 FACTORY TELEPHONE LOCATIONS (1)
		60	
15-300	RATHER QUIET RESIDENTIAL STREET, AFTERNOON		LARGE HALL WITH AUDIENCE TALKING (2)
		50	AVERAGE BUSINESS LOCATION (1)
15-500	MINIMUM NOISE LEVELS IN ENTIRE CITY DAYTIME: 10-20 MINUTE AVERAGE; 5 SECOND AVERAGE	40	AUDIENCE NOISE DURING PERFORMANCE IN LARGE AUDITORIUM (2)
15-500	IN MID-CITY NIGHT: 5 SECOND AVERAGE		
		30	AVERAGE RESIDENCE LOCATION (1)
		20	
		10	OBTAINABLE IN SOUND PROOFED ROOMS (2)

ABOVE DATA FROM REPORT BY NEW YORK CITY NOISE ABATEMENT COMMISSION

(1) DATA OBTAINED AT TELEPHONE LOCATIONS BY JOINT SUBCOMMITTEE ON DEVELOPMENT AND RESEARCH, NATIONAL ELECTRIC LIGHT ASSOCIATION AND BELL TELEPHONE SYSTEM.

(2) DATA OBTAINED BY ELECTRICAL RESEARCH PRODUCTS, INC.

Fig. 4—Levels of Noises

Characteristics of Hearing Affecting Noise Perception

The exact relations between the physical properties of a sound and its perception include psychological factors which are difficult to measure.[4] It is not the object of this paper to discuss these relations in detail, as a great deal has been published on this subject. Some of this work is referred to herein. The principal question of interest in the design of the sound meter is how the sensation of loudness is related to the physical properties of a sound, such as intensity, frequency, duration, and complexity of wave shape. These relations are indicated in the following.

The relationship between the loudness of a sound and its intensity and frequency is quite complicated. To produce a given loudness, low frequency tones require more intensity than those of higher frequency (500 to 5,000 cycles); and the loudness of low frequency tones increases more rapidly with increasing intensity than does that of the higher frequencies Curves showing the relation between the loudness of pure tones and their frequency and intensity are given in the Bibliography.[5,6]

The loudness of tones of very short duration, less than a few tenths of a second, is a function of the time during which the tone persists. The available information on this subject is rather meager. Fig. 6 indicates the relation between loudness and duration for short tones,[7] and indicates that the ear fully appreciates the loudness of a tone persisting for two-tenths of a second. More work along these lines, however, will be necessary, especially to determine how the relationship applies to complex sounds.

The relation between loudness and complexity of wave shape of sounds is extremely involved[8] and is by no means fully understood. It appears that for very low levels, the contribution from different frequency bands to the total loudness of a complex sound is approximately proportional to the squares of the pressures in these bands as compared to threshold pressure. At high levels, the loudness of a complex sound seems to be materially affected by the non-linear characteristic of the ear, and the relations become much more complex.

Fundamental Design

Consideration will now be given to the way in which the sound meter meets the requirements imposed on it by the characteristics of noise and hearing.

Intensity Range. While an extremely wide range would be necessary to cover all possible types of noise,

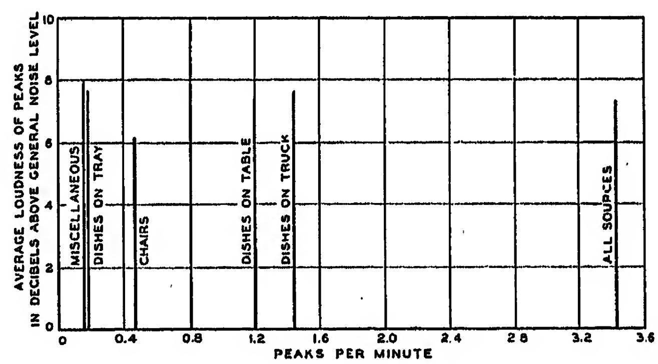

Fig. 5—Average Noise Peaks in a Restaurant Classified According to Sources

it is generally unnecessary for practical purposes to measure accurately extremely low or high noise levels. The sound meter has been provided with a range from about 10 to about 100 db. above reference sound level, which should be satisfactory in a very large majority of cases. It is direct reading in decibels above reference sound level.

Frequency Weighting. From the data given in the Bibliography,[5] the curves of Fig. 7 have been plotted to show how the shape of frequency-weighting curves

based on loudness would change with variations in the level of the sound being measured. On this account, consideration has been given to the provision of an adjustable frequency weighting in the sound meter. For general use, such refinement does not seem necessary, since it would appreciably change the results of only a very small proportion of the measurements. It will be seen from Fig. 7 that only at low frequencies are there large divergences between the different curves. Some experimental data are available on the differences in meter readings obtained with weightings corresponding to the 30- and 60-db. curves.

Source of noise	Difference in readings
Electric drill	0 db.
Electric fan, 1 ft. from pick-up	4 db.
Several telephone ringer bells	not greater than 1 db.

From consideration of the above and in view of the complications which a variable frequency weighting would entail, it seems desirable to employ a single weighting based on a loudness curve at a medium level. The sound meter weighting is based on the 40-db. level in the range 60-4,000 cycles, where experimental data are available. Outside this range its characteristic does not differ greatly from that of the threshold of audibility.

Duration Characteristics. The response of the meter to sounds of short duration approximately simulates that of the ear, as can be readily demonstrated by watching the movements of the meter needle and listening to the variation of the noise being measured. In such a test it will be observed that the visual and aural impressions are approximately synchronized.

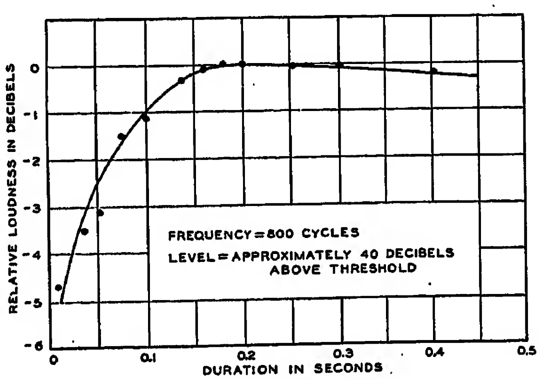

FIG. 6—RELATIVE LOUDNESS OF TONES OF SHORT DURATION

Data taken from article by V. Békésy in *Physikalische Zeitschrift* XXX, 1929, p. 118

The output meter used gives a full deflection on pulses of noise lasting about 0.2 second or more. It does not overshoot more than about 0.5 db. With these characteristics there is no difficulty in reading noise peaks to within ± 1 db. The measurement of non-recurring rapidly varying noises has been greatly facilitated by the use of an output meter having a long and easily readable db. scale.

The following test results illustrate how the readings of a meter are affected by its dynamic characteristics. Certain peaked noises were measured by the sound meter and by another meter identical except for its dynamic characteristics, which were as follows: when a pure tone was applied, the second meter took about 5 seconds to come within 0.5 db. of its ultimate steady-state value. The difference in maximum readings on

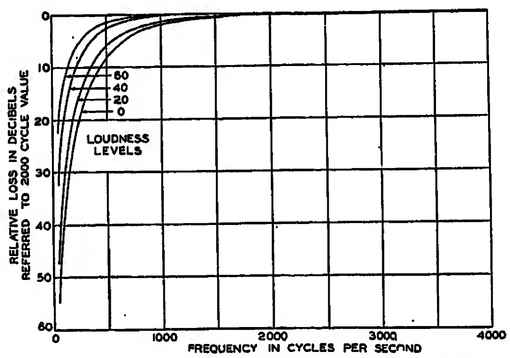

FIG. 7—FREQUENCY WEIGHTING CURVES BASED ON EQUAL LOUDNESS AT VARIOUS LOUDNESS LEVELS

Data taken from article by B. A. Kingsbury, "A Direct Comparison of the Loudness of Pure Tones," *Phys. Rev.*, April 1927, p. 588

these noises was as follows, the slower meter reading lower.

Type of noise	Approximate difference in maximum readings
Short blast from automobile horn	25 db.
Sharp blow on large metal can	30 db.
Single short piano note	15–30 db.
Hammer blow on metal plate	25 db.
Cough	20 db.

Response to Complex Waves. Taking account of present knowledge as to the way in which the ear combines components of a complex sound, practical considerations have been controlling in the design of the meter. The rule of combination employed by the meter is known and simple, and is not a poorer approximation to the performance of the ear, as far as this is known, than other practicable rules. The rectifier employed is of the full-wave type, and approximately follows a "square law;" that is, the meter readings are approximately the same when a sinusoidal voltage is impressed on the rectifier as when there is impressed a steady complex voltage having the same r. m. s. value. With a square law device, the meter indication is a function merely of the average impressed power, and not of wave shape. With devices following other simple laws, the meter indication, in general, is a function of the impressed wave shape, even for waves differing only in the phases of their components. Since phase relations would be changed in the sound meter circuit, any dependence on phase would be arbitrary. With rectifiers not of the full-wave type, the meter indi-

cation may depend on which way the connection between pick-up and amplifier happens to be poled.

Practical Design

In addition to the above, a number of practical requirements has been met in the design of the sound meter.

Portability. The general usefulness of the sound meter depends to a large extent upon its portability, as in most cases of noise measurement it is necessary to take the meter to the location of the noise. In the interests of portability, a light and efficient type of amplifier has been designed, and the battery supply has been reduced to the minimum consistent with reasonable life. The sound meter proper consists of two units, which together are portable by one man.

From the standpoint of both portability and cost, the analyzing equipment was built as a separate attachment.

Linearity. In designing a sound meter to cover a wide range of levels, and especially with an amplifier using a frequency-weighting network having a high loss at low frequencies, special precautions are necessary to reduce non-linearity to a minimum. Non-linearity in the sense used here means the failure of the meter to read correctly differences in sound level; it may be due either to feed-back from a high level part of the circuit to a low level part, or to overloading in circuit elements for which the transmission loss varies with level. It is by no means sufficient for the meter to measure single frequencies without overloading since many noises have very peaked wave shapes.

The sound meter amplifier has been designed so that non-linearity is negligible. It can handle without overloading pure tones as low as 20 cycles, and complex voltage waves having ordinary peak factors. Since measurements are seldom made near the upper end of the meter scale, there is for most noises a working factor of safety.

Shielding. Since sound meters may be employed to measure noise levels in the presence of strong electromagnetic or electrostatic fields, they should be sufficiently shielded to make the effects of stray fields negligible. This is particularly important when an analyzer attachment is used.

The sound meter is practically completely shielded against electrostatic fields. It was necessary to compromise between portability and complete protection from stray electromagnetic fields. The electromagnetic shielding is sufficient for ordinary purposes, and there are several features which compensate for the fact that it is not complete. In most locations where intense magnetic fields are encountered, the noise levels are so high that no material error should be introduced even by a moderate effect from such stray fields. Also, since the sound meter is readily movable, in many cases the effects of electromagnetic fields can be made negligible by properly orienting or locating the meter.

Method of Calibration. The primary absolute calibration of the sound meter was made by placing the sound pick-up in a 1,000-cycle free progressive wave at a point at which the pressure was determined from Rayleigh disk measurements.[9] This wave was essentially plane in so far as its effect on the pick-up is concerned. By means of the calibrating control on the meter amplifier, the gain was then adjusted to make the meter read the level of the 1,000-cycle wave in decibels above reference sound level. The meter reading for given acoustic pressures at other frequencies can be found from the frequency characteristic of the sound meter.

Great care has been taken to make the sound meter amplifier as stable as possible. In addition, in order to verify in the field that the meter is reading correctly, a means of making an over-all acoustic calibration has been included. The apparatus for this secondary calibration consists of a very stable type of telephone receiver, a small 2,000-cycle oscillator for energizing the receiver with a known current, and a means of coupling the receiver to the sound pick-up in order to produce a definite acoustic pressure on the pick-up diaphram. The frequency of 2,000 cycles was chosen since at this point the sensitivity of the sound meter is greatest, and the frequency characteristic is practically flat in this region. When the battery voltages are within their normal limits, calibration once a day is sufficient. The process of calibration is a very simple one, requiring only a few seconds.

Individual sound meters will have the same electrical characteristics within very close limits.

Analyzer Attachment

The frequency analysis of noise can be performed easily and accurately by means of electrical circuits. Means for making two types of frequency analysis are provided in the analyzer attachment, single-frequency analysis and analysis into frequency bands several hundred cycles wide.

For continuous-spectrum noises or other noises which have their frequency components very closely spaced, a single-frequency analysis is not practicable because of the insufficient resolving power of any available resonant circuits, and consequently wide-band analysis must be employed. These noises are those which either do not contain predominant single-frequency components or else are a mixture of noises from so many sources that the individual frequency components from any one source have lost their significance. Street noise, office noise from typewriters and computing machines, and noise from a large number of factory machines are examples.

In the development of the analyzer attachment, much assistance has been obtained from the work done by others on the development of harmonic analyzers, especially those for use in studies of inductive interference on telephone circuits.[10] In recent work con-

siderable effort has been exerted to design an instrument which is truly portable, and at the same time is very simple to operate.

The selectivity, sensitivity, and modulation requirements in an analyzer depend upon the types of noise to be analyzed and the purposes of the analysis. These requirements have been considered together since small components of a noise cannot be measured if other components, including undesired modulation products, affect the indication of the analyzer appreciably. When the analyzer is arranged for single-frequency analysis, the response curve in the immediate neighborhood of the tuned frequency will be sufficiently flat so that small variations in the frequency of the noise component being measured will not cause large variations in the reading of the indicating meter; this curve, however, will not be flat over so great a range that the accurate location of low-frequency components will be difficult or the discrimination against adjacent components suffer.

Uses

The field of usefulness of the sound meter is as yet only partially developed, as it is applicable in the solution of a great many acoustic and noise problems. Before considering the uses it is desirable to have in mind certain limitations.

Limitations. A sound meter cannot discriminate between noise and desired sounds, nor can it pick out one noise from a combination of noises unless this noise differs radically from the others. In case it is desired to measure a single noise whose source is known, sometimes all of the other noises may be stopped temporarily. For example, in order to be able to measure the noise of a certain machine in a factory room, it may be necessary to stop all other machines in this room, and also to exclude street noise and noise from other parts of the building. If the noise background is 20 db. or more below the noise from the source under investigation, it may be neglected. In other cases, a correction can sometimes be made by measuring the background separately.

Secondly, the amount and character of the noise due to a given source are affected by the acoustic properties of its surroundings. The principal acoustic properties which must be considered are the volumes of rooms in which measurements are made, the total acoustic absorption present, and the characteristics of this absorption over the audible spectrum. Allowance for these must be made when applying the results of noise measurements made in a particular location, such as a test booth, to predict the amount of noise from the same source in some other location.[11,12]

Another limitation is the effect of the location of the pick-up. Small changes in the distance from the source to the pick-up, particularly when the distances themselves are short, may result in large changes in the measured noise level. Certain types of noise are more directive than others; this must be taken into account in determining the best location for the sound pick-up.

General Noise Measurements. Many general noise measurements may be made with the meter without the analyzer attachment. Meters have been employed in telephone, municipal, and industrial surveys to determine points where noise conditions are most severe, and have yielded valuable data on the relative contributions of various sources to the total noise. Information of this type is essential in preparing any well-founded engineering plan for the reduction of noise.

The effect of noise on the efficiency of workers is a matter of growing public concern. Architects and builders are being called upon to plan their structures so that they will be quiet. The meter can be used to aid in ascertaining desired levels of noise in various cases, and to determine when these levels have been met. It can also be used to measure the noise produced by a machine, which will be of value in forecasting the noise conditions which will obtain in the building containing the machine and in related locations. Meter measurements may be made in residential neighborhoods prior to and after installation of power plants, substations, etc., where it is desirable to know the effect, if any, on the general noise level in the neighborhood caused by such installations.

As indicated in the selection of the name "sound meter," the meter can be used to measure sounds other than noise. In the recording and broadcasting of speech and music, valuable data may be obtained by the use of this instrument. Optimum sound levels may be determined for speech and music in various situations; studies may be made of the sound distribution throughout a studio; and meter determination of balance of orchestras appears possible, permitting standardization of methods with consequent saving in time.

Analysis. The analyzer may be used to obtain more detailed information about noises than can be obtained from over-all measurements. With this information it should be possible to make better estimates of particular effects. For example, analysis will generally be required to determine whether an undesired sound such as machinery noise will be covered up by the general noise, or to determine whether a desired sound, *e. g.*, a signal, will be audible through this noise.

In cases of noise reduction by the use of sound insulating or sound absorbing materials, determination of the frequency composition of the noise will permit a choice of materials which will be most effective at the most important noise frequencies.

The arrangement for single frequency analysis opens opportunities for the study of noise in detail. This feature is important in the case of machinery noise. A determination of the components existing in the noise produced by a machine may yield, by comparison with the mechanical features of the machine, accurate data as to the contribution to the total noise of each individual source in the machine. Such data have been

applied in the redesign of machines to reduce noise, and in many cases have permitted of carefully planned designs involving the minimum of changes to secure given results.

The control of uniformity of manufactured products by means of acoustic measurement offers a wide potential field of application of the sound meter. It is anticipated that the results in this field will be most valuable when careful single frequency analysis is made of the particular machine or process under study by means of special adaptations of the meter constructed for the particular problem.

Measurement of Electrical Quantities. The sound meter is to be provided with a suitable electrical input, so that it can also be used to measure electrical levels (or to analyze electrical quantities) in cases where the frequency weighting and other features provided are suitable.

Conclusion

This paper has outlined the general requirements for an indicating meter for measuring noise and has described a meter meeting these requirements which has been developed. In the design of this meter, great care has been exercised to have it both simple and portable. It is believed that this new meter will provide a valuable tool in studying a great many noise and sound problems.

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Discussion

For discussion of this paper see page 1071.

The Measurement of Machinery Noise

BY H. B. MARVIN*
Associate, A. I. E. E.

Synopsis.—The audio noise meter described in this paper is a portable instrument for measuring and analyzing noise and vibration of machinery. The analyzer obtains its high selectivity through the use of a mechanical "band pass filter" of four stages. Some test results and a discussion of the measurement problem are given..

* * * * *

INTRODUCTION

THE purpose of this paper is to describe a new instrument for measuring the loudness and frequency composition of noise and to present some typical test results. The description is preceded by a brief discussion of some aspects of the problem.

Noise is a comprehensive term which includes almost all sounds except speech and music. The usual noises created by machinery are recurrent and approximately steady as contrasted with the random character of street noise. They are caused by vibration of the machine as a whole or in parts. The vibrating parts radiate sound waves which reach the ear directly and also set up vibration waves which travel through the supports to floors, etc., which radiate sound waves. The noise perceived is usually a mixture of the direct and indirect radiation.

In order to control noise, an understanding of the causes and mode of transmission is necessary. It is in this field that measuring instruments are especially helpful. Information may now be obtained with instruments of the strength of the source of noise, and of its magnitude and quality in space. Coincidence of noise frequency with rotational frequency of machines and with vibration frequency due to magnetic forces may be established and serve as a basis for design changes. The effects of noise on the nervous systems of human beings are also important but satisfactory methods of measurement have not yet been developed.

UNITS AND METHODS

It is desirable to measure vibration amplitudes of the machine, the loudness of the noise, and the frequency and loudness of each component. The measurements should include vibration at various points on the machine and its support, and loudness at various points in space around it. The response of the instrument should correspond with the loudness heard.

The sensation of loudness corresponds more or less to brightness, heaviness, coldness, sweetness, etc. The method of measurement of all these sensations has long been a problem. The natural unit of sensation is the smallest perceptible increment, but this kind of unit is difficult to establish and reproduce. This is because the relation between sensation and exciting stimulus (measured in physical units) is not a simple one. It is approximately a logarithmic one over part of the range. Accordingly a logarithmic scale in which loudness is expressed as the number of decibels[1] above threshold of audibility is here used. This choice of threshold or zero loudness as a reference point is chiefly because results are then comparable with published data on room and street noise.[2]

Dependence of the sensation of loudness on frequency for various intensities has been shown by B. A. Kingsbury[3] as a series of contours of equal loudness. The contour which at 700 cycles is 30 decibels above threshold of audibility of the average ear has been used as the basis of the response-frequency characteristic of the noise meter for total loudness. In the analysis of noise, frequency-weighting by the instrument is not necessary, since only one frequency is present at a time. The loudness of each component is measured separately and referred to the equal loudness contours. This method gives greatest sensitivity.

Much information can be obtained by analysis of noise which is not obtained from total noise tests. In the case of machinery noise, high selectivity or ability to separate adjacent harmonics is essential because the noise contains harmonics of rotational frequency which is often 60 cycles or less. This requirement led to the adoption of a mechanical type of filter which offers the possibilities of a high degree of selectivity with small size and weight.

The portable audio noise meter to be described is capable of making the above measurements. The method involves conversion of sound into electric current by a microphone, amplification, detection by a sensitive a-c. milliammeter, and calibration by comparison with sounds of known loudness.

DESCRIPTION OF AUDIO NOISE METER

The complete analyzing noise meter is contained in three cases as shown in Fig. 1. One case contains the measuring amplifier and has space to hold the sound

*General Electric Co., Schenectady, N. Y.

1. *Decibel—the Name for the Transmission Unit*, W. H. Martin, JOURNAL A. I. E. E., Vol. 48, p. 223, March 1929.

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3. "A Direct Comparison of the Loudness of Pure Tones," *Physical Review*, 2nd Series, Vol. 29, p. 588, 1927.

pick-up and tripod. Another contains batteries. The third contains the analyzer. This arrangement was chosen so that the meter may be used with or without the analyzer. Fig. 4 illustrates the circuits.

The sound pick-up consists of a condenser microphone

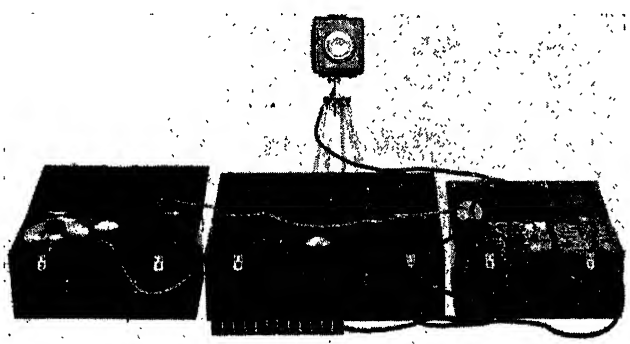

Fig. 1—Audio Noise Meter Set Up for Analysis

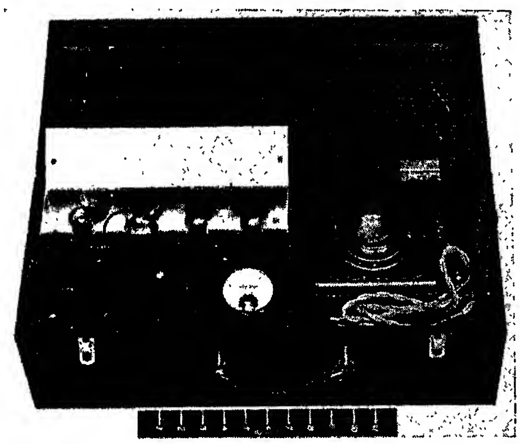

Fig. 2—Audio Noise Meter—Amplifier Unit

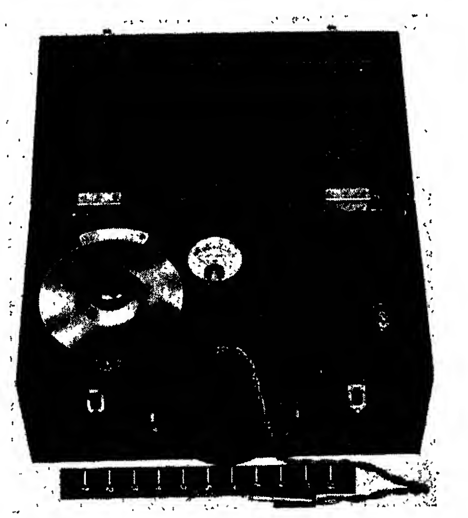

Fig. 3—Audio Noise Meter—Analyzer Unit

and a three-stage amplifier enclosed in a separate metal case, shown mounted on a tripod in Fig. 1 and packed for carrying in Fig. 2. Power for the amplifier is obtained from the battery unit.

The amplifier unit consists of a four-stage amplifier with calibrated output meter of the rectifier-milliammeter type, a calibrated attenuator, and an ear compensation network. Scale and dial are marked in decibels. Fig. 6 shows response of the noise meter for constant loudness tones of different frequencies. Tests have shown the readings to be closely proportional to loudness over the range of the instrument.

The analyzer shown in Fig. 3 may be connected between sound pick-up and amplifier to obtain harmonic analysis of noises. This analyzer has been designed to have high selectivity to obtain complete separation of

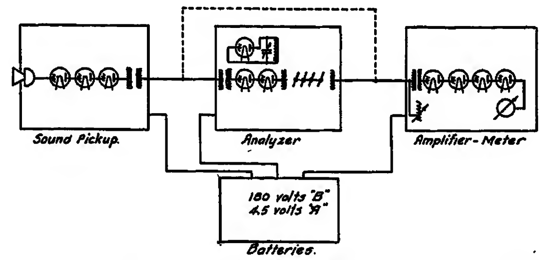

Fig. 4—Schematic Diagram of Audio Noise Meter

harmonics, yet sufficiently broad at resonance so that small variations of speed or frequency do not seriously affect operation. The results are accomplished by an electro-mechanical filter as shown schematically in Fig. 5. The complex current to be analyzed is passed through windings on the poles of the driving magnet and a sinusoidal current from a variable frequency vacuum tube oscillator is passed through an exciting winding. The resultant flux in the air gap produces a torque on an armature or vibrator equal in frequency to

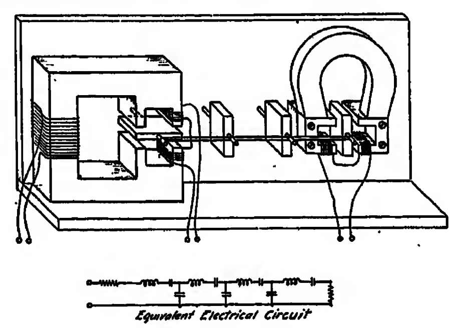

Fig. 5—Mechanical Filter Used in Analyzer

the difference between the complex wave and the analyzing sine wave and proportional to the product of their amplitudes. The moment of inertia of the armature and the moment of compliance of its supporting stem are designed to give angular vibration resonance at 6,000 cycles. Four vibrators are tuned to the same frequency and coupled in tandem by wires under tension. A permanent magnet and associated pole windings are mounted in such relation to the fourth vibrator that its motion induces in the windings a voltage proportional to the velocity of angular vibration. Mass and compliance of the vibrators and tension of the

coupling wires are adjusted so that the combination has characteristics similar to an electrical "band pass filter."

In the equivalent circuit of Fig. 5, inductance represents moment of inertia, series capacitance represents moment of compliance of vibrator stem, shunt capacitance represents longitudinal compliance of coupling

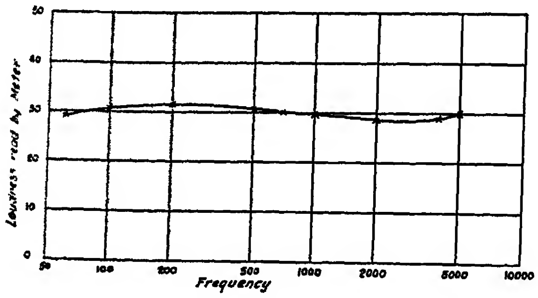

FIG. 6—RESPONSE OF AUDIO NOISE METER TO TONES OF 30 LOUDNESS

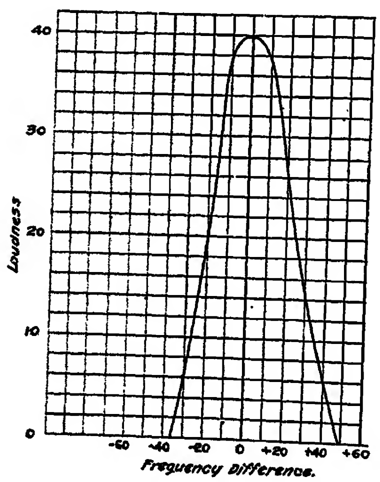

FIG. 7

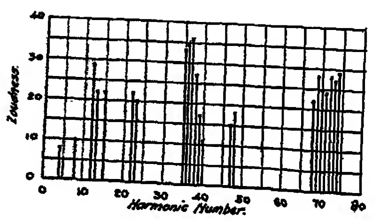

FIG. 8—NOISE ANALYSIS OF SMALL SYNCHRONOUS MOTOR, ¼ HP., 1,800 R. P. M., 60 CYCLE, THREE-PHASE, 220 VOLTS, NO LOAD

The fundamental of the harmonic series is rotational frequency, 30 cycles per sec.

wires, and resistance represents electromagnetic damping. Fig. 7 shows the measured selectivity of the filter. Since forty loudness units correspond to 100 fold in current change, it is seen that the current drops to 1 per cent of its resonant value with a change of less than 50 cycles in frequency, while five-cycle changes have negligible effect.

In operation the frequency of the oscillator is varied slowly from 6,000 to 11,000 cycles. Whenever the difference between this and the frequency of a harmonic in the complex wave equals 6,000, the analyzer responds. The frequency range is therefore 5,000 cycles maximum. The minimum is at present 30 cycles. This heterodyne method of analysis has an important property; namely, a selectivity which is the same for all harmonics.

The combination of analyzer and amplifier units has

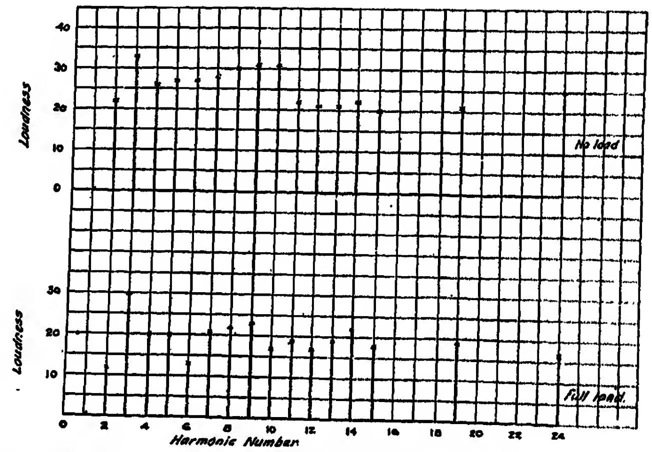

FIG. 9—NOISE ANALYSIS OF 300-WATT TRANSFORMER. FUNDAMENTAL OF THE SERIES IS 120 CYCLES PER SEC.

a broad field of usefulness. It may be used directly on electric circuits of suitable voltage for harmonic analysis where phase angles are not needed and is particularly useful on higher harmonics where analysis from oscillograms is difficult. In addition to sound analysis, it may be used with attachments for the analysis of mechanical vibrations.

In the measurement of vibration, a device, shown in Fig. 10, is held against the machine and connected to

FIG. 10—VIBRATION PICK-UP

the amplifier. The vibration generates a voltage proportional to its velocity. This vibration pick-up is substituted for the microphone so that both total vibration and harmonic analysis measurements are possible. Since the noise is more often proportional to velocity than to amplitude of vibration, this measurement gives an idea of the strength of the noise source and may be used to supplement or substitute for sound measurement.

Results

The uses of an audio noise meter are manifold. For some work, a total noise test is sufficient as for example comparing noise levels in factories, test booths, etc., and acceptance testing of machinery. If tests are made with the idea of changing design, harmonic analysis is most useful. Fig. 8 shows an analysis of noise from a small synchronous motor, in which the noise arises principally from higher harmonics of rotational frequency which is 30 cycles. Fundamental, second and third harmonics are below threshold while harmonics as high as the 74th produce appreciable noise. In synchronous apparatus nearly all noises are multiples of rotational frequency but in induction motors and other variable speed machines there are usually two series, one referred to power supply frequency and one to rotational frequency.

Analysis of hum from a 325-watt 60-cycle 110-:1100-volt transformer is shown in Fig. 9. Core clamps were loosened and the applied voltage was raised 20 per cent above normal in order to increase the amount and complexity of the noise. Here again the harmonics fall in one series having a fundamental of 120 cycles per sec. All harmonics up to the 15th are present in a measurable amount but the fundamental is inaudible. This would probably not be surmised in a listening test owing to imagination, which would supply a fundamental in order to be logical. The effect of loading the transformer is to decrease the total noise, but the analysis shows that some harmonics notably second, sixth, and tenth are weakened much more than the average.

Calibration

Laboratory calibration is made by comparison with primary sound standards on sine-wave tones of variable loudness and frequency. The instrument contains voltmeters for checking battery condition but in addition a convenient scheme is used for an over-all one point check which can be made in the field. This is accomplished by plugging a telephone receiver used with the noise meter into the output circuit and placing the receiver against the microphone. The attenuator having been set at a predetermined point, the amplification is adjusted by an auxiliary control until the noise meter is on the verge of self-oscillation. Since the oscillation is due to acoustic feed-back, the amplifier is again stable when the receiver is removed and is adjusted to a standard sensitivity.

Conclusions

Although knowledge of the sense of hearing is incomplete, it is felt that the instrument described serves a very useful purpose in studying machinery noise, the manner of its generation and transmission, its composition and magnitude.

A new analyzer having a high degree of selectivity and convenience of operation has been developed and combined with sound and vibration pick-up devices so that quantitative analyses can be made on electrical, mechanical, and acoustical waves.

Acknowledgment is made of the able work of Messrs. M. S. Mead and T. M. Berry in the development of the noise meter and analyzer, and of C. D. Greentree in the development of the vibration pick-up.

Discussion

For discussion of this paper see page 1071.

Induction Regulator Noise

BY J. P. FOLTZ* and W. F. SHIRK*

Associate, A. I. E. E. Non-member

IN recent years designers of machines have been confronted with the necessity of making them operate with a minimum of noise. This has been brought about by a general rebellion of the public against unnecessary noises. The Noise Commission in New York City is both an effect and a cause to bring about further action on the subject. For many years there has been some agitation along this line, but lately it has become an important factor affecting the sale of the machines.

At the same time the design of quiet machinery has been made more difficult due to a tendency to reduce the size and weight of machines. This makes it necessary to work electrical and magnetic materials harder, figure stresses higher and allow greater deflections. With the improvement of electrical and magnetic properties of materials it is possible to build machines smaller from an electrical point of view. This produces a weaker structure mechanically which will vibrate more since the forces involved have not been changed or may even have been increased under certain conditions. It is necessary then for the engineer to look to improvements in mechanical design which will overcome these difficulties that are being aggravated by the trend in electrical design.

One of the most important prerequisites to the study of noise in induction regulators is the establishment of a method whereby quantitative measurements of noise can be made. Surroundings have a very large affect on the values of sound intensity obtained by sound measurement for any particular machine, so for consistent results all sound measurements would have to be made under identical conditions, as regards not only background noises, but also surfaces in the vicinity. Induction regulators, however, are usually too heavy to handle readily without heavy cranes and it is therefore quite inconvenient to move them to a place suitable for making sound measurements.

In order to eliminate many of these difficulties it has been found desirable to measure surface vibration instead of sound produced. That the surface vibration bears a quantitative relation to the sound produced can be seen from the fact that in regulators there are no fans or other moving parts which might act on the air directly to produce a sound; therefore, the sound must all be radiated from some vibrating surface. By following this idea all the need for special surroundings is eliminated and direct comparison of machines is possible on the test floor during working hours and with a minimum expenditure of time. This principle also makes it possible to make representative measurements on the various elements of the machine tracing the vibration to its source and determining the effect of different conditions on the source.

The noise produced by induction regulators and transformers is quite simple in character as compared with noise produced by rotating machinery. The noise, exclusive of clicking of relays during the period of voltage adjustment is radiated directly from the tank walls, cooling tubes, and boxes housing the control panels which fasten directly on to the side of the regulator, and has a fundamental frequency double that of the supply voltage. There may be harmonics of the fundamental present depending on the supply voltage, but they are usually not large where the voltage supply is reasonably pure.

Experience gained on small single-phase regulators in the past has indicated that invariably noise could be reduced by a well-balanced gap and small clearances in the bearings. More recent experience, however, gained with larger machines, has shown that there are several types of vibration, especially in larger regulators, which no amount of accuracy as to fits and centering will cure.

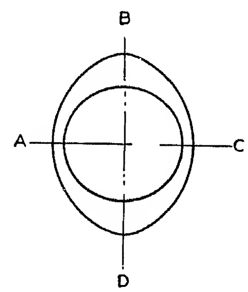

FIG. 1—DEFORMATION OF STATOR INTO AN ELLIPTICAL SHAPE BY THE FORCES OF THE FIELD—SHOWN EXAGGERATED

According to the source, regulator vibration may be divided under three headings. In its elementary form an induction regulator consists of a stator of hollow cylindrical shape built of steel punching rings stacked to a considerable length and assembled in a frame, a rotor separated from the stator by a small air gap and held concentric by means of bearings fastened to the stator at either end. This whole assembly is immersed in a tank of oil for cooling and insulation. Both stator and rotor are slotted to provide space for the coils. The coils on the rotor are usually the primary and are connected across the supply line.

The force due to the flux across the gap is radial and is balanced. Its only effect then is to deform the stator into an ellipse, in the two-pole regulator, with minor axis corresponding to maximum of flux wave. This

1. Both of the Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

deformation is shown much exaggerated in Fig. 1. In the case of a single-phase machine the position of the deformation does not move but its amplitude changes with time since the field is alternating. Since the parts are not polarized, both halves of the cycle produce the same deflection in the same direction and the resulting vibration has a frequency double that of the line. In the case of a three-phase machine the value of the field does not change but the axis or maximum point moves around the gap at synchronous speed. In this case the

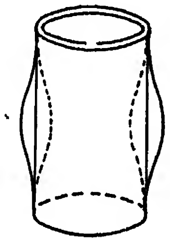

Fig. 2—Vibration of Stator Under Radial Forces of Field—Shown Exaggerated

The drawing is not quite accurate since the end portions vibrate some but the increased rigidity due to the end frames prevents the ends vibrating as much as the center

deformation does not change magnitude but rotates in space with the field. The result of this deformation into an elliptical shape is a vibration of the stator in a radial direction. This may be termed "hoop vibration" to distinguish it from the other types to be mentioned later. This vibration is transmitted directly through the oil to the tank. Fig. 2 gives an exaggerated view of this motion.

If the rotor is out of center or eccentric with regard to the stator (as shown exaggerated in Fig. 3) the forces

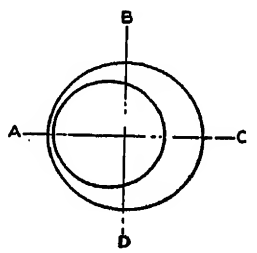

Fig. 3—Exaggerated Sketch Showing Rotor out of Center in Stator

on either side of the rotor are not balanced and there is a force tending to vibrate the rotor and stator as beams supported at the ends. This vibration is transmitted to the tank partly through the top and partly through the oil directly from the stator.

A third type of vibration, evident in some single-phase regulators, is torsional. Torsional vibrations are carried to the cover through the worm and segment. While this type does not contribute much to the noise directly, if of considerable magnitude, it is liable to cause impact between worm and segment with a resultant rattle that is quite objectionable.

These types of vibration are the same frequency but it is necessary to distinguish between them when the question of source or cause of vibration arises.

The more important type of vibration in large three-phase two-pole regulators is produced by the elliptical deformation of the stator under the action of the field. In the case of four-pole machines the field acts at four points with the result that the effective rigidity of the stator is increased and the deflection decreased. Most machines, however, are of the two-pole type, since they are more economical of floor space. Then this type of vibration becomes the most important in larger machines. In small machines the stator rigidity is high enough to limit this vibration to a point where the other types become of importance.

In designing regulators it is desirable to calculate the probable amount of vibration that will be produced in the finished machine. It is possible to do this approximately by the usual theoretical formula provided certain assumptions and approximations are made.

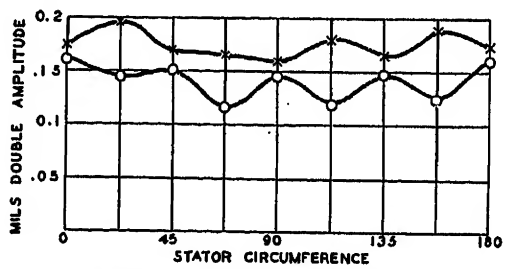

Fig. 4—Relative Motion Between Core and Frame

X—Vibration amplitude on stator
O—Vibration amplitude on frame
Punching bars at 0-45-90-135-180 degrees

In the case of hoop vibration the annular rigidity of the stator is the factor controlling the amplitude, so the amplitude of vibration is the same as the static deflection under the load produced by the force of the field. In the case of a three-phase regulator the field is fairly well distributed and it may be considered as having a sine wave of distribution. Then for a two-pole regulator the deflection can be calculated approximately by the following expression:

$$= \frac{q R_1^3 R_2^*}{18 E I}$$

q = maximum magnetic force—lb. per inch of circumference.
R_1 = radius to neutral axis of ring.
R_2 = radius at air gap.

In computing I it is necessary to correct for the fact that the stator is built from punchings. In this example I is calculated as 75 per cent of I for a solid ring of the dimensions of the stator exclusive of teeth.

*"Etude Meneanique Et Uswage Des Machines Electriquet," by H. De Pistoye, p. 560.

Again the I of the frame should be included but the frame and punching are not rigidly connected so the two moments of inertia will be added. Fig. 4 gives comparative readings on frame and core of a large regulator operating at rated voltage and frequency. This shows the flexibility between frame and core. When computing q_0 it is necessary to correct for the effect of slotting. A discussion of this problem is given in the appendix.

$q = 0.014\, B_m^2\, L\, F$

L = length of core

F = factor to correct for slotting

For a representative three-phase two-pole regulator the following values have been computed using a value of F for the punchings 75 per cent of the I of a solid ring.

$R_1 = 13$ in.

$R_2 = 9.6$ in.

I punchings 114 in.^{4}

I frame 4 in.^{4}

I total 118 in.^{4}

$q = 210$ lb.

$= 0.07 \times 10^{-3}$ in. deflection.

Double amplitude as calculated $= 2 \times 0.07 = 0.14 \times 10^{-3}$ in. The measured double amplitude (average of several readings) was 0.19×10^{-3} inches.

If the rotor is slightly eccentric in the air gap the forces of the field on either side of the stator do not balance and the rotor is deflected toward the narrower gap. This same action also bends the stator as a beam. When the field changes this deflection takes the form of a vibration having a frequency double that of the supply voltage. This vibration adds to the hoop vibration on one side and subtracts on the other in the case of a three-phase, two-pole machine. The vibration of the rotor is carried to the covers and thence to the outer tank surface. If the rotor was very eccentric in the gap this vibration might become bad but it is quite possible to keep the eccentricity low enough in practise on large regulators to keep this type of vibration within limits.

Torsional vibrations of the rotor, due to considerable flexibility in the shaft and worm are controlled by the mass reaction of the rotor. This type of vibration is of more importance in small regulators, or large regulators on 25-cycle service. The frequency of the torsional vibrations is double the frequency of the line as with all the other types and the amount of frequency reaching the outer surfaces is usually not large so ordinarily it does not directly contribute much to the noise. However, in the worm and segment there is a system which has a variable spring constant so if this torsional vibration becomes large, higher frequencies are introduced due to impact between worm and segment. These lie in the range of greater ear sensitivity and are very objectionable. The introduction of flexibility between rotor and segment or between worm and cover eliminates this difficulty preventing the vibration reaching the cover and preventing impact between worm and segment.

While the vibration has its origin in the stator core it must pass through the oil and move the tank wall before it can be radiated as sound. The oil is nearly incompressible and serves to carry the vibration to the tank. The tank however, does not follow the core. It appears to have a number of locally resonant regions on the surface whose positions are not at all definitely determined. These do not necessarily locate themselves symmetrically on two similar machines or even on the two sides of one machine. They even quite frequently shift position without any apparent reason so that it is seldom possible to check amplitude measurements made on a particular point on the tank. As a result, in order to compare machines, it is necessary to take a number of readings in both a vertical and horizontal plane and obtain an average.

The presence of cooling tubes further complicates the action of the tank wall. Tubes are liable to be resonant in themselves. This should be guarded against not only because of the noise caused, but because due to the large amplitudes that occur at resonance it is likely to fatigue the weld where the tube enters the tank resulting in a leak. In these tubes one may find anything from nearly rigid support to nearly perfect flexibility at end welds. In order to be safe then it is necessary to work outside the region between these two extremes. This resonance region can be computed by consideration of weight of tube full of oil, inertia of cross section and length. It is then the problem of the uniformly loaded beam. Where the tubes are supported upon one another or on the tank at any place other than the end, the problem becomes indeterminate analytically.

With information gained from vibration measurements and some knowledge of the magnetic, electrical, and fundamental mechanical conditions in the general type of structure represented by the induction regulator, it is possible to classify regulator noise according to its source in the regulator proper, determine the relative importance of each class in different sizes and types of regulators and even calculate approximately what to expect from a new design. Each portion of the regulator may be approximated by some mechanical system whose characteristics can be determined. Then checking with actual measurements on finished machines it is possible to correct any erroneous mathematical assumptions as well as obtained correction factors which will bring the calculated values closer to the actual values obtained by measurement and make it possible to calculate accurately performance of new designs as regards noise.

The authors wish to acknowledge their indebtedness to Mr. J. P. DenHartog for theoretical work in connection with stator hoop deflection and to Mr. L. G. Tubbs for suggestions relating to the experimental tests.

Appendix

In calculating the force between stator and rotor in an induction regulator it is necessary to take into consideration the effect of the slotting of both stator and rotor. The effect of the slots is to increase the reluctance of the gap. The flux does not decrease, however, since the voltage induced by it in the primary must balance the supply voltage. The result is an increase of energy in the gap or an increase of force between the stator and rotor. The exact amount of this force will vary with the relative position of teeth on the stator and rotor. By use of the fundamental equation of potential magnetic energy and Carter's coefficients for calculating reluctance of a gap between toothed surfaces, it is possible to obtain a factor representing the average over the pole face of the increase of force due to slotting or due to concentration of flux in the gap over a tooth.

If W represents the potential magnetic energy in the field between any two magnetic bodies separated by an air gap and r is the reluctance of the gap:

$$W = k\, r\, \phi^2$$

where k is a constant depending on the units used.

Suppose the gap is decreased a differential amount $d\,g$; this will change the reluctance by $d\,r$, then the change in energy

$$d\,w = k\,\phi^2\,d\,r \qquad (1)$$

Also if f is the force between the two bodies

$$d\,w = -f\,d\,g \qquad (2)$$

From (1) and (2)

$$f = -k\,\phi\,\frac{2\,d\,r}{d\,g} \qquad (3)$$

The sign indicates that when the body moves in the direction of the force the field does work and hence loses energy.

In the case of two smooth surfaces separated by an air gap where the gap is uniform

$$r = C\,g$$

and

$$\frac{d\,r}{d\,g} = C \qquad (4)$$

When the surfaces are slotted, the reluctance will vary not only with the gap but also with the dimensions and number of slots. If the increase in reluctance due to slotting is taken care of by means of Carter's coefficients then

$$r = C\,v$$

where v equals Carter's coefficient times the actual air gap. Then

$$\frac{d\,r}{d\,g} = C\,\frac{d\,v}{d\,g} \qquad (5)$$

when $\frac{d\,v}{d\,g}$ approaches unity we have the condition of smooth surfaces.

Then if f_1 and f_2 represent the force between the two surfaces with and without slots respectively

$$f_1 = -k\,\phi^2\,C \qquad \text{from (3) and (4)}$$

$$f_2 = -k\,\phi^2\,C\,\frac{d\,v}{d\,g} \qquad \text{from (3) and (5)}$$

The change in force

$$-f_2 f_1 = -k\,\phi^2\,C\left(\frac{d\,v}{d\,g} - 1\right)$$

Using Carter's coefficients to determine v

$$v = \frac{g^2 + \frac{b}{5}}{g + \frac{b}{5} - \frac{b^2}{5\,c}}$$

$$\frac{d\,v}{d\,g} = \frac{2g\left(\frac{g}{2} + \frac{b}{2} - \frac{b^2}{5\,c}\right) + \frac{b}{5}\left(\frac{b}{5} - \frac{b^2}{5\,c}\right)}{\left(g + \frac{b}{5} - \frac{b^2}{5\,c}\right)^2} = v$$

where b and c represent slot dimensions as shown in Fig. 5.

In order to find the increase of force due to slotting one member, it is necessary to evaluate $\frac{d\,v}{d\,g}$ for both

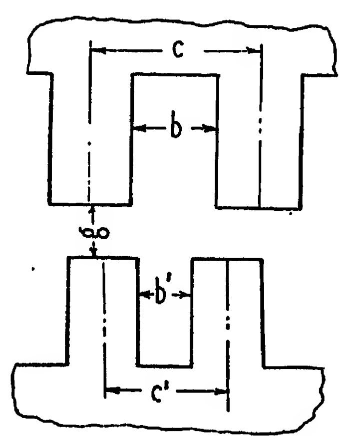

Fig. 5

stator and rotor. Then according to the use of Carter's coefficient, the factor by which the force calculated for uniform gap must be multiplied is $\frac{d\,v}{d\,g}$. A similar factor exists to correct for slotting the other member, so the total correction factor

$$f = v_s\,v_r$$

where v_r and v_s represent values for rotor and stator respectively. The average total force then between the rotor and stator, where both are slotted, is the force calculated for smooth surfaces having a similar gap and flux density, multiplied by F.

Discussion

For discussion of this paper see page 1071.

Magnetic Noise in Synchronous Machines

BY QUENTIN GRAHAM*, STERLING BECKWITH*, and FRANK H. MILLIKEN*
Associate, A. I. E. E. Non-member Non-member

Synopsis.—Magnetic noise in synchronous machines is shown to be of two principal kinds. One is a vibration in the tooth frequency range which usually occurs whether load is present or not; the other has twice line frequency and is dependent on load current. The vibrating part of the structure is usually the stator but may be the rotor and in nearly all cases of troublesome noise mechanical resonance is present. Some measurements of vibration on noisy machines are given.

WHILE magnetic noise in synchronous machines, particularly of the low-speed class, has often been a source of annoyance both to designers and operators, the literature on the general subject of noise is notably free from any discussion of the specific causes of noise in this class of machines. It is proposed in this paper to point out some of these causes and to show the extent to which noise may be controlled by the designer.

When the noise problem in synchronous machines was first investigated several years ago there was a great deal of speculation concerning the particular part of the machine that produced the principal vibration. Suspicion was cast on the stator teeth, on the overhanging tips of the poles, and on the poles as a whole. It finally became evident, however, that one of the principal sources of trouble was the vibration of the stator as a unit. Any condition which caused the stator to change from a circle to an ellipse periodically was likely to produce noise. Further investigation has revealed other sources of noise, particularly in the rotor, which may be almost as important as that in the stator.

The most frequent cases of stator vibration arise from flux distributions which, because of unequal magnetic pull on all axes, cause the stator to assume an elliptical shape. This requires, in effect, a two-pole flux distribution superposed on the normal distribution. It will be shown later that there are several ways in which such a flux distribution can be set up. In addition, it is also possible to have a four-pole distribution (or any larger number) which will tend to distort the stator at four (or more) points. Since the stator structure is much more rigid with respect to forces of this kind than to the two-pole force, they are not such frequent sources of trouble.

Rotor vibration takes more forms than stator vibration. One possibility is for the rotor rim between arms of the spider to move in and out, either in phase or 180 degrees out of phase with the segment between the succeeding spider arms. Another possibility is for the poles to vibrate circumferentially. A third mode of vibration occurs when parts of the spider rim vibrate axially. This last type of motion resembles that occurring in gears.

Under certain conditions it is possible to have magnetic forces tending to pull the rotor out of center. This requires a distribution of force having only one wavelength in the circumference of the machine. It might be called, by analogy, a one-pole distribution and is similar to what occurs when the air gap is unequal. Such forces are found in induction motors[1] due to combinations of rotor and stator harmonics, but are seldom encountered in synchronous machines. They would occur if odd numbers of slots were used, but since the number of slots is almost invariably an even number, especially in large machines, this type of vibration rarely occurs and will not be discussed here in detail.

CONSIDERATION OF PERIODIC FORCES

Two important classes of noise or vibration with reference to frequency have been noted and these will be discussed under separate headings. The first is tooth frequency noise, usually occurring at no-load and independent of armature current, but sometimes proportional to or increasing with armature current. Although called tooth frequency, it may be slightly greater or less than tooth frequency, due to the rotation of one member. The second type of noise is not present at no-load, but depends upon the armature current for its existence. The frequency is pole frequency, that is, twice the line frequency, and is thus much lower in tone than the other type of noise. It is, in fact, usually spoken of as a vibration and while still in the audible range, it is more often felt than heard.

It is realized that noises of other frequencies may exist but as they are relatively unimportant they will not be considered here.

Tooth-Frequency Forces. In order to explain the cause of tooth-frequency noise, it is convenient to consider the change in flux of a single pole as it moves over one stator tooth pitch. Clearly there is some point as in (*a*) Fig. 1, where the flux is a maximum and some point as in (*b*) Fig. 1, where it is a minimum. Thus the flux in each pole rises and falls slightly each time a tooth pitch is traversed and the magnetic pull between that pole and the stator goes through a corresponding cycle. If the number of slots per pole is an

*Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

1. *Quiet Induction Motors*, L. E. Hildebrand, A. I. E. E. TRANS., 1930.

integer, each pole on the rotor is passing through the same cycle at the same time. That is, the variation in magnetic pull is in time phase in all poles. If the rotor is centrally located the result is simply a variation in the radial pull which is the same across each axis.

Suppose now that the number of slots per pole is not an integer. Then the magnetic pull of all poles is not

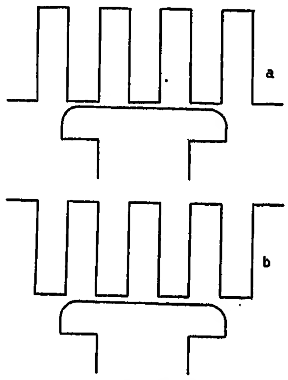

FIG. 1—POSITION OF MAXIMUM AND MINIMUM PERMEANCE

a. Position of maximum permeance
b. Position of minimum permeance

in time phase and it becomes necessary to examine the relative position of slots and poles in more detail.

The simplest case is that in which the number of slots per pole consists of an integer plus a fraction having one in the numerator, such as 5 1/7. It is evident that if No. 1 pole has maximum flux at a given instant, No. 2 pole, being 1/7 tooth pitch from the same relative slot position, will have slightly less flux. Each succeeding

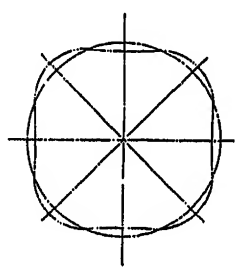

FIG. 2—DISTORTION OF STATOR DUE TO FOUR-POLE TYPE OF PULL

pole will have a little less flux than the preceding one until the minimum is reached, after which there will be a gradual increase. If the machine under consideration has fourteen poles there will be two areas diametrically opposite having high flux density and two areas on an axis at 90 degrees to these having low density. The result is a higher magnetic pull along one axis and a tendency to give the stator an elliptical shape. In the case of a 28-pole machine having the same number of slots per pole there would be four high density areas and four low density areas giving a distortion as shown in Fig. 2.

It now remains to see what happens to the magnetic pull or stator distortion as the field member rotates. Using the example above, it may readily be seen that when the rotor has moved 1/7 tooth pitch the point of maximum density has shifted from Pole No. 1 to Pole No. 2. When a tooth pitch has been traversed, the point of high density has moved seven pole pitches *as measured on the rotor*. With reference to the stator the distorting force has moved seven pole pitches plus one tooth pitch.

Thus it may be seen that a given point on the stator goes through a complete cycle from maximum pull through minimum pull and back to maximum, while the rotor moves slightly less than one tooth pitch. It is

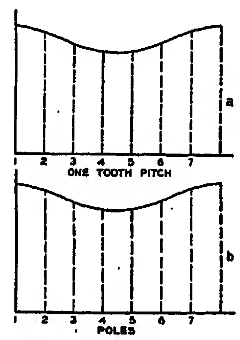

FIG. 3—VARIATION OF PULL ON ONE POLE

a. Variation of pull on one pole as it moves over one tooth pitch
b. Variation of pull-per-pole over a group of d poles (Case 1)

shown in Appendix 1 that for the general case the frequency of the deformation is: tooth frequency + twice line frequency [slots per pole − nearest integer].

It is also shown as Case 1 in Appendix 1 that in order to have a gradual distribution of density or pull over a group of adjacent poles, certain fractional numbers of slots must be used. With certain other fractional

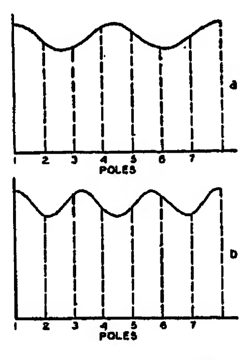

FIG. 4—VARIATION OF PULL ON ONE POLE (CASES 2 AND 3)

a. Variation of pull-per-pole over a group of d poles (Case 2)
b. Variation of pull-per-pole over a group of d poles (Case 3)

numbers it is shown that the poles having high density at a given instant are so alternated with poles of low density that the frame deformation is of a different form. The curves of magnetic pull plotted against pole position in these cases have two or more maximum points as shown in Fig. 4. To continue the example of a machine having 14 poles, the predominant component

of the pull now has the effect of a rotor of more than two poles. Due to the greater rigidity of the frame, a pull of short wavelength is usually unimportant. There may be a small component of pull which has a two-pole distribution, but its magnitude is so much less than the Case 1 distribution shown in Fig. 3b that there is seldom any trouble from noise. As a result, it is possible to classify fractional slot machines into those that are usually free from serious no-load noise (Case 2, Case 3, etc.) and those that are quite susceptible to it (Case 1).

Pole-Frequency Forces. It has previously been pointed out[2] that the armature m. m. f. waves of machines having fractional slot windings contain components of low harmonic order. With certain slot arrangements there may be a component wave of m. m. f., and flux, having a two-pole distribution. Other components may exist at the same time which are also of fewer poles than the actual number of poles on the machine. Considering first, for simplicity, a two-pole

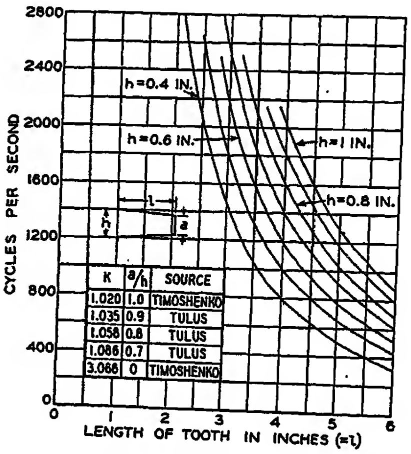

FIG. 5—RESONANT FREQUENCY OF STATOR TEETH FOR TANGENTIAL VIBRATION

distribution superposed on the main distribution of flux, we again have the requirement for elliptical distortion of the stator just as in the case described under no-load noise. The difference, however, is that in this case the two-pole field travels at the synchronous speed of a two-pole rotor and thus produces a vibration of the stator at twice normal frequency. Similarly, a subharmonic having a four-pole (or higher) distribution may distort the frame at four or more points and cause vibration. The frequency will be twice normal in all cases since the rate at which poles of the superposed distribution pass a given point on the stator is the same regardless of the number of poles.

While it is convenient to start from the assumption of a single two-pole component of armature m. m. f. in describing the effect of a two-pole pull, this simple case does not complete the story. It is shown in Appendix 2 and illustrated in Fig. 10 that when there exist any two components of flux differing in harmonic order by two, there will be a resulting modulated wave which gives the same effect as a simple two-pole distribution.[3] When the order of magnitudes is examined it is found that these modulated waves are really the important ones.

Since it is possible to have a two-pole pull due to a single sub-harmonic and at the same time to have combinations of two higher harmonics each giving an effective two-pole pull, it becomes necessary to combine the two or more sets in the proper phase relation. In some cases the several components may add in such a way as to partly cancel one another while in other cases they add to form a higher total.

The elimination of any one unwanted harmonic can be accomplished by changing the coil grouping[3] in two relatively simple ways—even though it is complicated somewhat in a machine with paralleled armature circuits by the fact that any change made must be duplicated in each parallel of every phase. The first method is merely to cut out coils whose electrical position coincides with the vector position of the harmonic to be eliminated. The second way is to shift coils from one phase group to the next adjacent group in such a way that the troublesome harmonic is decreased without reducing the terminal voltage of the machine appreciably.

MECHANICAL ASPECTS

The existence of various forces tending to distort the machine has been established. It has been pointed out that the most serious cases of trouble arise from the two-pole type of pull, since forces distributed in this way produce the greatest deflection. Experience, however, seems to show that nearly all cases of objectionable noise* occur when there is resonance between the magnetic forces and some part of the mechanical structure. For this reason even the four-pole or higher force distribution may result in trouble if the necessary resonant condition exists. It becomes of interest then to consider the possibilities of resonance.

Stator Vibration. It may not be entirely obvious that a rotating deflection of the stator can be affected by resonance. It can be shown however, that the rotating pull or deflection may be considered as the result of two stationary pulsating pulls which are 90 degrees apart in space and time. Thus resonance to a traveling elliptical deflection is quite possible. In practise, one of these components is usually damped considerably by the frame supports.

Tests made on a number of machines have shown that resistance to a distortion in the plane of the laminations is usually dependent entirely upon the frame or clamping plates and not upon the laminations. The

2. *The M. M. F. Wave of Polyphase Windings*, Graham, A. I. E. E. TRANS., 1927, Vol. XLVI, p. 19.

3. P. L. Alger and W. V. Lyon, discussion of *The M. M. F. Wave of Polyphase Windings*, Graham, A. I. E. E. TRANS., 1927, Vol. XLVI, p. 29.

*We are well aware of the difficulties of defining an objectionable noise. The term is necessarily relative and is used here to mean a noise considerably worse than that of average commercial machines for industrial plants or power stations.

laminations become so much dead weight attached to the frame but add nothing to its stiffness. The following formula developed by Timoshenko[4] for simple rings has been applied to complete stators in a number of cases and has given useful results, as shown by Table I and by the comparison between calculated and test results in Figs. 7 and 8.

TABLE I—NATURAL FREQUENCIES OF STATORS OF TYPICAL SYNCHRONOUS MOTORS FOR COMPRESSOR DRIVE

Frame size suitable for	Natural frequency of stator				
	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$
100 hp. 360 r. p. m.	190	540	1,030	1,670	2,450
150 hp. 277 r. p. m.	127	360	690	1,120	1,640
200 hp. 200 r. p. m.	85	240	460	750	1,100
400 hp. 200 r. p. m.	70	200	370	620	900
600 hp. 150 r. p. m.	54	150	290	470	690

$$f = \frac{1}{2\,\pi\,r^2}\sqrt{\frac{g\,E\,I\,i^2\,(1-i^2)^2}{\gamma\,A\,(1+i^2)}} \qquad (1)$$

where E = Modulus of elasticity.
γA = Weight of ring per circumferential inch.
r = Mean radius of ring.
I = Moment of inertia of frame (punchings considered as dead weight).
i = Wavelengths per circumference.
g = Acceleration of gravity.

Rotor Vibration. While it has been shown that stator vibration is of consequence only when there is a force tending toward elliptical distortion of the frame, rotor vibration may also be set up by a periodic variation in the magnitude of the normal radial gap force, such as that occurring in integral slot machines.

Rotor resonance is affected so much by the rotor arms that it is not susceptible to such accurate calculation as the stator. Two formulas,[5] however, have been found useful in giving the probable upper and lower bounds of resonant frequency for radial vibration of the rim. One, for a section of rim pivoted at both ends, is:

$$f = \frac{C_3}{2\,\pi\,r^2}\sqrt{\frac{g\,E\,I}{\gamma\,A}} \qquad (2)$$

Where g, E, I, r, and γA are the same as in formula (1), and α is the angle subtended by the rim segment.

$\alpha = 0 - 20° - 40° - 60° - 80° - 100°$

$C_3 = \infty - 321 - 78.5 - 33.6 - 17.8 - 10.65$

The other, for a section of rim fixed at both ends, is:

$$f = \frac{C_1}{2\,\pi\,r^2}\sqrt{\frac{g\,E\,I}{\gamma\,A}} \qquad (3)$$

Where g, E, I, r, and γA are the same as in formula (1), and α is the angle subtended by the rim segment.

$\alpha = 20° - 40° - 60° - 80° - 100°$

$C_1 = 504 - 124 - 53.8 - 29.2 - 17.9$

A formula for the resonant frequency of gears with different modes of rim vibration is also available,[6] and has been found useful, but requires so much empirical correction when applied to plate spiders that it will not be given or discussed here.

Stator Tooth Vibration. Tangential vibration of the stator teeth is often thought of as the simplest and most probable form of mechanical vibration causing tooth-frequency noise. If it be agreed that resonance is necessary for appreciable tooth motion, then Fig. 5 shows[7] that almost all machines are out of the range

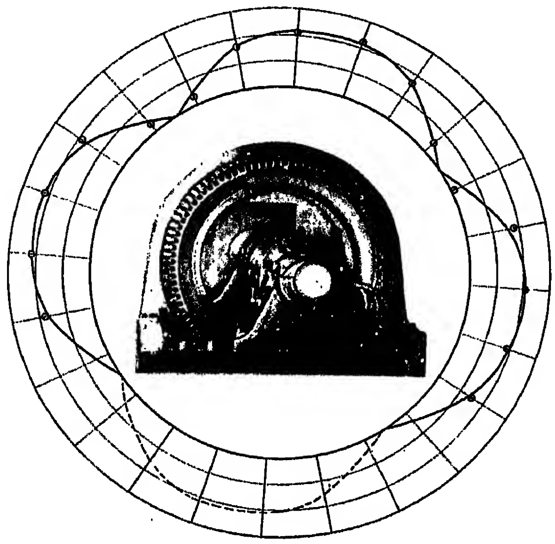

FIG. 6—MAGNITUDE OF VIBRATION AT VARIOUS POINTS AROUND THE STATOR

where tooth resonant frequency approaches tooth frequency. The assumption that resonance is essential is justified by past experience, and appears reasonable from a consideration of the stiffness of the teeth.

Test Results. Figs. 6 to 9 show the results of several vibration tests of machines having pole frequency vibration. The measurements were made by amplifying the resistance variations in a phonograph pick-up the needle of which followed the motion of the frame. The accuracy of this method is indicated by the consistency of the test points.

Fig. 6 shows the variation in the amplitude of the vibration at different points around the circumference. (The nodes could be located easily by feeling the frame.) The picture of the machine in the center of the figure is

4. "Vibration Problems in Engineering," S. Timoshenko, p. 297, Van Nostrand Co.

5. "Vibration of Frames of Electric Machines," Den J. P. Hartog, *ASME*, presented May, 1927.

6. "Natural Frequency of Gears," R. E. Peterson, *ASME*, presented December, 1929.

7. Resonant Frequency for Wedges and for Rectangular Teeth from S. Timoshenko, Vibration Problems in Engineering, pages 234 and 267. Factors for tapered teeth derived by E. A. Tulus of Westinghouse Elec. & Mfg. Co.

correctly oriented with the vibration curve. Measurements were made on the back of the punchings half way between the ends of the core although the results would have been the same had they been made at any point on the same axial line.

Fig. 7 shows the change of vibration amplitude as the supply frequency is varied. The calculated resonant

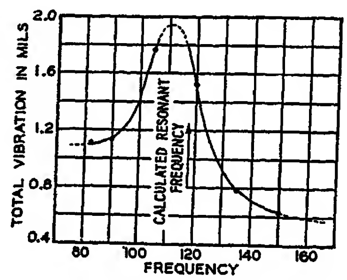

Fig. 7—Resonance Curve for Vibration of the Type Shown in Fig. 6

frequency for the frame is shown on the curve. In making the calculation the punchings were added as dead weight but the armature copper was not included since it is probable that the slot clearance, flexibility of end windings and damping effect of insulation may cause the effective mass to be zero or even negative.

Fig. 8 shows similar tests on a machine which was later braced to add stiffness to the structure. The reduction in vibration at the operating point (120-cycle vibration) is considerable. The frame of this machine was not weak in comparison with frames of similar sizes and types but happened to have a natural period which gave unfortunate results.

Fig. 9 is an exceedingly interesting curve showing a

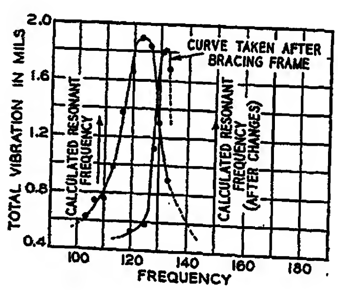

Fig. 8—Resonance Curve for Vibration of the Type Shown in Fig. 6

dip in the center which was found to be due to end-bell resonance at that point. The violent oscillation of the rather light end-bells absorbed sufficient energy to decrease the frame vibration at what otherwise would have been its region of greatest amplitude. Since calculation of the resonant point of a complex structure like an end-bell is practically impossible and since its likelihood of setting up objectionable overtones is great it illustrates the difficulties that face the designer in trying to predict his results.

Vibration of a stator in the tooth-frequency range has not been measured accurately although it has been observed and definitely identified. One machine exhibiting this type of vibration had eight distinct nodes (the four-pole type of pull). The number of slots per pole was such as to give a vibrating frequency of 635 cycles while the calculated resonant frequency of the frame for this mode of vibration was 645 cycles. The same frame was later tested with four-node (two-pole) vibration and a resonant frequency of 115 cycles was found. Since there is definite ratio between the resonant frequencies for the four-node and the eight-node type of vibration as shown by equation (1) it can be concluded that the resonant frequency for the eight-node case was actually 623 cycles. This value

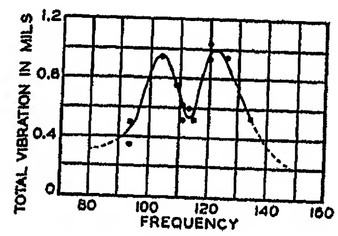

Fig. 9—Resonance Curve for Vibration of the Type Shown in Fig. 6

compared with the frequency of 635 cycles for the disturbing force indicates that a high state of resonance existed.

Appendix 1

Let the magnetic pull on each pole as it moves through one tooth pitch be represented by the curve of Fig. 3a. In the case of fractional slot windings each pole passes through the same cycle but not in time phase with all other poles. Let the curve, Fig. 3a, be divided in d parts where d is the denominator of the fraction expressing the slots per pole. Then at a given instant the pull of each pole can be taken from the intersection of the curve with one of the vertical lines. The particular point on the curve applying to any given pole is found from a consideration of the fraction expressing the slots per pole.

$$\text{Let, slots per pole} = \text{any integer} \pm \frac{n}{d}$$

There are several cases to be considered depending on the value of n.

Case 1; $n = \pm 1$. If $n = +1$ it is easily seen that adjacent poles will have values of pull taken from adjacent points on the curve in Fig. 3a. That is, between the center of pole No. 1 and pole No. 2 there is an integral number of tooth pitches plus $1/d$ tooth pitch. Thus the pull of pole No. 2 at any instant is the same as the pull of pole No. 1 will be after it has moved $1/d$ tooth pitch. A curve showing the pull of a group of d poles is given in Fig. 3b.

It can be seen that a similar condition would exist if $n = -1$.

Case 2; $n = \pm 2$. When $n = +2$ adjacent poles will have values of pull taken from every second division of the curve in Fig. 3a, since each pole is an integral number of tooth pitches plus $2/d$ tooth pitch from the pole next to it. The curve of pull for a group of d poles in this case takes the form shown in Fig. 4a. A similar curve is obtained if $n = -2$.

Case 3; $n = \pm 3$. *Case 4*; $n = \pm 4$, etc. Curves are obtained similar to Fig. 4 with three, four, etc., waves per group of d poles.

It is possible to have vibration in any of these cases, but experience indicates that it is usually the machines having Case 1 distribution that produce troublesome noise. Thus machines having a number of slots per pole consisting of an integer plus a fraction having numerator equal to plus or minus one are most likely to be noisy.

In Fig. 4b a smooth curve has been drawn through a very few points so as to give a curve which exactly repeats. This simplification of representing a few points by a single harmonic curve, even if the curve passes through all the points, is not quite exact. Furthermore, the assumption that the pull of each pole may be concentrated on its centerline is not strictly true. For these reasons the analysis must be taken as an approximate one which is convenient for illustration and which, fortunately, is close enough to the facts to give practical working rules. It may be possible, however, in Cases 2, 3, 4, etc. to have a small component of force distributed in space as in Case 1 (but traveling at a different speed) and, if a resonant condition exists, to have troublesome vibration. No such cases, however, have been observed.

Frequency of Vibration. For Case 1 distribution it is evident that the wave of force moves one pole pitch on the rotor while the rotor itself moves $1/d$ tooth pitch. When $n = +1$ the wave of force moves against the direction of rotation and when $n = -1$ it moves with rotation. Since the wavelength of the force wave is d pole pitches it is evident that the frequency of the force with reference to the rotor is $2 f s$ where f is line frequency and s is slots per pole. That is, it is tooth frequency. During one cycle with respect to the rotor the wave is carried forward or backward one tooth pitch by the motion of the rotor. Since there are $s\,d$ teeth per wavelength the frequency of the vibration with respect to the stator is increased or decreased by $1/s\,d$ times the rotor frequency. Or, stator frequency equals $2 f s \pm \dfrac{2f}{d}$.

For Cases 2, 3, etc., where there are n wavelengths in $s\,d$ teeth, stator frequency equals

$$2 f s \pm \frac{2 f n}{d}.$$

This can be expressed as:

Tooth frequency + twice line frequency [slots per pole − nearest integer] (4)

Appendix 2

Since any given harmonic of flux will not have the same effect in producing pull when it exists by itself as when it is superposed on other harmonics, as illustrated graphically in Fig. 10, it is necessary to consider all harmonics together and take cross-products into account in obtaining pull from flux. In the following analysis, however, only the harmonics cutting the stator with 60-cycle frequency will be considered, as the others can contribute nothing to the 60-cycle vibration phenomena observed. Thus the harmonics

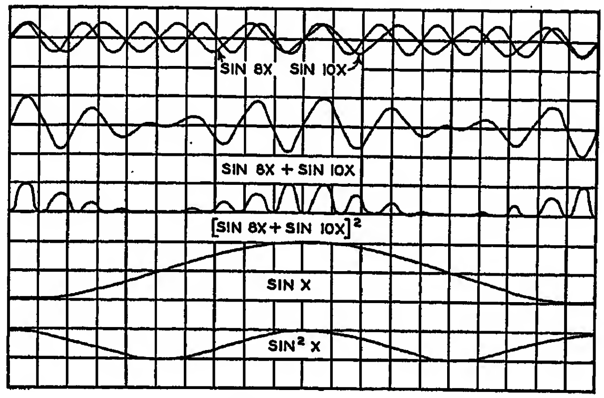

FIG. 10—COMBINATION OF HARMONICS

left to consider will be only the fundamental of field flux and harmonics due to the armature magnetomotive force.

The general Fourier equation for flux is:

$$\Phi = \sum_n A_n \sin\,(n\,(\theta + \beta_n) \pm \omega\,t) \qquad (5)$$

where n = order of harmonic ($n = 1$ for two pole harmonic)
θ = physical degrees
β_n = phase position of nth harmonic (physical degrees)
$\omega = 2\,\pi\,f$
t = time

As indicated[8] in Table II, the numerical summation is

TABLE II—POSSIBLE SUB-SYNCHRONOUS HARMONICS
(For three-phase machines with an even number of slots and 60-degree phase belts)

1. For machines with $10 + 12\,K$ poles,							
Possible harmonics are.....	+1,	−5,	+7,	−11,	+13,	−17,	+19, etc.
2. For machines with $14 + 12\,K$ poles,							
Possible harmonics are.....	−1,	+5,	−7,	+11,	−13,	+17,	−19, etc.
3. For machines with $8 + 12\,K$ poles,							
Possible harmonics are.....	+2,	−4,	+8,	−10,	+14,	−16,	+20, etc.
4. For machines with $4 + 12\,K$ poles,							
Possible harmonics are.....	−2,	+4,	−8,	+10,	−14,	+16,	−20, etc.

K = any positive integer, including zero.
Negative harmonics rotate rotorwise.
Positive harmonics rotate counter-rotorwise.
Machines with 6k poles are omitted since no two-pole pull exists in these cases.

8. Table II was derived from formulas developed in *The M. M. F. Wave of Polyphase Windings*, Graham, A. I. E. E. TRANS., 1927, Vol. XLVI, p. 19.

considerably simplified by the fact that all harmonics do not exist in any one machine.

Substituting the values $n = +1, -5, +7, -11, +13, -17, \ldots$ etc. (where + or − refer to direction of rotation) and squaring:

$$F^2 = \text{constant} \times \text{pull} = \sum_n A_n^2 \sin^2 (n(\theta + \beta_n) \pm \omega t)$$

$$+ \sum_n [2 A_n \sin(n(\theta+\beta_n) \pm \omega t) \sum_{n=1}^{n=\infty} A_{n+1} \sin\{(n+1)(\theta + \beta_{n+1}) \pm \omega t\}] \quad (6)$$

Where the second summation of the second term is performed for every value of n in the first summation (*i. e.*, it is a doubly infinite series).

Using the relations:

$$\sin^2 \alpha = 1/2 - 1/2 \cos 2\alpha$$

and,

$$2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

and neglecting the components that do not vary with time, it will be found that, for the values of n given above (+1, −5, +7, −11, etc.) the "two-pole" traveling harmonics of pull are:

$$- 1/2 A_1^2 \cos(2\theta + 2\beta_1 - 2\omega t)$$
$$+ \sum_n A_n A_{n+2} \cos\{2\theta + n\beta_n + (n+2)\beta_{n+2} - 2\omega t\} \quad (7)$$

Where n in the summation takes only the values 5, 11, 17, 23, etc.

Similarly, the four-pole traveling harmonics of pull are:

$$\sum_n A_n A_{n+4} \cos(4\theta - n\beta_n + (n+4)\beta_{n+4} + 2\omega t) \quad (8)$$

Where n takes only the values 1, 7, 13, 19, etc.

Harmonics producing 6, 8, 10, etc., pole pulls can be obtained by the same process.

For a machine in which the third or fourth series of harmonics is indicated in Table II, the process can be carried through in the same manner, and, for the two-pole pull, gives the following:

$$\sum_n A_n A_{n+2} \cos(2\theta - n\beta_n + (n+2)\beta_{n+2} - 2\omega t) \quad (9)$$

where $n = 2, 8, 14, 20 \ldots$ etc., in the summation.

For four-pole pull the components are:

$$- 1/2 A_2 \cos(4\theta + 4\beta_2 + 2\omega t]$$
$$+ \sum_n A_n A_{n+4} \cos[4\theta - n\beta_n + (n+4)\beta_{n+4} + 2\omega t] \quad (10)$$

Where $n = 4, 10, 16, 22$, etc., in the summation.

The above analysis applies to machines with an even number of slots. A machine with an odd number of slots, can be considered as half of a machine with an even number of slots, and the results determined and interpreted accordingly.

Discussion

For discussion of this paper see page 1071.

Elastic Supports for Isolating Rotating Machinery

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IN spite of the increasing refinement in the design of electrical machinery, the combination of high rotative speeds, smaller machine parts, and less massive buildings, makes the problem of noise constantly more acute. In general, two types of noise originate from electrical machinery; direct air noise, and noise transmitted through the machine foundations to the framework of the building in which the apparatus is installed.[1] Difficulties of the latter type may often be remedied by a properly designed elastic support between the machine and building.

Elastic supports can be made to isolate from two types of disturbance; shocks of any frequency typified by the automobile spring suspension, or from vibration of one or several known and constant frequencies. The supports described in this paper fall in the second class. The function of these supports is to *isolate*, not *absorb*, vibration.

The effect of such a support may be demonstrated with the apparatus shown in Fig. 1. The mass M,

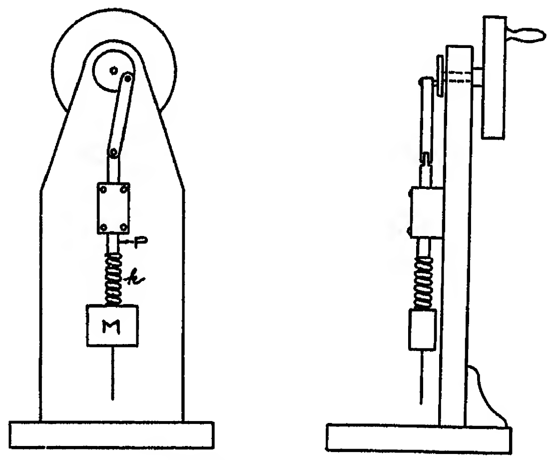

FIG. 1—DEMONSTRATION APPARATUS

constrained to move in a vertical direction, is supported through a spring, k, from the piston, P, which moves through a constant amplitude in approximately simple harmonic motion. M is elastically supported. At a very low speed the weight moves through the same amplitude and in phase with the driving member; with increasing speed the weight amplitude becomes larger until a maximum is reached at the critical speed of the system. Here the phase between the amplitude of the weight and the impressed force shifts through 180 deg., and at progressively higher speeds the weight amplitude becomes smaller, until, at a high speed, the weight remains practically stationary. Substantially the same phenomena is shown by a shaft while passing through its critical speed. The curve marked $b = 0$ in Fig. 2 shows the relation between amplitude (ordinate) and speed (abscissa). From this curve it can be seen that the elastic support is not useful until a speed well above the critical speed is reached.

Fig. 3 represents the general case in which a machine

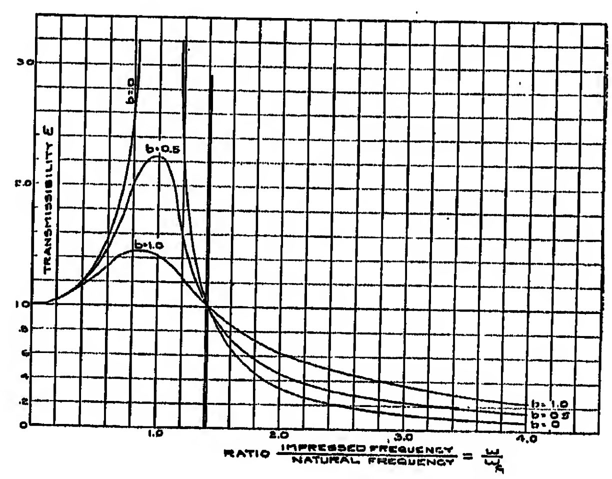

FIG. 2—TRANSMISSIBILITY CURVES

of mass M is supported by a spring of stiffness k, having a damping constant b. A sinusoidal force of maximum value F and angular velocity ω, is impressed on the mass. A certain proportion of F is transmitted to the base through the support and appears as a sinusoidal force of maximum value f. Call the ratio of the force transmitted to the force impressed the transmissibility ϵ. Appendix I gives in equation (4)

$$\epsilon = \frac{f}{F} = \frac{\sqrt{b^2\,\omega^2 + k^2}}{\sqrt{b^2\,\omega^2 + (k - M\,\omega^2)^2}}$$

When $b = 0$ this reduces to

$$\epsilon = \frac{1}{\left(\dfrac{\omega}{\omega_c}\right)^2 - 1} \tag{5}$$

where $\omega_c = \sqrt{\dfrac{k}{M}}$, the critical speed of the system.

As before the curve marked $b = 0$ in Fig. 2 gives the values of transmissibility, ϵ, plotted against a ratio of

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1. For references see end of paper.

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impressed frequency, ω, to the natural frequency of the system, ω_c. When the damping, that is, friction in the support, is increased, more force is transmitted to the foundation. Curves marked $b = 0.5$ and $b = 1.0$ show this effect plainly.

In our model (Fig. 1) the disturbing medium is an impressed amplitude on the spring, whereas in the actual case the disturbance is a force impressed on the mass. It can be shown, however, that the same expression for transmissibility is obtained in each case.

Several materials are available for elastic support. Metal springs may usually be designed to fit any given installation, but they require considerable space, and the ratio of stiffnesses in the various directions cannot be easily varied. Metal has an obvious advantage in its resistance to deterioration of all sorts.

Some materials such as cork, felt, and gelatinous substances, show a gradually decreasing thickness under load which solidifies the material, raising the stiffness, and hence raising the transmissibility of any support in which they may be used. Also, the damping is usually high in such material.

Rubber compounds containing a large percentage of pure rubber form a convenient substance for use in supports. Small volumes only are necessary, the natural life is fairly long, and may be extended with antioxidants. Oil slowly dissolves rubber, but protection can be arranged in most cases. Once its elastic properties are understood supports may be designed for nearly any combination of stiffness. Damping in rubber is not sufficiently high to cause increased transmissibility.

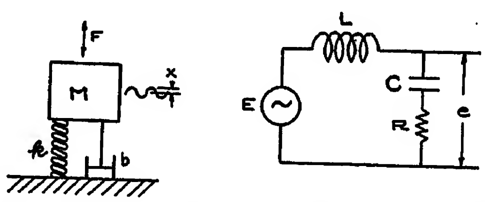

FIG. 3—ELASTICALLY MOUNTED MASS WITH ELECTRICAL ANALOGY

Damping in the support itself is detrimental since it must be designed to operate where damping raises the transmissibility. On the other hand damping in the foundation under the elastic support is beneficial. In the case of a lightly constructed building this damping would decrease resonance effects in the floors and walls, and decrease the amplitude at any frequency.

The noise and vibration of machines which may be transmitted through the foundation, and which may be isolated by an elastic mounting, fall into three general classes: unbalance, mechanical brush and bearing noise, and magnetic noise. Vibration due to unbalance occurs at rotational frequency which usually is near 1,200 r. p. m. or 20 cycles per second. These frequencies are too low to cause noise at the amplitudes ordinarily encountered, but may cause excessive motion of the foundations occasionally resulting in rattles.

Bearing noise is negligible in journal bearings. Ball bearings, however, cause considerable direct air noise and in some cases are the source of noise which is transmitted to the foundations. Brush noise transmitted to the foundations is not serious in the case of slip-ring motors.

Magnetic forces in the machine are the greatest source of noise. In a-c. machines the fundamental noise frequency is twice the applied electrical frequency or 120 cycles for 60-cycle machines. Harmonics of this fundamental frequency and other high frequencies also occur, but of relatively less intensity.

Elastic mountings used to isolate the above noises and vibrations usually do not constitute a simple system. In general, six natural frequencies of the machine on its mounting occur. These frequencies have been calculated by the expressions given in Appendix II and checked by observations. The modes of vibration of the machine at these frequencies are approximately as follows:

1. Angular motion about a horizontal axis parallel to the shaft.
2. Linear motion in a horizontal direction perpendicular to the shaft.
3. Angular motion about a horizontal axis perpendicular to the shaft.
4. Linear motion in a horizontal direction parallel to the shaft.
5. Linear motion in a vertical direction.
6. Angular motion about a vertical axis.

The elasticity of the mounting must be adjusted to the machine so that none of these resonant frequencies coincides with that of any disturbing force. If this occurs, the amplitude becomes large and the transmissibility high in the direction of resonance.

As described above, there is a wide variety of noise and vibration frequencies to be isolated, which range from the rotational frequency, the lowest, to the high magnetic disturbances which may be several hundred cycles per second. By inspection of the formula for the transmissibility, it is seen that if a mounting is designed to isolate a certain frequency, the higher frequencies will be isolated more effectively. For this reason the high frequencies, unless their amplitudes are greater than ordinarily encountered, do not present any difficulty.

The foundation upon which the machine is mounted affects the mounting design. Noise problems at the present concern installations in hotels, apartment houses, schools, and office buildings, where the foundation is usually a concrete floor. This floor is most sensitive to forces transmitted from the motor in a vertical direction which tend to bend the floor slab. For this reason the transmitted force normal to the floor should be given particular attention.

Damping, as has been described above, is beneficial in the foundation as it reduces the vibration amplitude of the floor which is the source of sound radiation. This is particularly true for high frequencies as they are damped more than the low frequencies.

Since the high frequencies are isolated best by the mounting and are also damped most by the foundation, there usually remain only two frequencies which are troublesome. These are the rotational frequency and

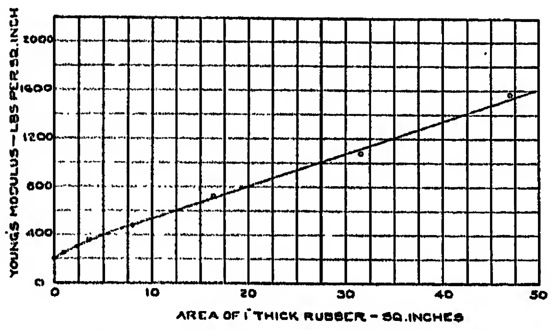

FIG. 4—VARIATION OF YOUNG'S MODULUS WITH AREA

the fundamental magnetic frequency. In estimating the transmissibility of a proposed mounting, the lowest disturbing frequency should be used as the impressed frequency in the calculation. The vertical resonant frequency of the mounting should be used as this determines the transmission normal to the foundation. The other five resonant frequencies, however, should also be comparatively low.

Elastically mounted machines may be separated into two classes depending upon whether they transmit torque to apparatus on a separate base or are self contained. Motor-generator sets are an example of the latter class. The stiffness of the support for these sets is not limited by the stability of the system against external forces such as belt pull. Hence the vertical natural frequency can be made sufficiently low to isolate the unbalance vibration of the machine, which is the lowest frequency of disturbance present. For these low frequency vibrations, which are below the audible range, a ratio of $\frac{\omega}{\omega_0}$ of 2 or 3 is sufficient.

A number of motor-generator sets has recently been mounted on rubber pads with success. In specifying rubber pads for a mounting of this kind account must be taken of the variable elastic properties of the material. Rubber is practically incompressible if entirely confined. When the material is compressed, the deflection is taken up by a change in shape rather than a change of volume. If the change in shape is restrained, the material becomes stiffer. For this reason Young's modulus in compression increases rapidly with an increase of the area of a rubber pad with respect to its thickness. If the area is very large and the rubber thin, the result is almost total confinement of the rubber and a correspondingly high stiffness. Fig. 4 shows a curve of Young's modulus in compression for 1 in. thick soft rubber as a function of the area.

The modulus of rubber also increases considerably with the deflection. For a given sample of rubber, the stiffness increases with load on the rubber. These variations of stiffness of rubber may be combined for the purpose of specifying the size of pad to be used under machines of various weights. Fig. 5 shows a useful arrangement of the data. This curve gives the area of a pad of rubber to be used for various loads to give a constant value of the vertical natural frequency of the system. This natural frequency is the criterion of transmissibility of the mounting. The curves given here are for 10 and 14 cycles. Similar curves can be drawn for other frequencies.

Four pads of suitable proportion to give the motor-generator set the proper vertical natural frequency are chosen from the above curve. These pads are slipped under the corners of the base between flat surfaces.

In the more general motor applications the mounting is of the first class as it must withstand belt pull. The mounting must have sufficient elasticity in the direction of the disturbing force to give satisfactory isolation, yet be sufficiently stiff to satisfy the requirements of belt operation. Mountings of this type have been used to isolate the torque pulsation of single-phase motors. The support can be made elastic in a torsional direction and stiff radially. This is an easy solution of the conflict between the requirements of elasticity for isolation, and rigidity against belt pull. The result is that the torque pulsation at 120 cycles is satisfactorily isolated. There remains some 120-cycle vibration in other directions which occurs both in single-phase and polyphase motors.

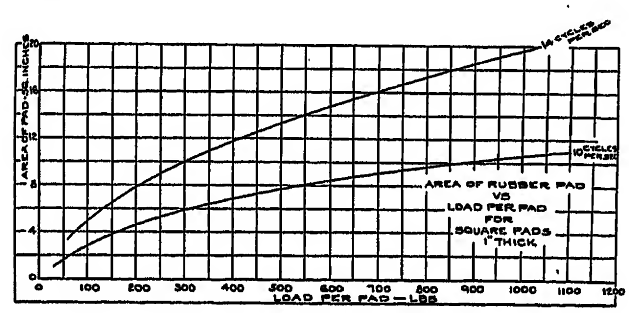

FIG. 5—AREA OF RUBBER PAD REQUIRED FOR VARIOUS LOADS AND FREQUENCY

The application of a mounting to polyphase motors is more difficult. The magnetic vibration in this type of machine may be in any direction, therefore the stiffness must be low in all directions in order to isolate the vibration. On the other hand, the mounting must be sufficiently stiff to withstand belt pull which usually makes it impossible to isolate the low frequency unbalance vibration. This vibration can be cured at the source by an accurate running balance.

Fig. 6 is an illustration of a mounting as worked out for a polyphase induction motor having a speed range from 50 to 100 per cent normal speed. To isolate the 120-cycle magnetic frequency the ratio of the impressed to the natural frequency should be about 8, or for 120-cycle vibration the natural frequency should be 15 cycles. This is in the usual range of rotational frequency and resonance may occur. The six natural frequencies of the mounting mentioned above were adjusted, by varying the size and disposition of the rubber, to be either above or below the range of running speeds.

This mounting has been compared with all of the usual types of mountings on a specially prepared sound box. This box consists of an elastically mounted concrete slab upon which the motor and mounting to be tested are placed. The slab serves as a roof to a box large enough for an observer to enter comfortably. The box sides and door are made of sound proof material which prevents the transmission of extraneous sounds. The only sound which can be heard inside the box is the radiated noise from the concrete roof, which is a measure of the transmission through the elastic mounting. Results of tests on the sound box show that this mounting compares very favorably in sound transmission with the mountings tested thus far.

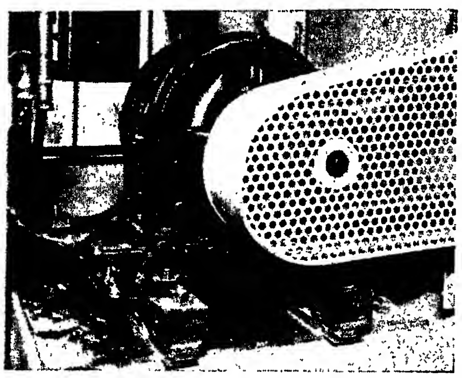

Fig. 6—Installation Under Motor Driving Ventilating Fan

Fig. 6 shows this mounting installed under an induction motor driving a ventilating fan in a large office building. Power is transmitted to the fan through a V-belt drive. The belt guard was not fastened to the motor as is usually the case for this would form a rigid contact with the foundations and transmit noise. Flexible leads were also used to prevent vibration being transmitted to the control panel and thence to the foundation. The steel plate underneath the elastic mounting is not necessary for its operation but is used in this particular case to make connection with the foundation bolts which were not located properly for this mounting. No noise transmitted from the motor could be detected in the floor. The motor itself was stable throughout the speed range, vibrating through a maximum amplitude of 2 mils at a low frequency, due to forces transmitted through the belt from the fan. This slight motion is not objectionable.

To summarize; elastic supports are useful only when the ratio $\frac{\omega}{\omega_c}$ is greater than 1.41. A ratio of 2 or 3 is sufficient for unbalance vibration, while a ratio of 7 or 8 is necessary for noise isolation. Damping in the elastic member of a support is undesirable since it causes increased transmission; on the other hand damping is beneficial when present in the foundations. If the mounting is designed to isolate the low disturbing frequencies, the higher frequencies will take care of themselves. In the general case, the mounting has six natural frequencies, none of which should occur within ten per cent of the running range. In addition, for machines which transmit torque, the mounting must be sufficiently rigid in the direction of belt pull to allow stable belt operation.

In conclusion we wish to express our gratitude to Mr. A. L. Kimball of this laboratory for the ideas and assistance which he contributed while this work was being carried out.

Appendix I

Derivation of the Expression for Transmissibility of an Elastic Support

Problems in mechanical sinusoidal motion may be solved, for the steady-state conditions, by setting up the analogous electrical circuit, solving that by the use of complex quantities, and then substituting the mechanical terms for their electrical analogies.[2] Table I shows these equivalents. This process involves the difficulty of transforming to the electrical system and back again, as well as the objectionable fact that the result appears in terms of velocity (current) rather than amplitude (charge).

Certain changes allow the problem to be solved directly in mechanical units as shown below.

$$i = \frac{E}{Z} \text{ or } v = \frac{F}{Z} = \omega x \tag{1}$$

Dividing by ω gives

$$x = \frac{F}{\omega Z} \tag{2}$$

Phase angles computed from (2) will be the complements of the proper phase angles. Multiplying ωZ by j to remedy this difficulty gives[3]

$$x = \frac{F}{j \omega Z} = \frac{F}{Z_m} \tag{3}$$

where Z_m = mechanical impedance, or amplitude impedance, which can be formed from the following components just as electrical impedance is set up from ω, L, C, and R.

TABLE I—ELECTRO-MECHANICAL EQUIVALENTS

Mechanical	Electrical
M...Mass........	L...Inductance
v....Velocity.....	i....current
x....Amplitude...	q...charge
b....Friction coef..	R...Resistance
$\frac{1}{k}$ ... $\frac{1}{\text{Stiffness}}$ ...	C...Capacity
Z....Impedance..	Z...Impedance
F....Force........	E...Voltage
Z_m..Mechanical impedance.	$j\ \omega\ Z$

Viscous friction (proportional to velocity) is represented by $j\ \omega\ b$.

Mass reaction is represented by $-\ M\ \omega^2$.

Spring reaction is represented by k.

Two general rules based on amplitude may be given for adding the mechanical components of impedance, just as two rules are given, based on current, for obtaining impedance in electrical problems.

1. When various components of a system have the same amplitude impressed upon them, their impedances are added directly to obtain the total impedance.

2. If it is possible for two or more parts of a system to divide an impressed amplitude between them, depending on their impedances, then the reciprocal of the total impedance is the sum of the reciprocals of the individual impedances.

The analogy with the series and parallel cases in electrical theory is obvious.

Referring to Fig. 3 the transmissibility[4] of the mounting, ϵ, equals $\frac{f}{F}$ where F is the maximum value of the impressed force and f is the maximum value of the alternating force on the foundation. In this case the mass, spring, and damping elements have the same amplitude impressed upon each, hence they are in series and their impedances add.

From (3) $$x = \frac{F}{j\ b\ \omega + k - M\ \omega^2}$$

Also $f = x\ (j\ b\ \omega + k)$

Solving for F and taking the ratio gives

$$\epsilon = \frac{f}{F} = \frac{j\ b\ \omega + k}{j\ b\ \omega + k - M\ \omega^2}$$

Taking the absolute magnitude yields

$$\epsilon = \frac{\sqrt{b^2\ \omega^2 + k^2}}{\sqrt{b^2\ \omega^2 + (k - M\ \omega^2)^2}} \tag{4}$$

When $b = 0$

$$\epsilon = \frac{k}{k - M\ \omega^2} \text{ or since } \omega_c^2 = \frac{k}{M}$$

$$\epsilon = \frac{1}{1 - \left(\frac{\omega}{\omega_c}\right)^2}$$

This expression becomes negative at values of $\omega > \omega_c$, indicating that F and f are opposite in phase. Since we are not interested in phase relations this sign may be reversed, giving

$$\epsilon = \frac{1}{\left(\frac{\omega}{\omega_c}\right)^2 - 1} \tag{5}$$

Appendix II

Resonant Speeds of an Elastic Mounting

The six resonant speeds of a machine resting on elastic feet may be calculated using the symbols given below.

k_x = Stiffness of the support in the X direction
k_y = Stiffness of the support in the Y direction
k_z = Stiffness of the support in the Z direction
I_x = Moment of inertia about the X-axis
I_y = Moment of inertia about the Y-axis
I_z = Moment of inertia about the Z-axis
M = Mass of the machine
x = Linear amplitude of vibration of the center of gravity along the X-axis
ϕ = Angular amplitude of vibration about the Z-axis
ω = Angular velocity of the impressed vibration
T = Maximum value of a sinusoidal applied torque.

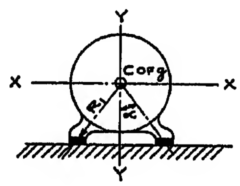

Fig. 7

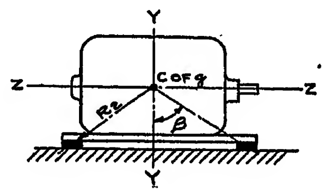

Fig. 8

Assuming symmetry of the feet about the Y-axis as shown in Fig. 7 the equation of linear motion along the X-axis for zero applied force is

$$(M\ \omega^2 - k_x)\ x + \phi\ R_1\ k_x \cos\alpha = 0 \tag{1}$$

The corresponding equation of angular motion about the Z-axis for applied torque $= T$

$$[I_z\ \omega^2 - R_1^2(k_x \cos^2\alpha + k_y \sin^2\alpha)]\phi + x\ R_1\ k_x \cos\alpha = -\ T \tag{2}$$

Solving equations (1) and (2) simultaneously for ϕ

$$\phi = \frac{-\ T\ (M\ \omega^2 - k_x)}{(M\ \omega^2 - k_x)\ [I_z\ \omega^2 - R_1^2\ (k_x \cos^2\alpha + k_y \sin^2\alpha\ )\ ] - R_1^2\ k_x^2 \cos^2\alpha} \tag{3}$$

Resonance occurs when the amplitude of vibration, ϕ, is infinite. Accordingly, setting the denominator of (3) equal to zero and solving for ω, we get two different numerical values, which are

$$\omega_1 = \sqrt{\frac{\omega_a^2 + \omega_b^2}{2} + \left[\left(\frac{\omega_a^2 + \omega_b^2}{2}\right)^2 - \frac{\omega_a^2\ \omega_b^2}{1 + \frac{k_x}{k_y}\cot^2\alpha}\right]^{\frac{1}{2}}}$$

and

$$\omega_2 = \sqrt{\frac{\omega_a^2 + \omega_b^2}{2} - \left[\left(\frac{\omega_a^2 + \omega_b^2}{2}\right)^2 - \frac{\omega_a^2\,\omega_b^2}{1 + \frac{k_x}{k_y}\cot^2\alpha}\right]^{\frac{1}{2}}}$$

where $\omega_a^2 = \dfrac{k_x}{M}$

$$\omega_b^2 = \frac{R_1^2\,(k_x \cos^2\alpha + k_y \sin^2\alpha)}{I_z}$$

ω_a represents the natural frequency of this system if the machine is constrained to move as a unit in the X-direction and is not allowed to oscillate torsionally about the Z-axis. ω_b is the natural frequency which would occur if the machine were allowed to oscillate about the Z-axis but prevented from moving linearly along the X-axis.

In a similar manner, two resonant frenquencies may be found in the Y-Z plane (see Fig. 8). They are

$$\omega_3 = \sqrt{\frac{\omega_c^2 + \omega_d^2}{2} + \left[\left(\frac{\omega_c^2 + \omega_d^2}{2}\right)^2 - \frac{\omega_c^2\,\omega_d^2}{1 + \frac{k_z}{k_y}\cot^2\beta}\right]^{\frac{1}{2}}}$$

and

$$\omega_4 = \sqrt{\frac{\omega_c^2 + \omega_d^2}{2} - \left[\left(\frac{\omega_c^2 + \omega_d^2}{2}\right)^2 - \frac{\omega_c^2 + \omega_d^2}{1 + \frac{k_z}{k_y}\cot^2\beta}\right]^{\frac{1}{2}}}$$

where

$$\omega_c^2 = \frac{k_z}{M}$$

$$\omega_d^2 = \frac{R_2^2\,(k_z \cos^2\beta + k_y \sin^2\beta)}{I_x}$$

There remain two resonances, one for linear motion along the Y-axis, ω_5, and another for angular motion about the Y-axis, ω_6.

These are,

$$\omega_5 = \sqrt{\frac{k_y}{M}}$$

$$\omega_6 = \sqrt{\frac{k_x R_1^2 \sin^2\beta + k_z R_2^2 \sin^2\beta}{I_y}}$$

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Discussion

For discussion of this paper see page 1071.

Noise Mitigation in Substations

BY E. A. BISHOP[1]

Associate, A. I. E. E.

IT is the intention of this paper to deal with the noise encountered in several types of substations and the methods employed to reduce or eliminate the noise. The following classes of substations will be considered in the order named: 230-volt direct current located in a downtown district; 600-volt direct current for traction service; 1,500-volt direct current for electrified suburban service and a-c. remote-controlled distribution substation.

Since the problem of noise due to converters is common to three of the types of substations mentioned it may be well to discuss it first. The noise emanating from converters is due to the following causes: d-c. commutation, windage, magnetic noises, a-c. brushes on slip rings, and vibration of the building housing the equipment. Commutation noise has two elements; (1) caused by brushes riding on the commutator bars, and is of a mild scraping nature, and (2) caused by the commutator slots passing under the brushes. With a commutator in good condition, the latter noise is not troublesome, but high mica or commutator bars causes a chattering noise which is rather objectionable. Windage depending upon the speed of the machine can be anything from a gentle murmur to a very high pitched and penetrating whistle. Magnetic noises are usually so low pitched that they are drowned out by the other noises present. Brushes on the slip rings contribute a rasping noise.

Vibration in substation structures is rather unusual and occurs chiefly if the unit, for any reason, runs open phase. When it occurs it is very pronounced, not necessarily adjacent to the unit but at points very remote from the unit. For example, in a substation located in the sub-basement of an office building, the vibration due to open-phase operation has been very pronounced thirteen floors above, yet the operator in charge was not aware of anything wrong. The noise encountered in any unit is in direct relation to the speed and frequency of the unit. For example, the slow-speed 25-cycle units were very seldom the cause of any serious complaints but with the advent of the higher speed 60-cycle units the complaints began to multiply very rapidly.

The first measure employed to reduce the noise coming from the 60-cycle units was to have the manufacturers provide as a part of the unit a semi-enclosure consisting of a sheet metal housing, with wire glass doors and windows, enveloping the d-c. end of the converter. Whenever any attempt is made to enclose a converter, a secondary problem of ventilation is immediately manifest. In fact, the two problems go hand in hand right through the entire consideration. Therefore, when considering noise mitigation it is necessary to consider ventilation. In the case of the semi-enclosures, the ventilation of the converters was accomplished by leaving an opening in the front end of the housing approximately the same size as the opening inside the commutator. Cool air was then drawn into the armature through the commutator, passed through the ventilating slots of the armature core and windings and discharged through a chimney on top of the enclosure. The chimney in some cases was connected by means of a duct to a point outside the substation. In others it simply discharged in the converter room. The semi-enclosures were first applied to lighting converters and made quite a reduction in noise. They were not quite satisfactory from a standpoint of either ventilation or accessibility for maintenance work around the commutator.

Three 600-volt railway converters with semi-enclosures were ordered, but when they were installed the enclosures were omitted as the hazard due to commutator flashover was great and it was not deemed advisable to take the risk. One of the units was installed in a new substation in an apartment district. As a result of serious complaints from neighbors the semi-enclosure was installed. It was felt that the noise reduction was not sufficient so a total enclosure consisting of one-half inch thick transite supported on a structural steel framework was installed. The inside of this enclosure was lined with one inch thick asbestos hair felt. A baffled chimney on top of the housing permitted the air to be discharged and deadened a large part of the noise.

From a noise reduction view point the total enclosure was rather successful since it eliminated the complaints, but the problem of heating was quite serious. Fortunately the layout was such that a blower to force air through the housing could be installed with slight expense. A new problem then presented itself. When flashovers occurred small particles of hot copper or carbon were discharged into hair felt lining. This material did not burn readily but would continue to smolder due to the draft of air, making it necessary to use a fire extinguisher. This was overcome by again lining inside the hair felt with one-eighth inch porous asbestos paper. The thin asbestos could not be applied to the baffles in the chimney so the baffles were omitted. Although the relatively poorer noise absorbing quality of the thin asbestos slightly decreased the over-all noise reduction, the net result was satisfactory.

The general outcome of the above was the development of the total enclosure. It was designed along two

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general lines by the two principal manufacturers. The first consisted simply of a cubical shaped housing completely enveloping the entire converter, the dimensions being such that the sides were a few inches clear of the machine bedplate at all points. A partition at the main field yoke divided the housing into two sections. For ventilation the air is admitted at the d-c. side of the unit through openings in the foundation. The air flows up past the commutator and d-c. brush rigging, into main and booster armatures, past the a-c. rings and brushes, and is discharged through a chimney located in the top on the a-c. side. A duct connects the chimney to the outside air. A blower is required either on the intake or discharge side to circulate the air.

The second design, while completely enveloping the converter conforms to the general shape of the unit, that is rounded off at the top. The method of ventilation is identical with the first design. The advantages of the total enclosure are: (1) better noise reduction, (2) better ventilation of the equipment, and (3) sufficient space available for maintenance and minor repairs. The disadvantages are: (1) complete disassembly for major repairs and (2) added cost due to elaborate ventilation facilities which are required.

In a recent installation the problem of noise being communicated to adjacent parts of a large downtown building had to be solved. In this case the situation was very critical. The least amount of noise or vibration would be very annoying to a large theater located immediately over the substation space. The following measures were taken to provide a quiet installation. The converters and transformers were supported on a cork insulated foundation to isolate vibration from the remainder of the structure. The blowers and motors were likewise mounted on cork foundations. The connection to the air exhaust ducts was made with a canvas joint. The main d-c. terminal connections were made using flexible laminated copper strips. To further eliminate the possibility of vibration being communicated along the main copper connections and in turn through the supports to the building steel, several flexible joints were made in these long runs. The converters of course, were completely housed. The results of the installation have been very satisfactory in every way. The installation has been in service two years and so far no complaints have been received.

In another recent installation, on company owned property, where cork insulated foundations were found to be impracticable, a one-inch space was left between the substation and the adjacent building. This space was filled with ground cork with a suitable binder. The principal purpose of this filler was not for its insulation value, as the air space would serve this purpose very well, but rather to prevent an accumulation of solid material such as brick bats, which would form a solid means of communicating vibration to the adjacent building.

At the present time, there is under consideration the problem of noise mitigation in 1,500-volt traction substations. The units installed consist of two 750-volt machines having their commutators connected in series. They have a speed of 600 r. p. m. and give rise to an extremely high-pitched penetrating noise. The noise consists principally of two elements, one due to commutators and the other to windage. The air emerging from the armature radial ventilating slots impinges on the field pole tips and is alternately cut off and allowed to flow freely as it passes the field poles. The resultant whistle is responsible for an objectionable portion of the noise. The manufacturer has designed a new form of pole tip which has its edges gradually rounded off thus eliminating the sudden cutoff of air. He claims that a very appreciable reduction in noise can be obtained by substituting the new pole tips. There is no authentic data on the amount of noise reduction obtainable by use of the new pole tips. This company has at present in operation slow-speed 230-volt converters equipped with both types of pole pieces and comparative tests are under way to determine the relative merits of the two types. Unfortunately these tests will not be concluded in time to include the results in this paper.

The alternate scheme for mitigating the noise is to totally enclose the two units with the resultant hazards of 1,500-volt flashovers and the cost of providing ventilating facilities in substations not readily adaptable to this type of ventilation. Other methods, such as noise absorbing walls surrounding the converters, have been tried but they have not proved successful.

Before leaving converting equipment it should be stated that from a noise mitigating standpoint, the mercury arc rectifier is the most satisfactory device so far developed. Its operation is silent except the hum in the transformer and the noise of the rotary vacuum pump. The latter device has been much improved since the original installations and is now satisfactory.

In a-c. distribution substations, the character of the noise produced is entirely different, consisting chiefly of 120-cycle hum inherent in transformers and induction regulators. The latter also produces a noise due to operation of the regulator motor control equipment. The noise in a-c. substations while not nearly as intense as in d-c. substations is considerably above the noise level of their neighborhoods especially in residence districts. In one substation, this noise was found to be transmitted through building steel to the outside walls and thence through to an adjoining apartment house. This difficulty was overcome by placing cork under the transformers and regulators, and replacing the rigid terminal connections with flexible ones.

Transformers installed outdoors for a remote-controlled substation have also been the cause of serious complaints. The transformers are, as a rule, installed in an open court adjacent to the building which houses

the switching equipment and regulators. One end and side of the court usually consist of brick walls with small openings for ventilation. The third side is a wire mesh screen containing the gate for admitting apparatus. The noise is confined to the court by placing sound absorbing felt over the wire mesh screen and by constructing a roof consisting of sheet metal arc-shaped sections having the under side covered with sound absorbing felt. These sections are arranged with respect to each other so as to baffle the sound waves and at the same time permit free passage of air.

In summarizing the situation at least as far as rotating equipment is concerned very little has been accomplished toward reducing the noise inherent in the machines. Practically all efforts up to the present time have been toward confining the noise to the equipment and involved with it is the serious problem of ventilation.

Discussion

MEASUREMENT OF NOISE IN ELECTRICAL MACHINERY
(Bailey)

INDICATING METER FOR MEASUREMENT AND ANALYSIS OF NOISE
(Castner, Dietze, Stanton, and Tucker)

MEASUREMENT OF MACHINERY NOISE
(Marvin)

INDUCTION REGULATOR NOISE
(Foltz and Shirk)

MAGNETIC NOISE IN SYNCHRONOUS MACHINES
(Graham, Beckwith, and Milliken)

ELASTIC SUPPORTS FOR ISOLATING ROTATING MACHINERY
(Hull and Stewart)

NOISE MITIGATION IN SUBSTATIONS
(Bishop)

R. E. Pumphrey: What is generally accepted to be one of the principal sources of magnetic noise in synchronous machines, namely, the periodic variation in magnetic pull as the poles pass the stator teeth, has been described by Messrs. Graham, Beckwith, and Milliken in a very clear manner. While it has been known for some time that such a variation in radial pull existed, the methods of calculating the extent of this variation have been laborious and unsatisfactory. Some three or four years ago in collaboration with Mr. H. D. Taylor we carried out a series of investigations to determine by actual test the magnitude of the forces involved, their relation to various design details of the pole face such as width, radius of pole arc, amortisseur winding slots, etc. The testing equipment consisted essentially of a section of stator core so supported that the force exerted on it by a single pair of poles could be accurately measured. Means were provided for preserving the proper air gap and rotating the poles past the stator teeth, recording readings of the pull for various positions. Some 90 odd combinations were tested and have provided the basis for an approximate quantitative analysis of the results to be expected from a given design.

Fig. 1 shows some of the results of these tests plotted in per cent pulsation against pole face width per stator slot pitch. Maximum pull always occurred with the pole center directly opposite either a stator tooth or slot. Pulsation was arbitrarily taken as the difference in pull between these two positions, and per cent pulsation was defined as this difference divided by the average. The effect of rounding the pole face, that is, of increasing the ratio of maximum to minimum gap is clearly shown.

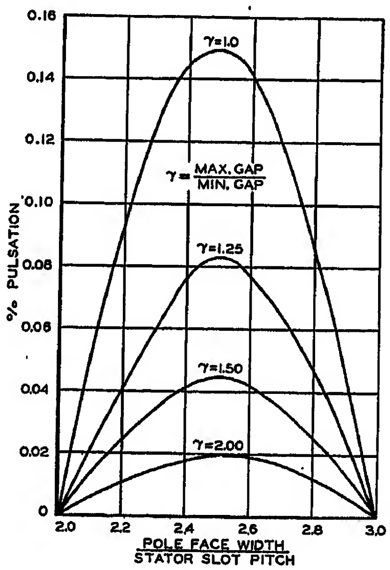

Fig. 1. Per Cent of Radial Magnetic Pull in Per Cent Average Pull for Various Pole Face Widths and Radii

E. J. Abbott: The availability of noise meters as portable and convenient as those described should greatly encourage the use of sound measurements of machinery noises. The desirability of having such apparatus can be especially appreciated by those who have transported bulky laboratory analyzers and amplifiers for field measurements.

Experience at the University of Michigan indicates that one or two weighting curves is hardly sufficient for the range of sounds likely to be encountered. We have measured noises from electric refrigerators in the neighborhood of 25 db. above the 1,000-cycle threshold, and also noises from large reduction-gear units which were from 90 to 100 db. above the 1,000-cycle threshold. Enormously different weighting circuits are required in such cases.

Even with convenient and accurate apparatus for measuring the magnitude of sound at a given point in space, the problem of rating machines for noise in terms of meter measurements is far from being solved. One would naturally assume that good comparative measurements on a group of similar machines could be obtained by placing the microphone at the same relative position for each machine. Experience shows that this usually is not the case. The discrepancies encountered in such cases are usually very much greater than the difference between the loudest and quietest machines in the group.

In case of the large reduction-gear units mentioned above, readings were made at 10 stations at corresponding positions on each side of eight identically constructed units. The value obtained by averaging the ten readings on each machine agreed perfectly with aural observations, but this was by no means true for the individual stations. For example, machine 2, which was the next to the loudest on the average, was the quietest of the group at microphone stations 1 and 10, and the loudest at stations 5 to 9. Machine 1, which was the quietest of the group, measured next to the loudest at station 10, and the quietest at several other stations. Machine 3, which was the fourth in loudness, measured the quietest of the group at stations 9 and 10, and next to loudest at stations 2 and 3. At station 6, machine 4, the noisiest of the group, measured almost the same as machines 1, 11, and 12 which were the quietest of the group. These results are by no means unusual, but rather the ordinary experience.

The causes of these large variations in sound from point to point and from instant to instant lie not only in the familiar acoustic standing wave patterns due to sound reflection from neighboring objects, which are troublesome enough; but also in the nature of the source itself, which is usually even more important. Machinery does not seem to vibrate as a whole at audio frequencies, but breaks up into a large number of small areas vibrating with different phases and magnitudes. Each of these radiates its own set of sound waves, the largest of which by no means come from the region nearest the original source of the noise. Insignificant variations in design, materials, construction, speed, load or other causes may entirely shift these patterns on the machine without greatly changing their average value. Hence the inconsistent readings obtained with a given microphone position on different machines.

The difficulties listed above can usually be overcome if time and conditions permit the averaging of a large number of readings. At the University we have had considerable success in obtaining such averages rapidly by installing the machine to be measured in a very reverberant room. A large moving metallic reflector serves to shift the sound energy continually, and by moving the microphone and using a slow-acting indicator meter to average-out short time variations in the intensity at the microphone it has been possible to obtain very satisfactory total noise measurements and frequency analyses.

Obviously this method is not applicable for impulsive noises, and in many practical cases it is not feasible to remove the machine to be measured to the laboratory. The frequency of the noise from most machines cannot be "warbled" several times a second to obtain an average, and the possibilities of moving the microphone often are limited if measurements must be made close to an irregular-shaped machine in order to reduce the effect of extraneous noise. In consideration of these practical aspects of sound measurement it appears that considerable remains to be done before machine noise measurement is on a production basis.

H. D. Taylor: Messrs. Graham, Beckwith, and Milliken have described a theory to account for tooth-frequency noise which is very similar to one developed by some of my associates and confirmed by several years' experience in designing and testing low-speed synchronous machines. Continual progress has been made—and is still being made—on this problem. But, generally speaking, it seems well established that the noise is usually due to vibration of the stator at about tooth frequency, and that the vibration is caused by pulsation of the radial magnetic pull of the field poles as they revolve past the stator slots and is independent of stator load current. Our experience confirms the authors' statement to the effect that the worst cases of noise are likely to be associated with pole and slot combinations giving a nearly but not quite integral number of slots per pole.

With respect to frequency of the noise, our analysis indicated that it should be a multiple of twice the line voltage frequency,—$2 f x$ (integer nearest to the number of slots per pole); and this rule was confirmed by a number of observations. The authors' rule gives slightly different results.

Supplementing the authors' illustration of the production of tooth-frequency vibration and noise by pulsation of the magnetic pull of the poles in moving over one tooth-pitch (Fig. 3), I should like to call attention to the magnitude of the forces involved. At ordinary magnetic densities, the average radial pull is about 25 lb. per square inch of pole face area,—which amounts to several hundred pounds per pole even for quite small pole pieces. Furthermore, for such design proportions as are illustrated in the authors' Fig. 1, the amount of pulsation may be as high as 20 per cent—that is, 10 per cent above and below the average. Considering the fact that vibration amplitudes of only a few tenths of a mil at the frequencies in question will produce large volumes of noise, it is not surprising that such forces have been making themselves heard.

In order to avoid noise from this source, there would seem to be two general courses of action: (1) to make the stator construction rigid enough to eliminate these almost microscopic deformations; or (2) to eliminate the fluctuations in magnetic pull by suitable control of the design details. Our experience has indicated that the latter course is likely to be more successful. We have made tests which show conclusively that by proper proportioning, pole pieces of almost any dimensions can be designed for practically zero pulsation; also, that the noise usually disappears when such poles are substituted in noisy machines of the type described by the authors.

We have observed a number of cases of other types of tooth-frequency noise in synchronous machines,—notably:

(1) Cases involving poles having little or no pulsation, but with a decided tendency to have the flux, and pull, swing from side to side of the pole piece,—the number of slots per pole being an integral number plus one-half.

(2) Cases intermediate between these and the type described by the authors, in which the flux-swinging action and the pulsation effect are cumulative.

(3) One case of tooth-frequency noise which was almost entirely dependent on load current in the stator, and which seemed to be associated with the use of a very nearly but not quite integral number of stator slots per pole *per phase*.

The first and second types responded to the same kind of treatment as I have described for simple pulsation; the load noise was eliminated by changing the number of stator slots.

C. E. Kilbourne: The paper by Messrs. Graham, Beckwith, and Milliken states that the stator is the chief source of the noise generating vibrations, but that the rotor may also contribute to the trouble. In a number of observations made on machines having several types of rotors, cast laminated steel plate, circular disk centers with welded rims, and keyed rims with structural centers, I have never found a case where the noise could be traced directly to the rotor. An analysis of the noise frequency and the stator vibration frequency has shown them to coincide and a stethoscopic analysis of the stator itself has always, with the exception of localized resonant areas, yielded an increase in the volume of noise as the stator core was approached. I should like to ask the authors if in their experience they have found cases where the rotors did vibrate and how they assured themselves that it was the rotors. The authors give some equations for the frequencies of rotor vibration and state that approximate resonant frequencies can be determined. I should like to ask if the approximations so obtained are close enough for actual use.

The paper shows that the effect of the traveling waves of force on the stator is to produce an elliptical distortion of the frame. It states Timoshenko's equation of such vibration as applied to a thin ring and uses the results to determine the natural frequency of the stator. This application is justifiable as a first approximation and gives good results for a small number of nodes; but as the number of nodes and consequently frequency increases, some of the assumptions made in the deduction of the equations, lead to results containing considerable error.

Timoshenko's equation is based on the assumption that the cross-sectional dimensions of the ring are small in comparison with the radius of its center line. A more general equation for the vibration of a homogeneous ring has been obtained by Mr. A. L. Ruiz as part of an investigation on stator vibration now in progress. This equation is limited only by the following assumptions.

1. A stator frame may be treated as a thick circular ring of uniform cross section.

2. The cross section is made up of punchings, air, and structural parts.

3. The neutral axis of the section passes through the center of gravity.

4. The energy of deformation of each cross section is negligible.
5. There are no damping or frictional losses.
6. There are suspended masses at a distance from the center of gravity which move with the section, adding to its kinetic energy, but not adding to its stiffness.

The first two assumptions are necessary to justify the application of the equation to a frame having feet. Experimental results have shown that these approximations do not introduce much error if the frame is of the type usually used in low-speed machinery. The next three assumptions seem justifiable from the physics of the problem. The last assumption is in accordance with the conclusions drawn by the authors of the paper, namely that the laminations add to the weight of the section but do not add to its stiffness. This statement must be modified as the frequency increases. At low frequencies, where the wavelength is long, it is practically true, but as the frequencies become higher, and the wavelengths shorter, the laminations do add stiffness to the frame.

The method of attack is to set up expressions for the potential and kinetic energies of deformation of a circular ring. The general expressions for these are:

$$V = a \int_0^{2\pi} \left(\frac{M_B^2}{2\,E\,I} + \frac{S^2}{2\,A\,E} - \frac{M_B\,S}{A\,E\,a} + \frac{Q^2}{2\,\mu} \right) d\,\theta \qquad (1)$$

which may be reduced to

$$\begin{aligned} V = a \int_0^{2\pi} \Bigg\{ & \frac{E\,I}{2\,a^4} \left(u + \frac{\partial \psi}{\partial \theta} \right)^2 + \frac{E\,A\,e^2}{2} \\ & - \frac{E\,I\,e}{a^3} \left(u + \frac{\partial \psi}{\partial \theta} \right) + \frac{\mu}{2\,a^2} \left(\frac{\partial u}{\partial \theta} - \psi \right)^2 \Bigg\}\, d\,\theta \end{aligned} \qquad (2)$$

and

$$\begin{aligned} T = a \int_0^{2\pi} \Bigg\{ & \frac{\rho}{2} \left[\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right] \\ & - \frac{M}{a} \left(\frac{\partial w}{\partial t} \right) \left(\frac{\partial \psi}{\partial t} + \frac{\partial w}{\partial t} \right) \\ & + \frac{J}{2\,a^2} \left(\frac{\partial \psi}{\partial t} + \frac{\partial w}{\partial t} \right)^2 \Bigg\}\, d\,\theta \end{aligned} \qquad (3)$$

Hamilton's Principle, namely

$$\delta \int_{t_1}^{t_2} (T - V)\, d\,t = 0 \qquad (4)$$

is then applied and the resulting expression is solved for a particular solution involving ω, which is $2\,\pi$ times the vibration frequency, and n which is the number of nodes in the circumference. The general expression is:

$$\begin{aligned} & \Bigg\{ \rho\,a\,\omega^2 (n^2 + 1) - \left(2\,M - \frac{J}{A} \right) \omega^2 - n^2 \Bigg(\frac{E\,I}{a^3} \\ & + \frac{\mu}{a} n^2 \Bigg) \Bigg\} \Bigg\{ - \frac{E\,I}{a^3} n^2 + \frac{J}{a} \omega^2 - \frac{\mu}{a} \Bigg\} \Bigg\{ - \rho\,a\,\omega^2 \\ & + \left(2 \frac{E\,I}{a^3} + \frac{\mu}{a} n^2 \right) + \left(\frac{E\,I}{a^3} + \frac{E\,A}{a} \right) \Bigg\} + 2\,n^2 \Bigg\{ \Bigg(M \\ & - \frac{J}{a} \Bigg) \omega^2 + \left(\frac{E\,I}{a^3} + \frac{\mu}{a} \right) n^2 \Bigg\} \Bigg\{ 2 \frac{E\,I}{a^3} + \frac{\mu}{a} \Bigg\} \Bigg\{ - \rho\,\omega^2 a \\ & + \left(2 \frac{E\,I}{a^3} + \frac{\mu}{a} n^2 \right) \Bigg\} + n^2 \Bigg\{ - \rho\,a\,\omega^2 + \Bigg(2 \frac{E\,I}{a^3} \\ & + \frac{\mu}{a} n^2 \Bigg) \Bigg\}^2 \Bigg\{ - \frac{E\,I}{a^3} n^2 + \frac{J}{a} \omega^2 - \frac{\mu}{a} \Bigg\} - \Bigg\{ \Bigg(M \\ & - \frac{J}{a} \Bigg) \omega^2 + \left(\frac{E\,I}{a^3} + \frac{\mu}{a} \right) n^2 \Bigg\}^2 \Bigg\{ - \rho\,a\,\omega^2 + \Bigg(2 \frac{E\,I}{a^3} \\ & + \frac{\mu}{a} n^2 \Bigg) + \left(\frac{E\,I}{a^3} + \frac{E\,A}{a} \right) \Bigg\} + n^2 \Bigg\{ 2 \frac{E\,I}{a^3} \\ & + \frac{\mu}{a} \Bigg\}^2 \Bigg\{ \rho\,a\,\omega^2 (n^2 + 1) - \left(2\,M - \frac{J}{a} \right) \omega^2 \\ & \qquad - n^2 \left(\frac{E\,I}{a^3} + \frac{\mu}{a} n^2 \right) \Bigg\} = 0 \end{aligned} \qquad (5)$$

This expression is unwieldy to handle though perfectly possible of solution. Numerical substitution for a given value of n will yield a cubic in ^{2}, the smallest positive root of which is the desired result.

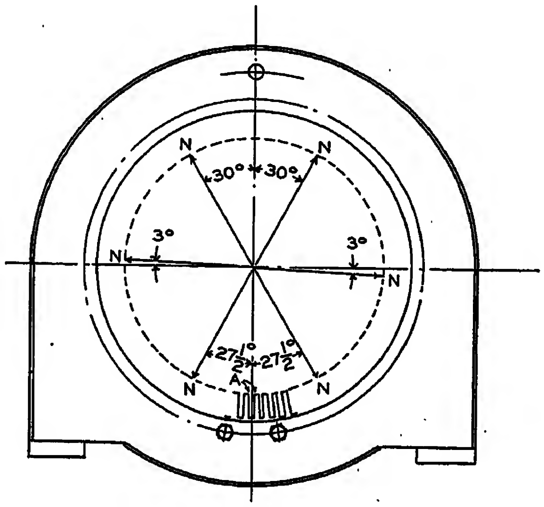

FIG. 2—NODE LOCATION FOR SIX-NODE VIBRATION, FORCE APPLIED AT A

A number of approximations which approach Timoshenko's assumption and eventually reduce to it, considerably simplify the labor of calculation. If we treat the motion as being inextensional letting $e \doteq 0$, the result is:

$$\begin{aligned} & \left\{ \frac{M^2\,a}{\mu} - \frac{J\,a}{\mu} (n^2 + 1) \right\} \omega^4 + \Bigg\{ \frac{\rho\,E\,I}{\mu\,a} n^2 (n^2 + 1) \\ & + \frac{J}{a} (n^2 - 1)^2 + 2\,M\,(n^2 - 1) + \rho\,a\,(n^2 + 1) \Bigg\} \omega^2 \\ & \qquad - \frac{E\,I\,n^2}{a^3} (n^2 - 1)^2 = 0 \end{aligned} \qquad (6)$$

If in addition shearing deformation is neglected, there is:

$$\begin{aligned} & \left\{ \frac{J}{a} (n^2 - 1)^2 + 2\,M\,(n^2 - 1) + \rho\,a\,(n^2 + 1) \right\} \omega^2 \\ & \qquad - \frac{E\,I\,n^2}{a^3} (n^2 - 1)^2 = 0 \end{aligned} \qquad (7)$$

If the dead weights are also neglected there is:

$$\left\{ \frac{J}{a}(n^2 - 1)^2 + \rho\, a\,(n^2 + 1) \right\} \omega^2 - \frac{E\, I\, n^2}{a^3}(n^2 - 1)^2 = 0 \tag{8}$$

And finally if rotary inertia is neglected the solution becomes Timoshenko's equation:

$$\rho\, a\,(n^2 + 1)\,\omega^2 - \frac{E\, I\, n^2}{a^3}(n^2 - 1)^2 = 0 \tag{9}$$

Details of the solution are not given here as it is hoped that when the investigation is complete there will be sufficient useful information to make publication of the whole advisable.

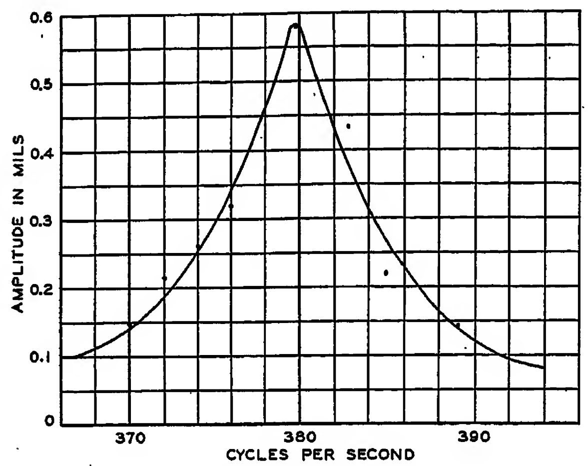

FIG. 3—RESONANCE CURVE FOR SIX-NODE VIBRATION

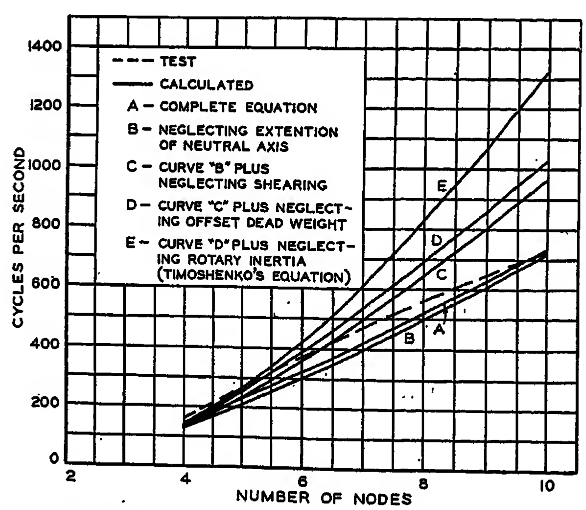

FIG. 4—COMPARISON OF TEST AND CALCULATED NATURAL FREQUENCIES

In our study of the phenomena a number of commercial stator frames complete but without coils were set up in a laboratory and vibrated by means of a magnet excited from a variable frequency source. Applied frequency was varied from 50 to nearly 1,000 cycles and whenever a resonant point was detected a detailed study of the frame was made. With resonant frequencies applied a stethoscopic examination of the punchings, side plates, and wrapper plate showed that the frame was vibrating as a whole, and the nodes were thus located more accurately than they could have been by hand. Amplitude-frequency curves taken at these points showed the nature and range of the resonant condition. In all cases the lower numbers of nodes were located as if the frame were a true ring, thereby justifying the first two assumptions. As the frequency was increased the effect of the feet became more pronounced and a shifting of the nodes from symmetrical positions resulted.

Results obtained on a frame used for about 200-hp., 300 r. p. m. motors are shown in a number of figures. Fig. 2 shows the location of nodes for six-node vibrations. The results of amplitude frequency measurements for the 4- and 6-node vibrations are shown in Table I and the latter plotted in Fig. 3. The general shapes of these curves check well with similar results obtained by the authors of the paper. A plot of resonant frequencies against number of nodes was made and compared with the theoretical results obtained from equations (5), (6), (7), (8) and (9). This is shown in Fig. 4. It will be noted that as the approximation approaches the true equation, the error decreases. Therefore, it is logical to expect that the use of equation (9) will generally lead to considerable error.

TABLE I—RESONANCE CURVES

Four-node frequency cycles/sec.	Vibration amplitude corrected to constant force $\frac{1}{1000}$ Inch	Six-node frequency cycles	Vibration amplitude corrected to constant force $\frac{1}{1000}$ Inch
156	0.046	370	0.145
158	0.167	372	0.216
160	0.890	374	0.260
161	1.388	376	0.320
162	4.33	377.5	0.430
163	1.48	379.5	0.580
165	0.980	382.8	0.434
166	0.553	385	0.215
170	0.206	389	0.142
174	0.117		
Maximum force	33.0 lb.	Maximum force	68.0 lb.

In conclusion, I wish to express my belief that frame resonance is an important contributor to synchronous machine noise, and that the authors have made a valuable addition to our knowledge of the noise problem.

NOMENCLATURE

a = radius of neutral axis of ring.
θ = angular coordinate of any point P on the neutral axis.
u = normal displacement of p, positive inwards.
w = tangential displacement of P.
$\frac{\psi}{a}$ = slope of deformed curve when the frame is in vibration (shear neglected).
e = extension of the neutral axis.
t = time.
ρ = mass of ring and added weight per unit length of arc of neutral axis.
I = area moment of inertia of ring section.
J = mass moment of inertia of unit cross section of ring and added mass, about the center of gravity of the ring section.
M = moment of mass per unit length of added mass about center of gravity.
T = total kinetic energy of vibration.
V = total potential energy of deformation.
E = Youngs modulus.
G = modulus of rigidity.
A = cross-sectional area of ring section.
k' = shear factor for cross section.
μ = $k' A G$
M_B = bending moment at P.
Q = shear at P.
S = tension along neutral axis at P.

J. A. Jackson: It seems to me that we are not going to completely solve our noise troubles in buildings until there is a much closer cooperation between the electrical industry and the architects and consulting engineers. There has been a strong tendency within the last few years to reduce the cost of buildings, the result being to make them much lighter and with much thinner walls, floors, and ceilings. Little attention seems to be paid to the fact that this makes them vibrate more readily and act as sounding boards. Therefore, the problem of preventing objectionable noises in inhabited space is a more difficult one. This fact does not seem to be fully recognized by many of those in the architectural and building engineering professions, with the result that when a noise complaint does arise the machinery manufacturer is asked and expected to bear all expense for curing the trouble. I think it is generally agreed that rotating electrical machinery and, for that matter, most other rotating machinery cannot be made absolutely noiseless without incurring a prohibitive cost and being unduly large and, that all manufacturers are now doing a good job in making their equipment as quiet as is commercially possible.

It is the duty of the architects and engineers to so design and acoustically treat their buildings so that such noises as do remain in the electricl machinery will not become objectionable. Electrical engineers should do all they can in the necessary cooperative and educational work with the architects and building engineers to bring about this much desired condition.

P. E. Stevens: Vibration and noise have always existed in power plants, but with the adoption of the lighter forms of switchboard manufactured of steel, these have come to have a more than unpleasant significance. We have had in one case a false operation of a relay which dumped a load of some 2,000 kw. Another instance may be cited in which the false operation of the relay dumped a load of 120,000 kw. The false operation of the relay was caused by vibration in the panel of the switchboard. The vibration in the building was not of such a magnitude as to cause any complaint. It was just the ordinary vibration which is common in the modern power plant. Drastic steps were taken to remedy these conditions. In one case the relays were moved to an entirely different switchboard. In another case, they were set up first to twice the gap and later to four times the gap at which they were designed to operate.

The false operation of relays is not the only difficulty which attends vibration in a switchboard. The graph drawing instrument and the integrating instruments suffer inaccuracies in their measurement and require frequent attention to keep them in operating condition. In some cases jewel bearings have been injured and broken. Our organization felt that this was a matter which called for a rather thorough investigation, the purpose of which should be to determine the source of vibration and the mode of propagation from the source to the switchboard; and if possible the magnitude of vibration under which instruments would operate successfully. In order to make the study as broad as possible we invited and secured the cooperation of Sargent and Lundy of Chicago. A group of five men was organized into a surveying committee. In ten stations visited measurements were taken of vibration of noise; tests of frequencies; and a very searching examination for minute vibrations which are too small to register on the usual vibrometers. In one case, records were also taken with a vibrograph. There were probably over five thousand observations, which have yet to be assembled in proper form for determining the parallels and divergencies in the various indications.

Definite measurements indicate that noise is accountable in certain cases for at least a part of the vibrations in switchboards and sometimes in much heavier structures. It is, therefore, necessary that a measuring instrument which is to be fully useful in surveying the effects of noise in a power plant, should not only measure the effect upon the human ear or the auditory effect, but should also be prepared to measure its probable vibration effect on the physical elements of the plant.

It is a well-known fact that a resonant condition will greatly magnify any impressed vibration. In the study of one of our plants we found three conditions in our switchboards; when the board was first delivered the blank panels had a vibration frequency of approximately 26 cycles per second. This is below the frequency of the vibrations impressed by the machine which operated at 1,800 r.p.m. or 30 cycles per second, but is too close to this frequency for a satisfactory quieting of the vibrations. The board was found to vibrate violently in certain places but no measurements were made. The manufacturer suggested the addition of certain stiffeners. These were applied and this condition changed the vibration period of the panels to somewhat over 40 per second. This is sufficiently above the frequency of the machine so that a rather small vibration might have been expected except for the fact that many of the panels are loaded with equipment which tends to lower their frequency. This is the purpose of the panel, that is, to support equipment. Further study of the natural frequency of the panel revealed the fact that the relays which had suffered most from the vibration were almost exactly the correct weight to bring the frequency down from somewhat over 40 to 30 per second, or in perfect resonance with the frequency of the impressed vibration from the turbine.

The relative magnitude of some of these vibrations is shown in the following figures. In one of the plants the turbo generator had a maximum transverse vibration of 1.4 mills (thousandths of an inch). In general the floor of the operating room had smaller vibrations than this, or a maximum transverse vibration of one mill. The floor immediately around the switchboard and on which the switchboard rested had a maximum transverse vibration, that is to say, a vibration normal to the plane of the panels of the board of 0.4 mills. The switchboard relay panels had a maximum vibration of 20.0 mills at the point before mentioned as being in resonance.

Trouble is also experienced with the vibration of the modern type of steel office furniture. A desk on the floor of the room in which the vertical vibration is 0.4 mills was found to have a vibration at the point where the chief operator was expected to write of 4.5 mills. This was rather a poor place on which to write but the effect on the telephone was even worse. It broke up the speaker's voice into a series of beats which were absolutely unintelligible to the listener at the other end of the wire. It is suspected that some of this vibration might be due to air borne waves or noise. The human ear does not appreciate air borne waves of low frequency although many can hear distinct sound vibrations as low as 28 per second. There is no reason to believe that because these low-frequency atmospheric vibrations are not heard that they may not be present and may not influence structures which may be in resonance with their frequency. At one station measurements were made on the switchboard in the operating room, following which the doors between the operating room and the turbine room were opened. An increase in the sound readings taken with a Western Electric 3-A audiometer was 30 per cent. The increase in the amplitude of vibrations of the panels was approximately 20 per cent.

Noise conditions present a practical problem calling for immediate and perhaps drastic steps to overcome the conditions which have arisen from the step-by-step reduction in weights of mechanical equipment and structures. The manufacturers are not being accused of skimping their work but the reduction of weights is a natural evolution brought about by the necessity for less weight per kilowatt if machines of the vast magnitude of today are to be manufactured.

Designers of power plants, engineers, and designers of machinery and equipment should assemble into an organized body which will attack this problem and correlate the point of view of the two interests involved.

E. H. Hull and W. C. Stewart: Mr. P. E. Stevens in his discussion describes the excessive vibration of a switchboard panel due to forces transmitted to it both through the air and through the foundations. Such vibration can be limited by the proper use of an elastic mounting. Fig. 2 of the paper shows the transmissibility ϵ as a function of the ratio of the disturbing frequency to the natural frequency of a mounting. ϵ may be either the ratio of the force transmitted to the foundation to the disturbing force or the ratio of the amplitude of an elastically mounted body to that of a vibrating foundation.*

The latter case, of course, applies to the mounting of a switchboard panel to isolate it from foundation vibration. The switchboard, as originally installed, had a natural frequency of 26 cycles, which presumably was due to elasticity in the mounting of the panel. This gives a frequency ratio of 1.15. Referring to curve marked $b = 0.5$ in Fig. 2 of the paper, this ratio gives $\epsilon = 1.8$. The panel was then stiffened until the natural frequency reached 40 cycles. This gives a frequency ratio of 0.75 and again $\epsilon = 1.8$. Therefore, the motion of the panel would be exactly the same after stiffening as it was originally. Further stiffening would reduce the panel amplitude slightly until for an infinitely rigid mounting $\epsilon = 1.0$ or the panel has the same amplitude as the floor.

Thus no amount of stiffening could reduce the panel amplitude below that of the foundation. On the other hand, by decreasing the mounting stiffness and lowering the natural frequency to 10 cycles, the frequency ratio would be 3.0 and ϵ would be reduced to 0.24. This means that the panel amplitude would be about ¼ of that of the foundation.

If the panel vibration is due to air borne forces alone, a very rigid mounting would be desirable, providing that resonance occurred somewhat above the disturbing frequency, as the motion of the panel would be zero for infinite stiffness. In the case of large air forces, where the foundation vibration is negligible, relays and instruments may be supported elastically from the panel, as the air force on them would be small due to the small area. If still better isolation is desired, the instruments could be elastically supported within cabinets fastened to the panel. The elastic support would isolate the vibration of the board from the instruments and the cabinet would protect them from air forces.

*See "Influence of Damping in the Elastic Mounting of Vibrating Machines," by E. H. Hull presented at the Applied Mechanics Meeting of the A. S. M. E., Purdue University, June 16, 1931

Sterling Beckwith: The rule for frequency of the stator vibrations given by Mr. H. D. Taylor is the same as the rule developed in the paper, but has been further simplified. This simplified rule is: vibration frequency = twice line frequency [integer nearest to the number of slots per pole].

Fig. 1 of the discussion by Mr. R. E. Pumphrey is very valuable, although one must not regard it as indicating a "cure-all" for any type of machine, as the ratio of maximum gap to minimum gap, and the number of slots per pole usually cannot be varied to reduce per cent pulsation without undesirably affecting other machine characteristics such as load loss, pole-face loss, machine size, core loss, telephone interference factor, etc.

Noise due to resonant vibration of parts of the rotor has been identified only by "circumstantial evidence" as it is exceedingly difficult to measure actual vibration of a rotating structure when vibration amplitudes are only a few mils or less. The circumstantial evidence may be listed as follows:

1. Changing spider construction on a line of machines built of the same parts has been found to affect probability of certain types of noise in the machines.

2. Six slots per pole has been found to cure tooth frequency noise on a certain size machine with a six-arm spider, which machine is noisy with four and a half slots per pole; whereas, on a similar, but slightly smaller machine with a four-arm spider, six slots per pole was noisy and four and a half slots per pole was quiet. The formulas for rotor resonance given in the paper indicated that the above results could have been anticipated.

3. Where tooth-frequency noise has been due to the stator, the vibration has been easily detected, so that when similar machines are noisy without any type of stator vibration of a comparable order of magnitude, the rotor must be the source of the trouble.

The work of A. L. Ruiz in obtaining an exact mathematical solution for resonance frequency of different modes of stator vibration is very valuable. The graphical summary in Fig. 4 shows at a glance both the relative error in the simple formula and the accuracy of the complete formula. It is hoped that the possibility of publishing the details of Mr. Ruiz' work and the results of the complete investigation of which this work is a part will become a reality in the near future.

Lightning

Characteristics of Lightning—Induced Voltages—Direct Strokes—Coordination—Transmission Line Design

BY F. W. PEEK, Jr.*
Fellow, A. I. E. E.

I. Introduction

IT is the purpose of this paper to show how the various characteristics of lightning can be obtained from measurements on lines, and how the results can be applied in design. *Oscillograms of traveling waves give a remarkably complete and easily read story of the life and characteristics of the lightning which produces them.* For example, such oscillograms give data from which the height, the time of discharge, the current and the voltage of the clouds producing them can be estimated. Induced voltages on any given transmission line can be found from these data. Direct stroke and induced voltages can be determined from direct-stroke currents, etc. Many measurements have been made during the past five years on the lines of the Pennsylvania Power & Light Company, the American Gas & Electric Company and elsewhere.

In making the analytical study, curves are plotted showing how voltage, time, and current are numerically interrelated over a wide range, for comparison with measured values from which the actual limits must be finally determined. Lightning measurements are very difficult; many are known to be accurate, while others are still uncertain. It is not possible, for instance, to fix the range of direct-stroke current until further measurements are available. Although some numerical values will probably be modified and their relative importance changed, as the reliability of field measurements is improved, the methods of analysis will remain unchanged and valuable in interpreting and utilizing data. This study should also point the way for future experiments. To avoid confusion, the derivation of the mathematical equations has been confined to the appendix or references in the Bibliography. Some actual designs are made for purpose of illustration.

II. Voltages on Lines and Methods of Controlling Them

Induced Voltages. Since voltages on lines are caused by induction or direct strokes, it is of practical importance to be able to estimate the numerical value of these voltages for different tower designs. When voltages occur by induction they are of a polarity opposite to that of the cloud and can be calculated by the following formula[1,2,3]

$$V = g\, h\, \alpha \qquad (1)$$

Where V is the voltage above ground, g is the gradient in volts per foot established by fairly good measurements to be about 100,000. α is a factor depending upon the time of cloud discharge and distribution of bound charge and is always less than unity. Values of α calculated from the rate of cloud discharge and used in former calculations are given in Fig. 1. The exponential rate of cloud discharge used in these curves probably more nearly represents actual conditions than any other. α is the factor for the induced voltage during the forming period at the origin, while α' is for the resulting free traveling wave.[4] In order to use these curves to determine α, and thus the induced voltages possible or probable, it is necessary to have

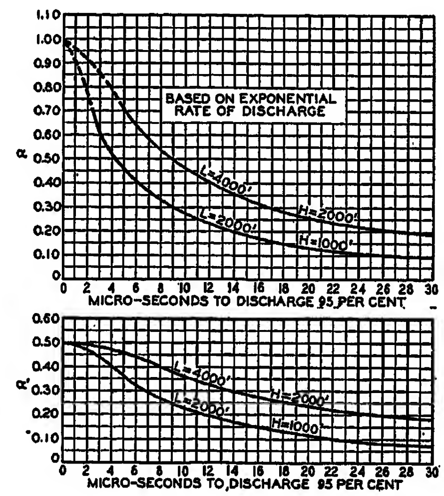

Fig. 1—α for Induced Voltages (Exponential Discharge)

measured values of the time of the cloud discharge. As already mentioned, cathode-ray oscillograms of traveling waves of induced lightning voltages taken reasonably near the source of the disturbance, and not badly distorted by sparkover, losses or reflections give practically the complete life history and characteristics of the stroke. In fact, the time from zero to crest is a measure of the length of bound charge, while the time from crest to zero on the falling wave is the time of cloud discharge. The equivalent distance for one-half the time to crest also approximates the height of the cloud.

There is a large number of such oscillograms with voltage and polarity available. Fig. 2 (inset) is a typical wave shape for a large group of these oscillograms. From this oscillogram the time of discharge is 11 microseconds. The length of the bound charge is 4 micro-

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1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

seconds or 4,000 ft., while the height of the cloud is

$$\frac{4{,}000}{2} = 2{,}000 \text{ ft.}$$

When the waves become distorted the front is not of value in obtaining information on the cloud height or length of bound charge since the values are too high. The lengthened tail would indicate a longer time than actually obtained. However, an approximation of the time of discharge is twice the time from crest to half voltage on the tail. Only the first loop is of interest for this purpose. A wave often persists for a number of loops due to reflections, etc. The complete wave is important from the standpoint of internal stresses in apparatus. The total length of a direct-stroke wave, if not completely chopped by an insulator sparkover, would indicate the time of discharge.

Measurements of a large number of oscillograms of lightning voltages obtained in Pennsylvania and Ohio are given in Table I and plotted in Fig. 2. This curve includes all voltage oscillograms not causing line sparkover. The majority of the waves in Fig. 2 are of (+) polarity and probably induced. A bound charge varying from one to several thousand feet in length and a cloud height varying from 500 to several thousand feet are indicated. The voltage of the lightning bolt can be estimated by multiplying the height by the cloud in feet by 100,000 the gradient. This gives 150,000,000 for a 1,500-ft. cloud. While great accuracy is not claimed, these data give the best available information on the time of cloud discharge. The time for the voltages to reach maximum on a short antenna is also a measure of the time of cloud discharge. Very little data are available from this source, but ten readings give a cloud discharge time of less than 5 microseconds.

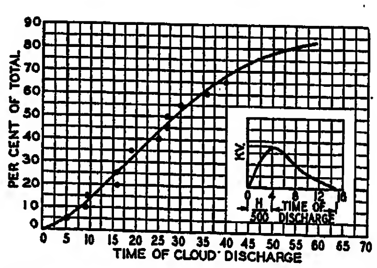


FIG. 2—MEASURED TIME OF CLOUD DISCHARGE

All 1929 Pennsylvania and Ohio oscillograms equal to or over 100 kv. and not coincident with trip-out

It is important to establish the time of discharge accurately by field tests.

Using these measured values of time, values of α varying from 0.1 to more than 0.4 are indicated from Figs. 1 and 2. These values of α substituted in equation (1) give induced voltages as high as 2,000,000 for storms over the usual line. See Figs. 2 and 6. Current corresponding to α can be found in Fig. 3.

The voltage of the free traveling wave is lower than the voltage during the forming period by the ratio of $\frac{\alpha'}{\alpha}$. Traveling waves are also rapidly reduced by attenuation. The voltages in Table I, measured at a considerable distance from the disturbance are therefore much lower than at the source. Since the object of Fig. 2 was to determine the time of cloud discharge, low-voltage values not causing sparkovers and as free as possible from distortion were purposely selected.

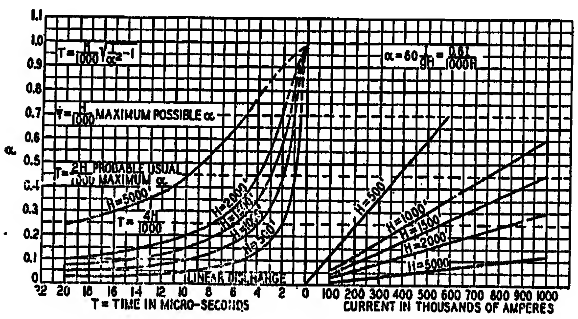


FIG. 3—VARIATION OF α WITH LIGHTNING CURRENTS AND TIME OF CLOUD DISCHARGE FOR DIFFERENT CLOUD HEIGHTS

(Based upon point cloud and linear rate of cloud discharge)

When the time and manner of the discharge is fixed the current is also fixed. In this study two types of discharge, exponential and linear, approximately representing respectively the maximum and minimum current range, are used. An example is of interest. Assume a cloud 1,000 ft. high and a 10 microsecond time of discharge. For exponential discharge, α from Fig. 1 is 0.28. The corresponding induced voltage on a line 50 ft. high is 1,400 kv. See Fig. 6. The current corresponding to α from Fig. 3 is about 400,000 amperes. *The current requirements are thus quite large and confirmation by actual current measurements is necessary before final conclusions are drawn.* In this connection, the fusing effects of high currents for short times shown in Fig. 14 are of interest. See Section III. For a 10 microsecond linear discharge, α from Fig. 3 is 0.10, and the corresponding current value is 170,000 amperes. For a given current, the induced voltages for both types of discharge are the same but the time for the linear discharge is one-third that of the exponential.

The range of α can be arrived at by starting from direct-stroke current measurements instead of time. In fact, certain direct stroke and induced voltage relations are interdependent and one cannot be changed without consideration of the other. An instrument which records only when the current is considerably above the maximum value possible by induced voltages, was developed to differentiate between induced and direct-stroke currents in towers. As the readings are proportional to the current in the tower, the use of the instrument has been extended to current measurements.[12,14] While readings as high as several hundred thousand amperes are indicated, further calibration is

TABLE I—MEASURED TIME OF CLOUD DISCHARGE
American Gas & Electric Company—1929
Penn. Power & Light Company—1929

Oscillogram number	Measured time of cloud discharge μ sec.	Voltage	Polarity	Approximate per cent: Waves over 50 μ sec.	Approximate per cent: All waves	α (From Fig. 1)
(1) Waves not Coincident with Trip-out (See Fig. 2)						
901282 Penn.	5	220	−	8	5	0.70
901241 "	9	120	+	15	10	0.50
901292B "	9.5	110	+	23	15	0.48
457A Ohio.	16	245	+	31	20	0.32
901179 Penn.	16	160	+	38	25	0.32
901252B "	18	100	+	46	30	0.28
901283 "	19	208	+	54	35	0.27
901183 "	25	180	+	61	40	0.21
901178 "	27	245	−	69	45	0.20
460 Ohio.	27	108	+	77	50	0.20
901189A Penn	30	180	+	85	55	0.19
462 Ohio.	36	190	+	92	60	0.17
901187 Penn.	40	260	+	100	65	0.15
901252A Penn.	Over 50	105	+		70	
901189B "	"	140	−		75	
901194 "	"	810	−		80	
901198 "	"	150	−		85	
901226 "	"	105	−		90	
901230C "	"	120	−		95	
901205 "	"	127	−		100	
(2) Waves Coincident with Trip-out						
901190 Penn.	11	280	−	10	8	0.43
901193 "	15	740	+	20	17	0.33
901188 "	16.5	530	+	30	25	0.30
901285 "	18	630	+	40	33	0.28
901231 "	20	330	+	50	42	0.25
901235 "	26	260	+	60	50	0.21
901197 "	27.5	190	+	70	58	0.20
901200	30	260	+	80	67	0.19
901240 "	32	300	+	90	75	0.18
901275 "	37.5	500	+	100	83	0.17
901201B "	Over 50	740	−		92	
901222 "	"	155	−		100	
(3) Waves Possibly Coincident with Trip-out						
901214 Penn.	2	135	−	12.5	7	
901191B "	5	250	+	25	14	0.70
901249 "	12	390	+	37.5	21	0.40
901191A "	21	600	−	50	29	0.25
901199 "	21.5	180	−	62.5	36	0.24
901186 "	22	330	+	75	43	0.24
901203 "	30	260	+	87.5	50	0.19
901251A "	49	390	+	100	57	0.13
901185 "	Over 50	360	+		64	
901251B "	"	100	+		71	
901202 "	"	100	−		79	
901220B "	"	330	−		86	
901254A "	"	220	−		93	
901272B "	"	600	−		100	

necessary. This instrument has shown the polarity of most direct strokes to be (−). One measurement of a direct-stroke current from a different source and made in an entirely different manner indicates 160,000 amperes and about 30 microseconds time of discharge.[11] For a 1,000 meter cloud discharge, α is 0.10 from Fig. 1. The current corresponding to $\alpha = 0.10$ and $H = 1{,}000$ (from Fig. 3) is 170,000 amperes, a remarkably good check. However, further data are necessary before definite conclusions can be drawn.

Values of α, based upon linear discharge, for various currents and cloud heights are given in Fig. 3. The corresponding values of time for cloud discharge are also given in the curve on the left hand side of the figure. Values of current for exponential discharge can be obtained by finding α for the given time from Fig. 1 and reading the amperes from the right hand curve in Fig. 3. It is of interest that a value of α of 0.7 obtains when the time of cloud discharge is equal to the time for a wave to travel from cloud to ground. This should be the maximum possible value of α. The actual maximum may be 0.45 or that corresponding to the time required for the wave to travel from cloud to ground and return. *The extent that this range of values occurs in practise can only be determined from observations.* A further examination of the curves shows that very low values of α can be obtained for special cases. In an example frequently cited as a proof that only very low induced voltages are possible, the special case of 200,000 amperes and a 5,000 ft. cloud height was taken.[5] This gives a value of α (from Fig. 3) of 0.024 or a voltage of 120,000 on a 50-ft. line. The calculations were really made as in equation (1). The assumption of a 5,000-ft. cloud is largely responsible for the low values. The time of cloud dis-

charge for this particular case is 200 microseconds. The wide range of possibilities illustrated in Table II shows that it is unsafe to generalize from a special case.

TABLE II—VARIATION OF INDUCED VOLTAGES WITH DIFFERENT ASSUMPTIONS AS TO CLOUD HEIGHT AND LIGHTNING CURRENT

Cloud height feet	Lightning current amperes (assumed)	α (calculated)	Induced voltages kv. (calc.)	Approximate time of discharge μ sec. (calc.)
LINEAR DISCHARGE Line 50 ft. High (No Ground Wires)				
500	600,000	0.70	3,600	0.5
1,000	600,000	0.36	1,800	3.
2,000	600,000	0.18	900	11.
500	200,000	0.24	1,200	2.
1,000	300,000	0.18	900	6.
2,000	300,000	0.09	450	22.
*5,000	200,000	0.024	120	200.
Line 80 ft. high (No Ground Wires)				
500	600,000	0.70	5,760	0.5
1,000	600,000	0.36	2,880	3.
2,000	600,000	0.18	1,440	11.
1,000	300,000	0.18	1,440	6.
*5,000	200,000	0.024	190	200.

$$T = \frac{H}{1000}\sqrt{\frac{1}{\alpha^2}-1} \qquad \alpha = \frac{0.6\,I}{1000\,H} = \frac{60\,I}{g\,H}$$

Cloud height feet	Lightning current amperes (assumed)	α (calculated)	Induced voltages kv. (calc.)	Approximate time of discharge μ sec. (calc.)
EXPONENTIAL DISCHARGE Line 50 ft. High				
500	200,000	0.25	1,250	6.
1,000	410,000	0.25	1,250	11.
2,000	840,000	0.25	1,250	21.
Line 80 ft. High				
500	200,000	0.25	2,000	6.
1,000	410,000	0.25	2,000	11.
2,000	840,000	0.25	2,000	21.
500	90,000	.125	1,000	12.
1,000	195,000	.125	1,000	22.
2,000	400,000	.125	1,000	45.

H = cloud height in feet.
I = current in amperes.
g = volts per ft.—100,000.
*Special case frequently cited (See Bibliography No. 5).

Examples of induced voltage values based upon assumed currents are given in Table II for a 50-ft. and 80-ft. conductor height (from Figs. 1 and 3).

Equation (1) becomes

$$V = g\,h\,\alpha\,\omega \tag{2}$$

when the ground wire protective ratio factor ω is added. Since

$$g = 100{,}000$$
$$V = 100{,}000\,h\,\alpha\,\omega$$

V = lightning voltage
h = average conductor height in feet (in dry, sandy, or rocky country with high resistance grounds the effective h may be much higher than indicated by the height of line)
α = usually less than 0.5
ω = 0.5 or less, depending upon ground wire arrangement.

The polarity of the induced voltage is opposite to that of the cloud.

A large variation of α and induced voltages is possible for the values of current and cloud height assumed. The lightning sparkover for a 230-kv. line with 14 suspension insulator units and the ½/5 wave is about 1,950 kv.; the corresponding voltage for the 1½/40 wave is about 1,300 kv. The effects of the ½/5 laboratory wave correspond to steep or short lightning waves; and the effects of the 1½/40 laboratory wave to the less abrupt and longer lightning waves. It is believed that the 1½/40 waves apply more generally to the lower voltages. Table II shows that under certain assumptions induced voltages caused by nearby strokes to ground could exceed the line insulation strength and cause sparkover of high-voltage lines. It also shows that under other assumptions nearby strokes could occur without causing sparkover on such lines. This

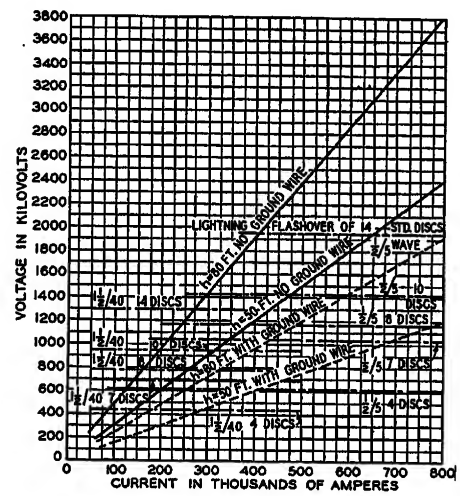


FIG. 4—VOLTAGES INDUCED ON LINES 50 FT. AND 80 FT. HIGH FOR VARIOUS LIGHTNING CURRENTS FROM A 1,000-FT. CLOUD

seems to account for conflicting reports or observations on the magnitude of induced voltages caused by nearby lightning strokes. With ground wires the above voltages could be reduced to one-half or less.

Table II and Fig. 4 indicate that a line insulated for 230 kv. (14 to 16 units), and without ground wires, could have occasional sparkovers by induction; that it is unsafe to assume induced voltages negligible until more data are available; that the addition of ground wires should eliminate sparkovers caused by induction; that higher lines or lines of the same height but with less insulation should have sparkover unless special ground wire arrangements are made.

A ground resistance of less than 50 ohms is desirable from the standpoint of induced voltages. For this resistance the ground wire efficiency is reduced in the order of 25 per cent. It is indicated later that resistances as low as 5 ohms might be desirable from the standpoint of direct strokes.

Fig. 4 shows the induced voltages on lines 50 ft. and 80 ft. high corresponding to various assumed lightning

currents from a cloud 1,000 ft. above ground. The sparkover voltages for different line insulation for the ½/5 and the 1½/40 waves are also indicated.

Fig. 5 shows the number of insulators necessary to resist induced lightning voltages calculated for various currents from a cloud 1,000 ft. high. These curves show that the proportion of sparkovers due to induced voltages would be expected to increase rapidly as the line insulation is reduced below 14 units when the insulation strength is based upon the ½/5 wave. The beneficial effect of the ground wire is also shown.

In Fig. 6 induced voltages are plotted for different values of time of cloud discharge. Fig. 6 is of especial interest because by comparing with Fig. 2 the possible voltages due to measured values of time of cloud discharge can be obtained. Based upon the measured time of discharge, a cloud 2,000 ft. high and a line 50 ft. high, 10 per cent of the discharges could induce a voltage over 2,500 kv.; 20 per cent over 1,600 kv.; 40 per cent over 1,150 kv.; and 60 per cent over 800 kv. The data given in Table I were, in most cases, for waves originating a number of miles from the oscillograph. Due to distortion, the actual time is shorter than measured. Most of the waves in Fig. 2 were (+) and probably induced. The voltages at the source were probably over twice the measured values. A similar study was made of the front of waves.[10] The data are

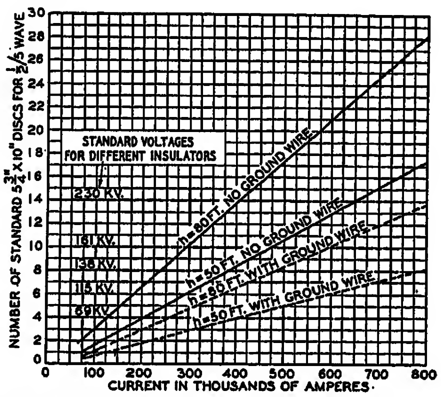


Fig. 5—Number of Standard 5¾-In. by 10-In. Suspension Insulators

Necessary to resist induced lightning voltages caused by various currents from a 1,000-ft. cloud

given in Table IV. An average cloud height of 1,500 ft. is indicated, with a range of 500 to 5,000 ft.

It can be concluded that:

1. Low lines are desirable.
2. Ground wires are desirable.
3. As the insulation is reduced or the conductor height increased, induced voltage sparkover becomes increasingly important.
4. Induced voltage troubles can be eliminated by ground wires.
5. Measurements made from cathode-ray oscillograms give data for estimating the induced voltage on any line. Further measurements are necessary to establish numerical values definitely.

Direct Strokes

Chance of Being Struck. The chance of a line being struck increases with increasing height of line. On a flat plane without projections, a tower may be hit when the distance from the projection of the storm center is less than four to ten times its height. This factor is based on the assumption that the nearest object will be hit and has been called the *direct hit ratio, r.*[3,6] Tests

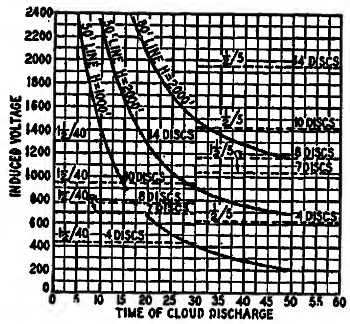


Fig. 6—Voltages Induced on Lines 50 Ft. and 80 Ft. High for Different Values of Time of Cloud Discharge

made on models in the laboratory check this ratio. The higher the cloud compared to the line, the greater this factor. The ratio r for different cloud-tower ratios is given in Table III.

TABLE III—DIRECT-HIT RATIO

$\frac{\text{Cloud height}}{\text{Tower height}} = \frac{H}{y}$	Direct-hit ratio r
50	10
40	9
30	7.5
20	6
10	4

Danger zone when storm center is over a strip $2\,r\,y$ ft. wide with tower as center.

The direct-hit ratio r is approximate and corresponds to the (−) cloud. The danger zone is less with the (+) cloud. This seems to be due to the fact that corona streamers from a (+) point (tower) are of much greater length, and thus more directive than from a (−) point (tower). For a (+) cloud the direct-hit ratio of four corresponds to an H/y ratio of about twenty. For laboratory tests see Bibliography reference 3, p. 289.

An example of the use of Table III follows:

With a ground wire or tower 50 ft. high and a cloud 1,000 ft. high

$$\frac{H}{y} = 20 \qquad \text{and } r = 6$$

A direct stroke may go to the ground or to a 50-ft. tower when the projection of the storm center comes within 300 ft. of the tower. In this case, a direct hit may take place to the tower when it is above a zone 600 ft. wide with the line as center. For a 100-ft. tower and a 1,000-ft. cloud this strip would be 800 ft. wide.

$$\frac{H}{y} = \frac{1,000}{100} = 10 \qquad r = 4$$

$$4 \times 100 \times 2 = 800 = \text{width of danger zone}$$

The hazard is thus greater for the taller tower.

When there are projections on the plane the chance of a given object being hit is very greatly reduced as shown in Fig. 7. Fig. 7a shows that a lightning stroke may begin to hit the tower when x and y become equal. Passing to Fig. 7b, a tree half the height of the tower and half way between it and p would take the strokes since

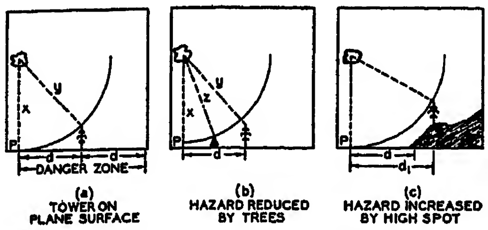


FIG. 7—LIGHTNING HAZARDS

the distance z would then be shorter than the distances y and x. The direct-hit ratio is approximate since it also depends upon polarity.[3] See note on Table III.

It is thus apparent that trees of even moderate height, within several tower lengths of the line, very greatly reduce the danger zone and the chance of a direct stroke to the line. Furthermore, many lines are brought through a right-of-way with trees practically equal to the line height. The hazard of direct strokes to such lines should be very small. It is a good reason for a low tower since the hazard is reduced to a much greater extent than would be indicated by the direct reduction in height. This follows because the chance of being

TABLE IV—PERCENTAGE OF LIGHTNING WAVES OF VARIOUS FRONTS MEASURED ON TRANSMISSION LINES

Per cent of waves	Measured wave front (μ sec.)	Estimated front at origin (μ sec.)	Cloud height	Impulse ratio for breakdown[10] on rising front of waves (Time from Column No. 3) Insulators	Gap
20	1	0.5	500	2.2	2.8
40	2	1.0	1,000	1.9	2.2
50	3	1.5	1,500	1.7	2.0
60	4	2.0	2,000	1.6	1.9
80	7	3.5	3,500	1.5	1.6

hit depends largely upon the relative heights of the line and secondary objects. It should be pointed out that a tree can direct a lightning stroke or "attract" lightning as readily as a metal tower because the charging takes place with small currents over a relatively long time. It also follows that the chance of any particular tower being struck does not depend upon its footing resistance. The resulting voltage on the insulators following a hit does, however, depend almost directly on this resistance.

Fig. 7c illustrates a line on a high point. If storms pass over such a point the hazard may be great. A consideration of the hazard of a line being struck emphasizes the importance of a tower relatively low compared to trees, etc.

Line Voltages Due to Direct Hits. Usually the tower is more likely to be struck than the line. When a tower is struck the resulting voltage from line to tower or ground wire, or across the insulator string, is

$$V_s = K\,I\,R \qquad (3)$$

Where K is a constant depending upon the tower and ground wire configuration, I is the current in the tower, which will be less than the current in the stroke by the current in the ground wire, and R is the tower footing resistance; 0.80 may be taken as an average value of K for purpose of calculation.[7]

This formula shows the importance of keeping the tower footing resistance very low if immunity from direct-stroke sparkovers is to be hoped for. It was shown above that a line with ground wires and 14 to 16 insulator units should be free from sparkovers due to induced voltages. It is now of interest to test such a line from the standpoint of direct strokes to the tower.

Assume I = 100,000 amperes in the tower

K = 0.8 and R = 40 ohms

then $V = 100{,}000 \times 40 \times 0.8 = 3{,}200{,}000$ volts

The resistance R measured by a megger or at low voltage will be somewhat greater than the high-voltage lightning value. A heavy direct stroke with a 40-ohm tower footing resistance would thus cause a sparkover. Furthermore, it would be necessary to keep the footing resistance to about 20 ohms to prevent sparkover at 100,000 amperes on a line insulated with 14 units. See Fig. 8. Thus:

$$V = 100{,}000 \times 20 \times 0.8 = 1{,}600{,}000 \text{ volts}$$

If 400,000 amperes obtains in direct strokes a 5-ohm tower footing resistance would cause 1,600 kv. Since a given current is associated with a given time of dis-

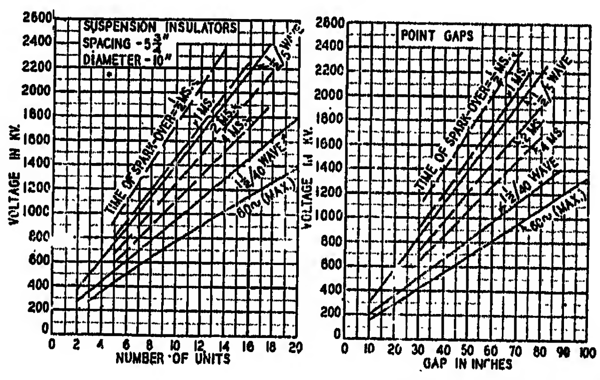


FIG. 8—LIGHTNING AND 60-CYCLE SPARKOVER OF INSULATOR AND POINT GAP

charge, it is possible to estimate the current and thus possible direct-stroke voltages from the oscillograms discussed above.

It is important to establish the actual range of direct-stroke currents by field measurements. For high currents a low resistance is necessary to make a line immune from outages due to direct hits at the tower.

The maximum insulator voltage at the tower struck is limited by the footing resistance but the tail of the

wave is rapidly reduced by reflections on the ground wire from the towers on either side. This reduction begins when the wave on the ground wire has traveled to the other tower and back. For a 1,000-ft. span the time is $\frac{2 \times \text{span}}{1{,}000} = 2$ microseconds; for a 500-ft. span it is 1 microsecond. The effect is that of applying a shorter wave across the insulators for the shorter span. The sparkover voltage is greater for the shorter waves. This is one advantage of the short span. Curves for

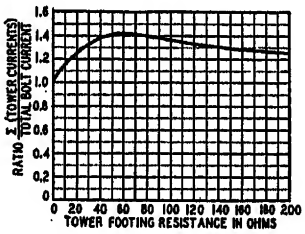


Fig. 9—Calculated Relation of Sum of Tower Currents to Total Current in Lightning Bolt for Tower with Different Footing Resistances

(For meter measuring maximum instantaneous crest currents)

sparkover in ½, 1, 2, and 4 microseconds are plotted in Fig. 8. The 1 and 2 microsecond waves correspond to the waves for 500- and 1,000-ft. spans respectively. It will be noted that the ½/5 laboratory curve falls between these values and should thus apply to direct-stroke sparkovers on the usual line. As already noted, when a tower is hit the total current of the stroke does not go directly to ground; part goes to ground through the ground wire to other towers. The division depends largely on the footing resistances. When the footing resistance is low, the total current of the stroke may be obtained approximately by adding the currents of the various towers. (See Fig. 9.)

It might appear that in certain cases a direct stroke could be steep enough to produce sufficient voltage between line and tower to cause sparkover even with zero footing resistance. This does not appear probable on a high-voltage line and the usual tower. On a 50-ft. tower the reflected wave would begin to reduce the voltage in about 0.1 microsecond. A front increasing at from 10,000,000 to 20,000,000 volts per microsecond would be necessary.

Location of Ground Wire to Shield Conductors from Initial Hits. The ground wire or wires should be at a sufficient distance above the line wires to "attract" the lightning and thus take the initial hit. The shielding effect of a ground wire depends upon its vertical height relative to the line wire, and to the horizontal distance between the projections of the two wires. If this horizontal distance is X and the vertical height of the line wire is h, the ground wire should have a vertical height greater than y.

y can be obtained from Fig. 10 or calculated from the equation

$$\frac{X}{H} = \sqrt{2\left(\frac{y}{H}\right) - \left(\frac{y}{H}\right)^2} - \sqrt{2\left(\frac{h}{H}\right) - \left(\frac{h}{H}\right)^2}$$

which gives y and h for equal hits. y should be at least 10 per cent greater than the calculated value for protection of h. Fig. 10 curves are for a cloud height $H = 1{,}000$ ft. While this position of the ground wire may shield the line wire from the initial hit, it does not follow that a side flash or sparkover will not take place from the struck ground wire to the line if certain other conditions are not fulfilled. This is so if the hit takes place at some distance from the tower and the effect is maximum for a hit at the center of the span.

Direct-Hit Wire—Protection from Side Flashes. The only difference between the effect of a direct hit at the tower and one somewhere on the ground wire is that caused by the time taken by the wave to travel from the point hit to ground and for the reflected wave to travel back. This is so because the point struck does not "know" there is a ground until the reflected wave returns. Since the reflected wave is of opposite polarity it begins to reduce the voltage at this point upon its arrival. If the front of the direct-stroke voltage is not steep enough to reach a dangerous voltage before the wave returns, sparkover will not occur between the ground wire and the line if the ground resistance is low enough. The time required for the return wave is one microsecond for every thousand feet of complete travel. The maximum distance to the ground would occur for a hit in the center of the span. There are several ways

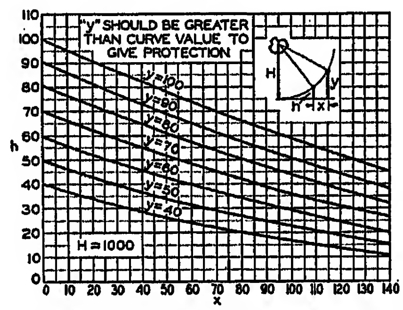


Fig. 10—Location of Ground Wire to Take Initial Hit

by which this factor can be taken care of, as for example:

a. Very high direct-hit wires.

b. Short spans or short distance between ground wire grounds.

c. A moderately high direct-hit wire and a moderate length of span.

d. By separate towers or lines to take direct hits.

With the high direct-hit wire the separation between ground wire and the line wire requires a longer time for the voltage to reach the sparkover value. Because of its height it is undesirable from the standpoint of "attracting" direct hits and not being very effective as a ground wire. With long spans 1,000 ft. or more and

abrupt direct strokes the separation between this wire and the line or ground wire at the center of the span should be 25 ft. to 50 ft. or more. Methods of stringing may permit greater clearance at the center of the span. These distances for different lengths of span can be obtained from Fig. 11. The *minimum distance required* on the assumption of 10,000,000 volts reaching the line is given at the intersection of the sparkover curve and the curve of voltage between ground wire and line wire.

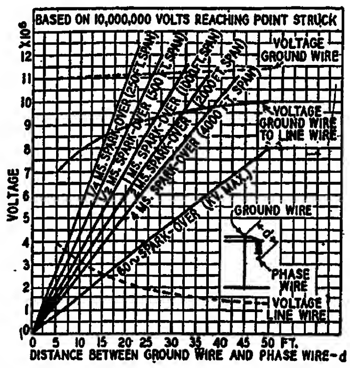


FIG. 11—MINIMUM CLEARANCE NECESSARY TO PREVENT SPARKOVER FROM GROUND WIRE TO LINE FOR A DIRECT HIT TO GROUND WIRE IN CENTER OF SPAN

The sparkover curve is based upon the point gap sparkover with voltage rising to breakdown value in the time for the wave to travel to the tower and back.[10,8] Usually it may not be necessary to design for a hit at the center of the span since the chances of such a hit are small. Further experience with direct-hit wires of varying heights on sections of operating lines is desirable.

Immunity from Outages Due to Direct Hits. This discussion shows that if lightning strikes the tower,

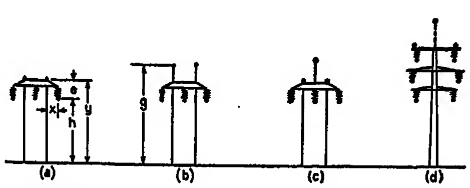


FIG. 12—TOWERS WITH INDUCED VOLTAGE AND DIRECT-STROKE PROTECTION

immunity from sparkover can be approached only with very low tower footing resistance and 230-kv. insulation (14 to 16 units). Direct-stroke wires in addition to the ground wires may be necessary. A line with comparatively short spans (500 ft.) and a direct-hit wire sufficiently high to take the initial strokes from the ground wires, as well as the line wire, may generally be more practicable than excessively high direct-hit wires. (See Fig. 12.) This same effect could be obtained on a long span line by placing a ground wire or direct-stroke wire sufficiently high to take the initial discharges on what is, in effect, a short span line. This could be accomplished by permitting the main towers to support the direct-stroke wire and using the intermediate towers for grounding purposes only.

Low-Footing Resistance. Low-footing resistance can usually be obtained by means of buried wires connected to the tower legs. These wires may be either extended out radially from the tower or extended directly from the towers along the line. Radial wires of moderate length should usually be more effective. These have been called counterpoise wires, a term having a somewhat different meaning in radio. However, the method used will be governed largely by cost.

Protection Against Direct Strokes When Towers Have High-Footing Resistance. In certain parts of the country where there is little or no soil and the towers are located in rock it may be very difficult or impossible to reduce ground resistances low enough to prevent direct-stroke outages. On the other hand, resistances low enough for the induced voltage ground wire may be practicable. When economic conditions permit, this can be taken care of by extending parallel lines at a distance x on either side of the sections of the transmission line.

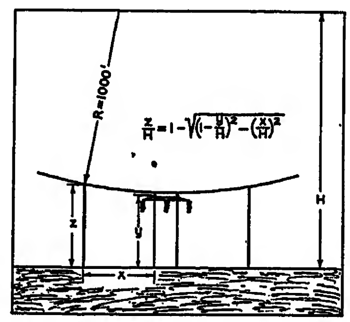


FIG. 13—SPECIAL TYPE OF CONSTRUCTION FOR PROTECTION AGAINST INDUCED VOLTAGES AND DIRECT STROKES

The object of these lines is to "attract" the stroke away from the transmission line and thus prevent it from being hit. For ideal protection a cloud directly overhead should be nearer the direct-stroke line than the main line as shown in Fig. 13. The minimum height of these lines Z, can be calculated from the formula

$$\frac{Z}{H} = 1 - \sqrt{\left(1 - \frac{y}{H}\right)^2 - \frac{x}{H}^2}$$

This line need not have low-footing resistance and the conductor need not be any larger than mechanical conditions require. This form of protection would in many ways seem ideal.

Another solution, possibly not economical, would be a very low line with a net fence and net top. Towers placed to take all of the direct hits could also be used for this purpose.[6]

Dischargers or Diverters. It is possible to prevent outages by some form of discharge device or diverter at the tower permitting the lightning to discharge freely to ground but preventing the power current from following. The requirements for such a device are much more severe for direct strokes than for induced voltages.

In the case of direct strokes the resistance must be very low. The fused grading shield is operating successfully in practise. Other devices have also been successful but they are still in the developmental stage. One weakness of such a device is that when located at the tower it would not necessarily prevent a power arc between line and ground wire due to a lightning discharge at the center of the span.

III. Operating Experience

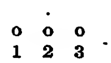
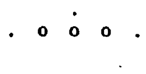
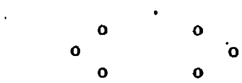
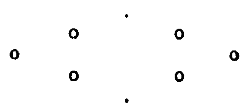
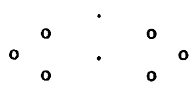
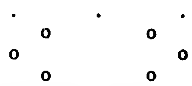
The Ground Wire. There is now overwhelming operating evidence of the value of the ground wire in reducing outages, operating reports generally indicating a reduction varying from two to one, to ten to one after the installation of ground wires. Probably better results will be obtained with more attention to location and grounds. Measurements of the protective ratio of ground wires on full size outdoor antennas give good agreement with the values in Table V obtained from laboratory measurements on models. These ratios are lower than those obtained mathematically where corona is neglected.

The value of the ground wire for direct-stroke protection has also been indicated. This applies to the ground wire as usually located, as well as the high direct-stroke wire, both of which have been experimented with on the lines of the Pennsylvania Power & Light Company. On one bad section of this line the outages were 6, 9, and 14 respectively in the years 1926, 1927, and 1928. During 1929 the ground resistances on this section were reduced from 50 to 150 ohms to values of the order of one ohm and the usual ground wire was unchanged. There were no outages on the section in 1929 and 1930. "Diverter" wires or high-ground wires erected about 50 ft. over the line wires on another section were struck without outages.[13]

Direct Strokes vs. Induced Voltages as a Cause of Outages. It is very difficult to decide definitely whether an outage is caused by induced voltages or direct strokes even when lines are equipped with instruments for that purpose. There is very good evidence that 70 per cent of the outages on the 220-kv. (14 to 16 insulator units) lines of the Pennsylvania Power & Light Company were caused by direct hits. A result of this order might be expected from the analysis above, particularly from the results in Figs. 2 and 6.

A very interesting analysis of outages on the 132-kv. lines of the American Gas & Electric Company has been made by Mr. Sporn.[9] Table VII in this paper shows that 8 outages out of 12 occurred where the footing resistance was less than 5 ohms. In the remaining 4 the resistances varied from 18 to 33 ohms. Similar data are given for other lines. These data would at first appear to be at variance with the conclusion that low-ground resistance is important, because more flashovers occurred on the towers of low-ground resistance than on those of high resistance. A possible explanation is as follows:

TABLE V—EFFECT OF GROUND WIRES ON LIGHTNING INDUCED VOLTAGES
(From Tests on Models)

Arrangement	Protective ratio	
	1 2 3	0.50 0.44 0.50
	1 2 3	0.40 0.34 0.40
	1 2 3	0.34 0.28 0.34
	1 2 3	0.28 0.25 0.28
	1 2 3	0.43 0.50 0.62
	1 2 3	0.34 0.39 0.48
	1 2 3	0.40 0.48 0.36
	1 2 3	0.35 0.35 0.43
	1 2 3	0.25 0.30 0.37

o indicates conductor
. indicates ground wire

$$\text{Protective ratio} = \frac{\text{Voltage with ground wire}}{\text{Voltage without ground wire}}$$

Average values for various relative conductor and ground wire distances that have been used in practise.
Includes corona effect.

PROTECTIVE RATIOS
ω
Showing Effect of Varying Distance between Line Wire and Ground Wire

h = 50 ft.

x	Values of y: 60 ft.	70 ft.	80 ft.	90 ft.	100 ft.	125 ft.
0 ft.	0.41	0.48	0.53	0.58	0.63	0.73
10 ft.	0.45	0.50	0.55	0.59	0.64	0.74
20 ft.	0.51	0.53	0.57	0.61	0.65	0.75

h = 80 ft.

x	Values of y: 60 ft.	70 ft.	80 ft.	90 ft.	100 ft.	125 ft.
0 ft.	.....	.....	0	0.39	0.43	0.52
10 ft.	.....	.....	0.39	0.41	0.45	0.54
20 ft.	.....	.....	0.44	0.45	0.47	0.56

For a single ground wire.
x = horizontal distance between projections of ground wire and phase wire.
y = vertical height of ground wire.
h = vertical height of line wire.

For the lower ground resistance values sparkover by direct strokes would not be expected, while for the higher values direct-stroke outages would be expected from application of equation (3). On the other hand, most of the footing resistance values are low enough so as not to affect materially the efficiency of the ground wire for induced voltages. Approximately equal effects would be expected from induced voltages as recorded in the tables. Thus 100 per cent of the outages could be accounted for by induced voltages and 33 per cent by direct hits at the tower. Therefore the cause of outages might be 66 to 100 per cent by induction or up to 33 per cent by direct hits and the remainder by induction. The percentage of direct-hit outages might be increased somewhat by side flashes from hits out on the span.

Lightning has been observed to strike near lines without causing induced voltages high enough to flashover insulation; it has also been observed to strike at a distance with simultaneous splintering on wood poles or insulator flashovers on steel towers. Because of the short duration of the phenomena, exactly what happens is frequently uncertain. However, as pointed out above, induced voltages would be expected to vary widely

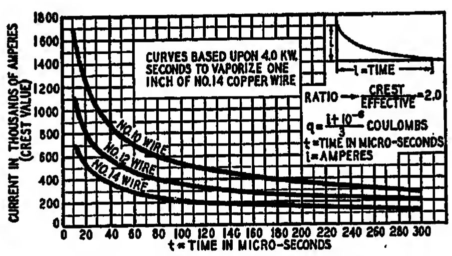


FIG. 14—CURRENT REQUIRED TO VAPORIZE WIRE FOR VARIOUS TIME LENGTHS OF CURRENT APPLICATION

with the character of the discharge. The reported observations are, therefore, not contradictory.

On July 16, 1930 a radio antenna of No. 12 copper wire was struck at Pittsfield, Massachusetts. One hundred feet of the wire was completely vaporized by the lightning current. The middle curve in Fig. 14 gives the calculated current necessary to vaporize a No. 12 copper wire for varying time.* It is of interest that this current is 800,000 amperes at 20 microseconds; 510,000 amperes at 50 microseconds and 112,000 amperes at 1,000 microseconds. No. 14 rubber-covered copper wire used on telephone circuits and house wiring is also frequently fused.

In order to give the various factors the proper weight in tower design it is important to obtain more data on how frequently the conditions necessary to cause high induced voltages occur during storms and also on the hazard of a line being struck.

*These curves are based upon 4.0 kw. seconds to vaporize 1 inch of No. 14 copper wire.

IV. TOWER DESIGN

Several different tower designs will now be considered for the purpose of illustrating the application of the above principles.

230-Kv. Line Conductors in Horizontal Plane

Assume that a single-circuit steel tower will be used with conductors in a horizontal plane and limited to an average height of 50 ft.

With two ground wires the general arrangement would be as in Fig. 12a. Take $\alpha = 0.45$ as the probable maximum value, then

Induced Voltages

$$V = g\,h\,\alpha = 100 \times 50 \times 0.45 = 2{,}250 \qquad (1)$$

or with ground wires when $\omega = 0.5$

$$V = g\,h\,\alpha\,\omega = 1{,}125 \qquad (2)$$

The footing resistance should be less than 50 ohms and preferably not over 25 from the standpoint of induction. From Fig. 8, eight insulator units would be sufficient with the ½/5 wave or twelve on the 1½/40 wave. Thus induced voltages are readily taken care of if ground wires are used.

Direct-Hit Voltages $V = K\,I\,R = 0.8\,I\,R \qquad (3)$

Take $I = 200{,}000$

then for $R = 25$ ohms

$V = 0.8 \times 200{,}000 \times 25 = 4{,}000{,}000$ volts $= 4{,}000$ kv.

This is a high voltage for which to insulate. Reduce the footing resistance, then

$$R = 12 \text{ ohms}$$
$$V = 1{,}920 \text{ kv.}$$

This requires 14 units on the ½/5 wave which seems reasonable for most direct strokes (See Fig. 8). However, a more conservative arrangement, until more information is available on the common direct-stroke currents, would be 5-ohm resistance and 14 to 16 units. It will be noted that up to the present the induced and direct-stroke requirements are the same except that the direct stroke necessitates a lower ground resistance. The clearance from the conductor to tower can be obtained from the point gap curve in Fig. 8.

Shielding the Line Wires from Direct Hits. It is now necessary to determine if the ground wires are high enough to shield the conductor from the initial hit. Referring to Fig. 10, for $X = 10$ ft. and $h = 55$ ft. at the tower, y must be greater than 60. Insulator lengths and other considerations have fixed y at 70 ft. The shielding condition is more than met without impairing the efficiency of the ground wires.

Direct Hits to Ground Wires at Center of Span. To provide against direct hits from the ground wire to the line at the center of the span consideration of a further condition is necessary.

If the span is 500 ft. the requirement is a minimum clearance of 20 feet. See Fig. 11. A somewhat greater clearance at the center of the span can probably be provided for without greatly reducing the ground wire

efficiency. Check the clearance decided upon with the protective ratio in Table V to see if the value is greater than 0.5 as used above.

With a 1,500-ft. span the requirement is greater than 29 ft. and the ground wire is reduced in efficiency.

From the calculation of the voltage due to direct strokes with a 5-ohm ground it is seen that 14 to 16 units are necessary. It would then appear that 230-kv. insulation is necessary where protection from direct strokes is desired. The tower in Fig. 12a seems to meet the requirements for 230-kv. insulation both from the usual induced voltage and direct-stroke standpoints, with a possible small factor of safety for mid-span hits on the longer span. It meets the induced voltage conditions for 138-kv. insulation but not all of the direct-stroke conditions. A one-ohm tower resistance would be desirable. *Operating experience is desirable with direct-hit wires of different heights on sections of lines exposed to lightning.* Good results have been obtained with the usual ground wire with low-footing resistance as noted above.

If longer spans are used or greater safety factor for mid-span hits is desired, the height of the ground wire may be increased as in Fig. 12b with 16-unit insulation. This could be done without too great reduction in the induced voltage efficiency.

For lines of lower voltage or at lower insulation a combination of direct-stroke wires can be used as in Fig. 12c. Figs. 12b and 12c have the disadvantage of increasing the hazard of being hit. For high insulation, the ground wires in Fig. 12c could be omitted, leaving only the single direct-stroke wire properly located to take direct hits.

Conductors in Vertical Plane.

A tower with a vertical conductor arrangement increases the difficulty of designing a line highly resistant to lightning. See Fig. 12d.

Induced Voltages. $V = h\,g\,\alpha\,\omega = 100\,h\,\alpha\,\omega$

$$= 100 \times 80 \times 0.45 \times 0.5 = 1{,}800 \text{ kv.}$$

would require 14 to 16 insulator units with ground wires for the ½/5 wave. See Fig. 8.

Direct-Hit Voltages. With 12-ohm footing resistance —200,000 amperes

$$V = K\,I\,R = 0.8 \times 200 \times 12 = 1{,}920$$

Requires 14 to 16 units for the ½/5 wave.

Obtain clearance to tower from gap curves (Fig. 8).

Shield Wire. The 15-ft. clearance between line and ground wires should be sufficient for shielding.

Direct Hits to Ground Wires at Center of Span. If desired, a direct hit wire can be provided as shown by dotted line.

With this tower the number of direct hits should be greater than for tower 12A.

Short Spans Important

Since the height of the direct hit wire must be increased with increasing span length, short spans between ground wire grounds are important. As already noted, short spans also increase the effectiveness of the insulation.

Special Method of Protection Against Direct Hits.

Where low-ground resistances are not possible, or are difficult to obtain, the arrangement in Fig. 13 could be used. In fact, if economically feasible it would be ideal for any condition since it takes the hit away from the tower. The usual tower is used with the ground wires that are necessary for induced voltages. The ground resistance of these wires need not be particularly low. At either side, at a distance from the line about equal to the height of the ground wire, shielding wires are erected and grounded at each pole. No care need be taken in obtaining low-ground resistance. The wire need not be larger than mechanical conditions demand. The object of these wires is to take the direct hits and thus prevent the line from being struck.

With the ground wire height $y = 60$ and the distance between ground wire and shielding wire $x = 60$, the minimum height of the shield wire is $z = 62$ from

$$\frac{Z}{H} = 1 - \sqrt{1 - \left(\frac{y}{H}\right)^2 - \left(\frac{x}{H}\right)^2}$$

The same effect is obtained by trees along the right-of-way of many lines. Z should be at least 10 per cent greater than the calculated value. Direct-stroke towers could also be used.

The towers shown in Figs. 12 and 13 are not intended for definite designs but merely for the purpose of illustration.

The above principles can be used in the design of lines with wood pole towers. With such towers advantage should be taken of the wood to increase the insulator sparkover. The lightning strength of wood has already been given elsewhere.[2,3]

V. Wave Shapes—Coordination

From the above analysis it appears that the steep wave-front effects, causing insulator sparkover on the more highly insulated or higher voltage lines, are approximated by the ½/5 wave. This applies particularly to direct strokes to the ground wire or to tower, or severe induced voltages. The ½/5 curve should then apply in determining the lightning strength under such conditions. When a direct stroke goes to a line wire, high sparkover values such as are caused by the ½/5 wave will occur but the resulting wave on the line is likely to be very long.

Lower induced voltages may have very long waves. While voltages of such waves will not usually reach sufficiently high values to cause sparkover on the highly insulated lines, they may be the predominant cause of trouble on the lower voltage lines, especially on distribution circuits. The 1½/40 wave approximates the effects of this type of lightning voltage as the lower extreme. Both waves should be considered in the design of the lower voltage lines.

The coordination of line insulation with apparatus

insulation has been so successful in practise that it will probably soon be in general use. In adjusting the sparkover values of the various insulations and gaps for purposes of coordination it is important to have the relative values hold for any possible lightning wave. Usually, if the coordination adjustment is made with the ½/5 wave, the same relative sparkover positions of the various insulators will remain the same for any wave. However, it may sometimes be desirable to know the relative voltages for a long wave as well as a short one. All conditions are well covered by tests with the ½/5 and the 1½/40 waves. A wave falling between the two and giving an impulse ratio of about 1.5 might also be useful. Further information on lightning strength for different waves can be found elsewhere.[10]

VI. Conclusions

1. The cloud height, the charge, the time of discharge, and the current and voltage of the bolt can be estimated from lightning oscillograms on transmission lines.

2. An independent check on oscillographic measurements can be obtained from direct-stroke current measurements.

3. Transmission line voltages due to induction and direct strokes can be calculated from the above information.

4. In some instances more accurate field measurements are necessary before final conclusions can be reached.

5. Line sparkover can be caused by either induced voltages or direct hits. As line insulation is increased, the direct hit becomes of increasing importance as the cause of sparkover.

6. Induced voltages can be reduced to safe values on high-voltage lines by ground wires. An important factor in direct-stroke protection is low tower footing resistance.

7. The general requirements for a safe line from the standpoint of direct hits or induced voltages are the same. They are; low lines, short spans, low tower footing resistance, and ground wires. However, direct strokes require lower footing resistance and higher ground wires.

8. The lightning strength of insulators for induced voltages and direct strokes can be estimated.

9. A lightning proof line seems feasible. Special designs are discussed.

10. Methods of tower design are illustrated.

11. More accurate field measurements are desirable to determine the numerical range of voltage, current, etc.

The valuable assistance of Mr. L. V. Bewley in developing formulas, making calculations and preparing the appendix is acknowledged.

Mathematical Appendix

Induced Strokes

$$e = \frac{(\omega)}{2}\int_0^t \left\{ f[x+v(t-\tau)] + f[x-v(t-\tau)] \right\} \frac{\partial F(\tau)}{\partial \tau}\, d\tau \qquad (1)$$

$f(x)$ = Distribution of voltage due to instantaneous release of bound charge.

$F(t)$ = Law of cloud discharge

$$= \begin{cases} \dfrac{t}{T} & \text{linear law} \\ (1 - \epsilon^{-\frac{t}{3T}}) & \text{exponential law} \end{cases}$$

T = Time of cloud discharge

$$(\omega) = \frac{1}{h_r} \frac{\begin{vmatrix} z_{11} & \dots & z_{m1} & h_1 \\ \dots & \dots & \dots & \dots \\ z_{1m} & \dots & z_{mm} & h_m \\ z_{1r} & \dots & z_{mr} & h_r \end{vmatrix}}{\begin{vmatrix} z_{11} & \dots & z_{m1} \\ \dots & \dots & \dots \\ z_{1m} & \dots & z_{mm} \end{vmatrix}}$$ = theoretical protective ratio for line wire r due to m ground wires

h = Heights of the conductors

$$z_{xx} = 60 \log_\epsilon \left(\frac{2h}{\rho} \right) = \text{self surge impedance}$$

$$z_{xy} = 60 \log_\epsilon \left(\frac{a}{b} \right) = \text{mutual surge impedance}$$

$$e' = \frac{(\omega)}{2}\int_0^t f[x+v(t-\tau)] \frac{\partial F(\tau)}{\partial \tau}\, d\tau \qquad (2)$$

= voltage of the free traveling wave

$$V = \alpha\, g\, h\,(\omega),\ \therefore = \frac{V}{g\, h\,(\omega)} \qquad (3)$$

$$V' = \alpha' g h(\omega),\ \therefore = \frac{V'}{G\, h\,(\omega)} \qquad (4)$$

$$I = Q \frac{\partial F(t)}{\partial t} = \text{Cloud discharge current} \qquad (5)$$

Q = Initial charge of cloud.

By (5) and (1)

$$e = \frac{(\omega)}{2Q}\int_0^t \{ f[x+v(t-\tau)] + f[x-v(t-\tau)] \}\, I(\tau)\, d\tau \qquad (6)$$

Front of free wave $\simeq L \simeq 2H$

Total length of free wave $\simeq (L + T)$

$$L = \frac{\int_{-\infty}^{+\infty} f(x)\, .\, dx}{f(0)} = \text{length of equivalent rectangular bound charge} \qquad (7)$$

Direct Strokes

Stroke at tower	Voltages	Stroke at mid-span
$\frac{2Rz_{11}E}{(z_{11}+2z_o)R+z_{11}z_o}$	$V_{g.w.}$ (tower)....	$\left(\frac{2R}{2R+z_{11}}\right)\left(\frac{2z_{11}}{z_{11}+2z_o}\right)E$
"	$V_{g.w.}$ (mid-span)...	$\left(\frac{2z_{11}}{z_{11}+2z_o}\right)E$
$\frac{2Rz_{1r}E}{(z_{11}+2z_o)R+z_{11}z_o}$	V_{line} (tower)......	$\left(\frac{2R}{(2R+z_{11})}\right)\left(\frac{z_{1r}}{z_{11}+2z_o}\right)E$
"	V_{line} (mid-span)...	$\left(\frac{z_{1r}}{z_{11}+2z_o}\right)E$
$\frac{2R(z_{11}-z_{1r})E}{(z_{11}+2z_o)R+z_{11}z_o}$	$V_{insul.}$ (tower)....	$\left(\frac{2R}{2R+z_{11}}\right)\left(\frac{z_{11}-z_{1r}}{z_{11}+2z_o}\right)E$
"	$V_{insul.}$ (mid-span)..	$\left(\frac{z_{11}-z_{1r}}{z_{11}+2z_o}\right)E$

z_{11} = Equivalent self surge impedance of all ground wires.
z_{1r} = Equivalent mutual surge impedance of all ground wires to any line wire r.
z_o = Surge impedance of lightning bolt.
R = Tower footing resistance.
E = Voltage of incident wave from lightning stroke.

In terms of the current in the tower the voltages are (for a strike at the tower)

$$V_{g.w} = V_{tower} = RI$$

$$V_{line} = RI\left(\frac{z_{1r}}{z_{11}}\right)$$

$$V_{insul.} = RI\left(\frac{z_{11}-z_{1r}}{z_{11}}\right)$$

Ordinarily $z_{1r}/z_{11} = 0.20$ so that
$V_{insul.} = 0.80\,RI$

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Lightning Investigation on 220-Kv. System of the Pennsylvania Power & Light Co., N. N. Smeloff and A. L. Price, A. I. E. E. TRANS., Vol. 49, p. 895, July 1930.

Discussion

For discussion of this paper see page 1146.

Lightning Discharges and Line Protective Measures

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Synopsis.—This paper describes the mechanics of the development of a lightning stroke in a cloud and the formation of a surge on a line. It shows that the energy of a stroke can vary between wide limits so that the actual surge on the line may be of practically any magnitude.

Fundamental types of high-tension line construction are analyzed to show the performance that would be obtained with them.

Experience obtained on the 220-kv. lines of the Public Service Electric and Gas Company is offered to substantiate the belief that induced surges are of relatively small importance and that the direct stroke is the criterion for good design.

The effects of line construction on the character of surges impressed on the terminal substation apparatus is discussed to indicate the degree of protection that would be required.

An understanding of the formation of lightning surges on transmission lines is essential to the study of the problem of protecting lines and substations against these surges. In the study of lightning protection, the magnitude and wave form of the surges formed on the line are of fundamental importance; attenuation also becomes important when considering the surges to which a substation may be subjected during any thunderstorm.

Complete data on the limiting value of lightning stroke potentials are not available, but from known data the approximate values of potentials that may be encountered may be estimated. The highest surge yet recorded on any transmission line occurred during the past season on a 110-kv. wood pole line in Arkansas. This surge was formed on the line as the result of a stroke of lightning which splintered the pole from top to bottom; a side flash from the transmission line jumped 25 ft. of air to a neighboring telephone line. The cathode-ray oscillograph, located four miles from the point at which the lightning struck the line, gave an oscillogram of a surge which reached its maximum of 5,000 kv. in one microsecond. From known data it has been estimated that, at the point struck, the potential due to the lightning stroke must have reached a value of approximately 15,000 kv. before flashover took place from line to ground over the wood pole. From laboratory data on creosoted wood poles and needle gaps it is estimated that the rate of rise of the potential due to the lightning stroke before flashover took place must have been between 10,000 kv. and 20,000 kv. per microsecond.

Mechanism of Lightning Strokes

To understand the factors that have a bearing on the limitation of the potential of the lightning stroke when it hits a line or, what amounts to the same thing, the maximum limit of the lightning stroke current, one must consider the nature of the thundercloud itself and the manner in which electric energy is stored in it. The thundercloud consists of small particles of water-vapor each carrying a minute charge of electricity. If a free negative charge is present it attaches itself to a particle of vapor and thereby loses mobility. A thundercloud retains its charge by reason of the fact that the particles of water-vapor repel each other and, since the size of the particles is maintained constant, remain suspended in the air by the action of upward components of the air currents.

Since there are very few free ions in the cloud atmosphere and since the mobility of the charged moisture particles is very small, the only way such a volume can be discharged is by ionization. This process in a cloud atmosphere may be extremely slow. During the formation of the lightning streamer very little current is drawn from the cloud, and therefore the volume of cloud atmosphere required to be ionized is small. As the head of the streamer progresses by ionizing the air space in front of it, a space charge of requisite value must be built up and maintained. This ionization of air requires energy and time. Since the energy must be obtained from the cloud itself, the rate at which the point of the streamer will travel will be governed by the rate at which cloud ionization can release energy.

Even if it were assumed that ionization streamers inside the cloud atmosphere could form with the velocity of light, it would require 10 microseconds to discharge completely a cloud volume one mile in diameter. However, the speed of formation of streamers is much slower than the velocity of light, probably being not more than one-twentieth of that value as determined from time lag measurements of sphere-gap breakdown. To discharge completely a spherical volume one mile in diameter would therefore take at least 105 microseconds. If a reflection from the ground over the lightning channel is considered, 5 microseconds should be added to the above values.

Some streamers have been noted which never reach ground as the charge in the cloud is not sufficient to maintain and propagate the streamer. It may therefore be inferred that lightning strokes to earth or to a

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structure will vary their severity over a wide range depending upon the amount of energy the cloud is capable of discharging through the lightning channel.

The energy in the streamer itself, until it reaches ground, a transmission line or any structure, is mainly potential energy because the motion of the charges in the channel is relatively slow as compared to the velocity of light and because the currents required to produce ionization are relatively small. When the earth or a structure is reached by the streamer its potential energy is instantly changed to kinetic energy and a current wave moving at the velocity of light flows into the earth or the structure. Negative reflections pass up the lightning channel, increasing the discharge current in the channel as they proceed, until they reach the cloud where they increase the gradient in the cloud atmosphere, causing the ionization streamer to penetrate further into the cloud volume and thus tapping more of the cloud energy to supply current to the channel. As the volume of ionization extends within the body of the cloud, the rate of discharge gradually decreases until the point is reached where the available energy is insufficient to stabilize the channel from cloud to earth and the current ceases to flow.

Formation of Line Surges Due to Lightning Strokes

Formation of line surges due to lightning strokes should be analyzed in accordance with the type of transmission construction used of which there are two general classes. In the first general class are included the wood pole lines and steel tower lines not equipped with ground wires. The second consists of steel tower lines equipped with ground wires. Three cases occur in this class; first, where inadequate ground wire protection is provided and the lightning stroke is allowed to strike the conductors; second, where the design of ground wires is adequate to prevent the contact of the stroke with any conductor but the conductor insulation is inadequate to prevent flashover either at the tower or mid-span; and third, where all strokes are terminated on the ground wires and the insulation of the conductors is adequate to prevent flashover.

Considering first surges occurring on transmission lines unprotected by ground wires, when lightning strikes a wood pole line unprovided with a spill-over gap at that point, the potential attained by the surge before the wood structure breaks down may be very high. Such a lightning stroke usually involves all conductors before the failure of the wood pole unless the clearances between wires are extremely large. When the structure flashes over, the potential is reduced quickly to a value which depends upon the surge impedance of the lines and of the wood pole and ground, considering the wood pole as a conductor to earth. The surge impedance of the wood pole and earth may range between 10 and 200 ohms. Assume a value of 50 ohms and a lightning stroke that will have a potential of such a value that, when it terminates on a conductor of the same surge impedance as the lightning channel, it will deliver a surge of 15,000 kv. crest and will rise at a rate of 15,000 kv. per microsecond. If the surge impedance of a lightning channel is 200 ohms the calculations for potential on the line are as follows:

$$\frac{V_a}{V_o} = \frac{2\,Y_o}{Y_o + 2\,Y_a + Y_c} \tag{I}$$

Potential at point of stroke $= V_a$
Dynamic lightning potential $= V_o$
Surge admittance of lightning channel $= Y_o$
Surge admittance of conductors involved $= Y_a$
Surge admittance of pole and ground $= Y_c$

$Y_o = 0.005$ mhos
$Y_a = 0.002$ mhos for single conductor
$= 0.004$ mhos for three conductors
$Y_c = 0.02$ mhos

Voltage reduction factor for stroke to three conductors:

$$\frac{V_a}{V_o} = \frac{0.01}{0.005 + 0.008} = 0.77$$

Voltage reduction factor for stroke to three conductors and ground:

$$\frac{V_a}{V_o} = \frac{0.01}{0.005 + 0.008 + 0.02} = 0.303$$

Steepness of surge on the line (0.77 × 15,000 kv.) is 11,550 kv. per microsecond.

If the line under consideration has physical characteristics similar to the Arkansas line,[6] flashover may be assumed to take place when the potential rises to 10,000 kv. or in 0.87 microseconds.

Potential of the conductor immediately after the pole flashes over will be

$$\frac{10{,}000 \times 0.303}{0.77} = 3{,}940 \text{ kv.}$$

This potential at the end of one microsecond will be

$$15{,}000 \times 0.303 = 4{,}550 \text{ kv.}$$

Fig. 1 shows the approximate shape and value of the surge that will be impressed on the line due to the lightning stroke and the flashover of the wood pole. When the surge reaches the next structure, flashover may take place there also but at a greater time lag, reducing the tail of the surge to a much lower value. Surges on the line resulting from lightning strokes of lower values are also shown in Fig. 2. It should be noticed that the lower the potential the longer will be the unchopped portion of the surge. This always will be true if the lightning stroke has plenty of energy and therefore, the values represented by the longer surges shown in the curves are the most severe types of waves due to lightning which may be encountered in a line of this type.

In the case of steel towers without ground wires the surge potential of the conductor after flashover of the

6. For references see Bibliography.

insulator will be that of the tower. One insulator string only may be flashed over if the surge impedance of the tower is low, since the surge traveling along the line that is struck induces a potential of the same sign on the other conductors thereby reducing the difference of potential between them and the tower. In horizontal

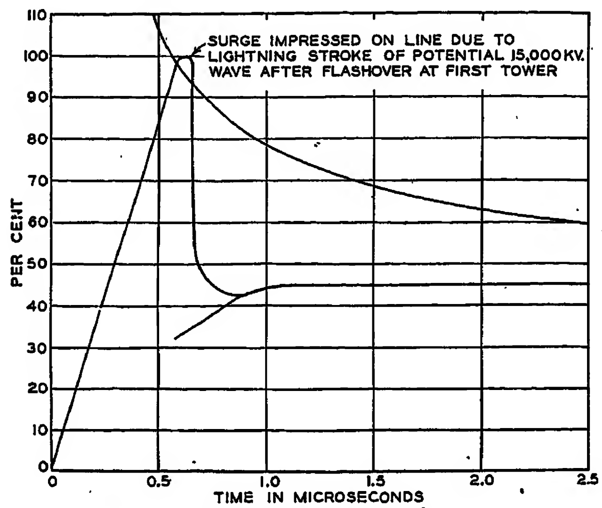

FIG. 1—FORMATION OF SURGE DUE TO LIGHTNING STROKE ON WOOD POLE LINE

construction, if an outside wire is struck, the other outside wire has the next higher probability of flashing over and if it should flash over the middle wire will generally be well protected from flashover. The likelihood of another flashover taking place at adjacent towers is

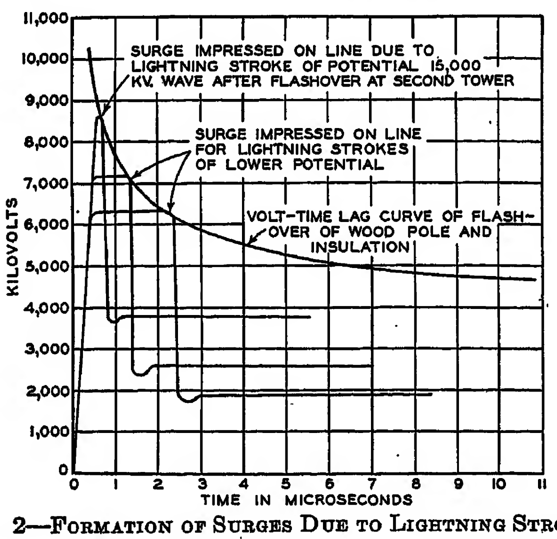

FIG. 2—FORMATION OF SURGES DUE TO LIGHTNING STROKE ON WOOD POLE LINE

dependent upon the intensity of the stroke and the impedance of the tower where flashover first occurred.

Considering the case of a steel tower line protected by ground wires, the primary function of the ground wire is to intercept the stroke of lightning and conduct the current to a tower through which it dissipates itself into the earth. To accomplish this the correct relationships of ground and line wires for both horizontal and vertical construction are shown in Figs. 3 and 4. For the horizontal configuration, the ground wire should be so placed that a line drawn through it and the outside conductor will form an angle with the vertical which is no greater than 20 deg. With angles greater than this, side flashes to the outside conductors are very likely. For the vertical configuration, the ground wires on the double circuit towers should be placed vertically above

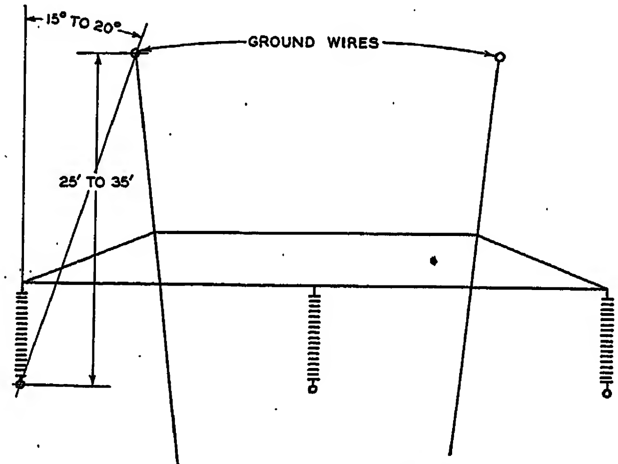

FIG. 3—HORIZONTAL CONDUCTOR CONFIGURATION SHOWING GROUND WIRE ARRANGEMENT

the outermost conductors. With both types of construction the spacing between the ground wire and conductor should be such that a stroke to the ground wire at mid-span will not cause a flashover between the ground wire and the conductor. Theory and limited practical experience indicate that this spacing for 1,000-ft. spans should be approximately 40 ft.

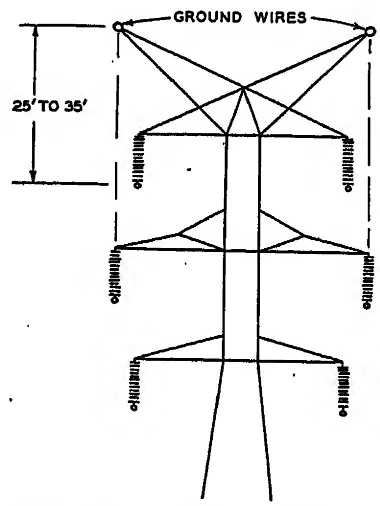

FIG. 4—DOUBLE-CIRCUIT TOWER SHOWING GROUND WIRE ARRANGEMENT

To evaluate the surge that is impressed on a line inadequately protected with ground wires and sufficiently insulated against tower potentials only, first consider the potential set up on the conductor when struck in mid-span before flashover takes place. The value of this potential V_a, may be obtained in the same manner as

described under the discussion of lines not equipped with ground wire. The insulator will flash over on the front of the wave at some value which is less than V_a, and the potential will be reduced to a value computed by formula (I) with the added admittance of the ground wires included. After flashover of the insulator, part of the surge is propagated over the ground wires and is reflected from the adjacent towers reducing the potential of the tower and conductor involved in the flash-

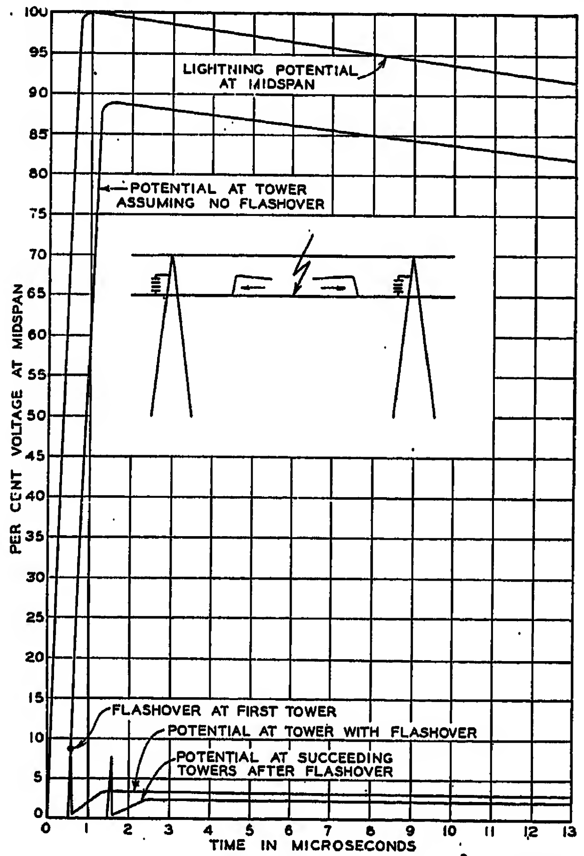

FIG. 5—POTENTIAL AT TOWER DUE TO LIGHTNING STROKE ON LINE CONDUCTOR AT MID-SPAN

over. Meanwhile the rest of the surge is carried away by the conductor and tower. An analysis of the form indicated above was made but with the stroke terminating on the conductor in the middle of the span. The results follow, being presented in simple numerical values and illustrative curves.

Assume a 15,000-kv. wave to be impressed upon the conductor in the middle of the span (Fig. 5). The ground wire will at the same time assume a similar potential wave with a magnitude determined by the coupling which in this case is 0.295. The surge will travel along the line until it strikes the tower. The front part of the wave which is below flashover will travel along the line, but with a slight reduction. The current drawn by the ground wire through the tower reduces the potential of the ground wire and allows a charge to form between the ground wire and the conductor. This reduces the potential of the surge traveling beyond the tower to 89 per cent of its original value which is not reduced at succeeding towers except by attenuation.

Assuming that flashover takes place when the voltage rises to 1,000 kv. so that the potential of the tower, ground wires and conductor now becomes the same, the rest of the wave will divide as follows:

A part will travel beyond the tower on the conductor, a part will travel in both directions on the ground wire, a part will travel over the tower into ground, and the remainder will be reflected over its original path. Thus the magnitude of the surge traveling over the ground wire is the same as that on the conductor following the flashover. At the succeeding tower, however, the potential of the ground wires is reduced to a negligible value. This change, through coupling with the conductor reduces the conductor potential for the section of the wave following flashover, to 63 per cent of its original value. Thus if the potential of the tower where flashover occurred rises to 159 per cent of the flashover

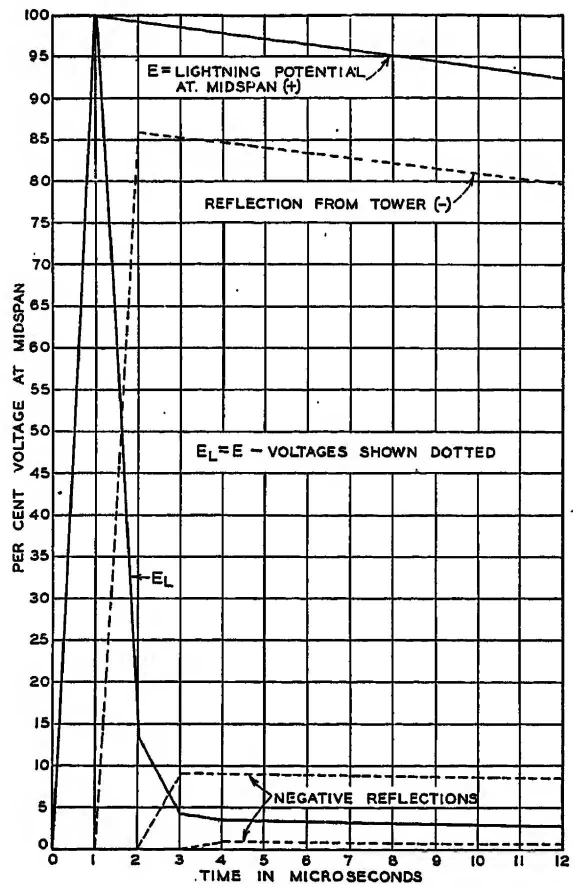

FIG. 6—LINE WIRE POTENTIAL AT MID-SPAN DUE TO STROKE ON LINE WIRE AT MID-SPAN

value of the insulator string, the insulators will flash over at only one of the succeeding towers.

Flashover at the first towers will set up reflections over the conductor and cause waves to travel back and forth over the ground wires. The potentials developed at various points are given in the following curves. Fig. 6 shows the potential (E_L) of the conductor at mid-span. This wave is a resultant of the lightning

potential impressed by the lightning stroke and the subsequent reflections from the towers. The reflections shown with dotted lines are of opposite polarity, thus subtracting from the impressed wave. The ground

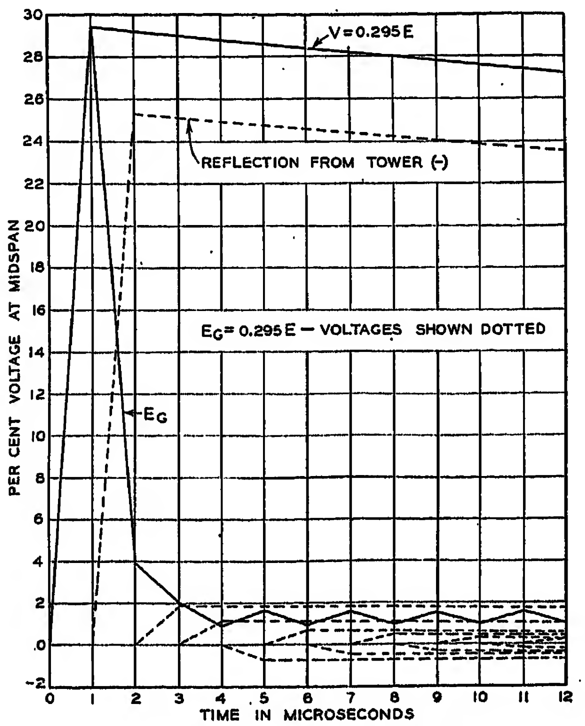

FIG. 7—GROUND WIRE POTENTIAL AT MID-SPAN DUE TO STROKE ON LINE WIRE AT MID-SPAN

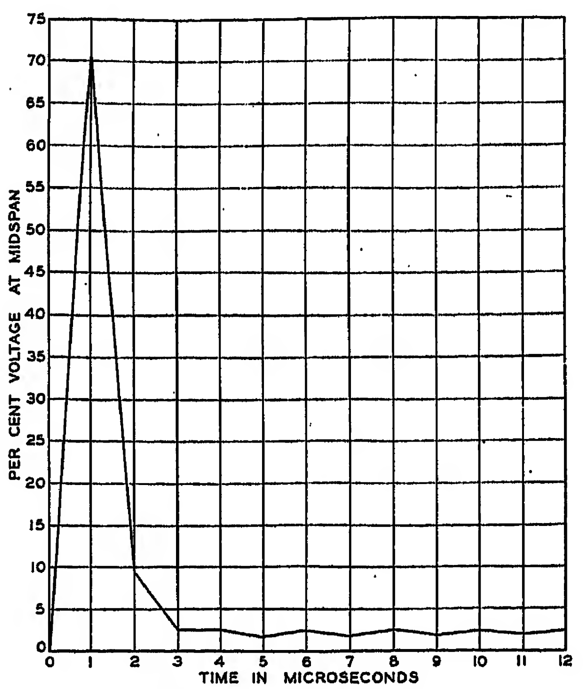

FIG. 8—VOLTAGE BETWEEN LINE WIRE AND GROUND WIRE DUE TO STROKE ON LINE WIRE AT MID-SPAN

wire potential at mid-span due to the surge on the conductor is as illustrated in Fig. 7. The first part of the voltage wave on the ground wire is generated by virtue of its coupling with the conductor. Traveling waves modify the latter part of the wave as shown by dotted lines. Fig. 8 shows the voltage between the ground wire and conductor at mid-span and is derived by subtracting E_G from E_L. It is quite obvious that high voltages exist at mid-span requiring a large separation between ground wire and line conductor to prevent flashover at that point.

The potential to which the tower will rise when flashover takes place is shown in Fig. 9. Incidentally this is the potential of the ground wire, tower and conductor at that point. The tower footing impedance was assumed to be 10 ohms. Fig. 10 shows the tower potentials but in this case the tower footing resistance was assumed to be 200 ohms. A comparison of Figs. 9

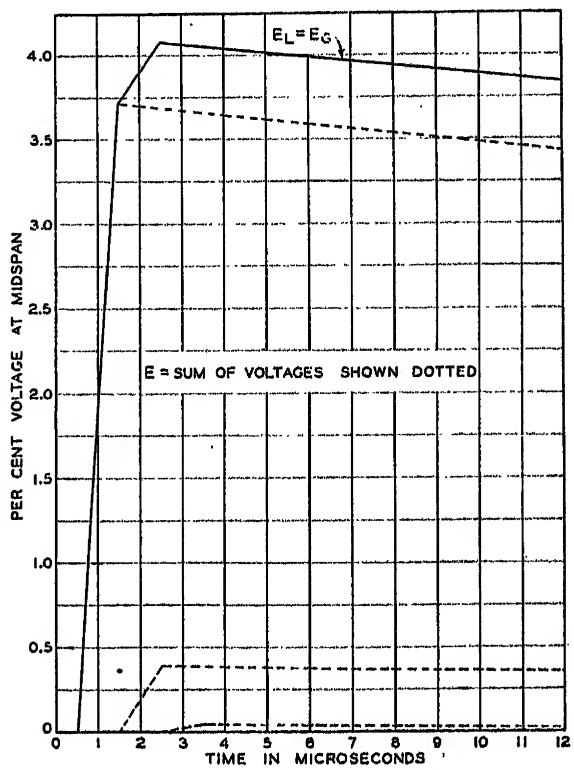

FIG. 9—POTENTIAL AT TOWER OF 10 OHMS FOOTING RESISTANCE DUE TO STROKE ON LINE WIRE AT MID-SPAN

and 10 illustrates the value of low tower footing resistance.

The case of a line properly protected with ground wires but with insufficient insulation is to be considered next. Inadequate insulation may be considered to exist at either of two points, at the tower or at mid-span. When the lightning terminates on the ground wires at mid-span the ground wires will be raised to a potential which may be computed by formula (I). This surge will travel to the adjacent towers where part of it will be absorbed by the tower, a part will continue on the ground wire, and a reflection will take place. With inadequate insulation at the tower, flashovers may take place at the adjacent towers on one or more conductors so that towers, ground wires, and affected conductors will assume the same potential. After

flashover occurs, reflections on the ground wires similar to those occurring in the case where lightning strikes conductors inadequately protected by ground wires, will take place from these towers. When the flashover takes place at mid-span from the ground wire to one of the conductors, the surge will travel over the ground wire and conductor to adjacent towers. A part of the surge on the ground wire will be absorbed by the tower, a part will continue on the ground wire and a reflection will take place at the tower as before. As in the case in which the conductor inadequately protected by ground wires is struck, the potential on the conductor will be of sufficient magnitude to cause flashover at adjacent towers. Because of the shorter arc path at the tower the 60-cycle arc is more apt to be maintained at the tower than in mid-span, flashover at the tower therefore being more easily identified than at mid-span.

When the stroke terminates on the ground wire in the middle of the span, the voltage on the ground wire is given in Fig. 11, while the conductor potential is shown

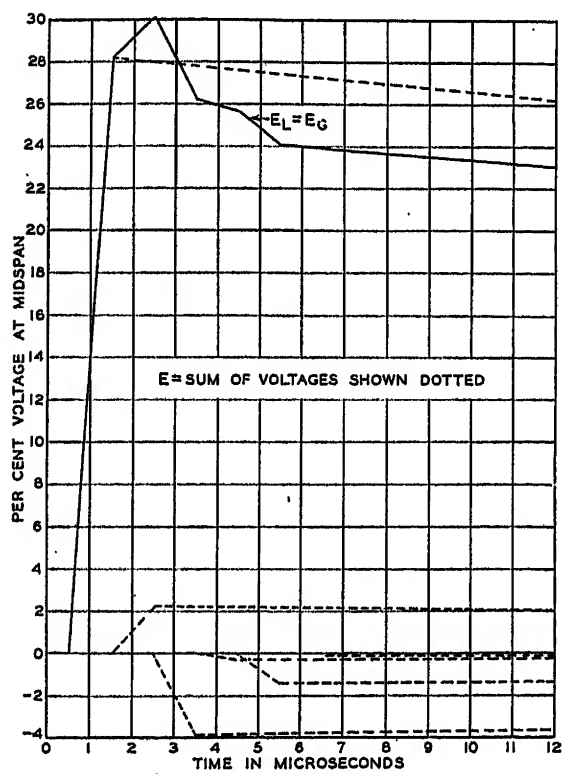

Fig. 10—Potential at Tower of 200 Ohms Footing Resistance Due to Stroke on Line Wire at Mid-Span

in Fig. 12. The difference in potential between ground wire and conductor is shown in Fig. 13. The tower potential will be of the nature shown in Fig. 14. The induced portion of the voltage on the conductor will vanish beyond the tower structures where the ground wire potentials become negligible. A flashover at the first tower impresses upon the conductor a traveling wave of the nature shown in Fig. 14. This wave will be reduced to only 63 per cent of the tower voltage beyond the afflicted section, and as such, subject to attenuation, will continue along the conductor until it finally enters a substation.

Should the spacing between the ground wire and conductor be insufficient, a flashover will take place at mid-span impressing upon the conductor the same potential as the ground wire. A voltage wave of the nature shown in Fig. 11 will be transmitted by the conductor. This potential will exceed the potential of the tower by an amount dependent on the magnitude of the reflection of the surge on the ground wire, will

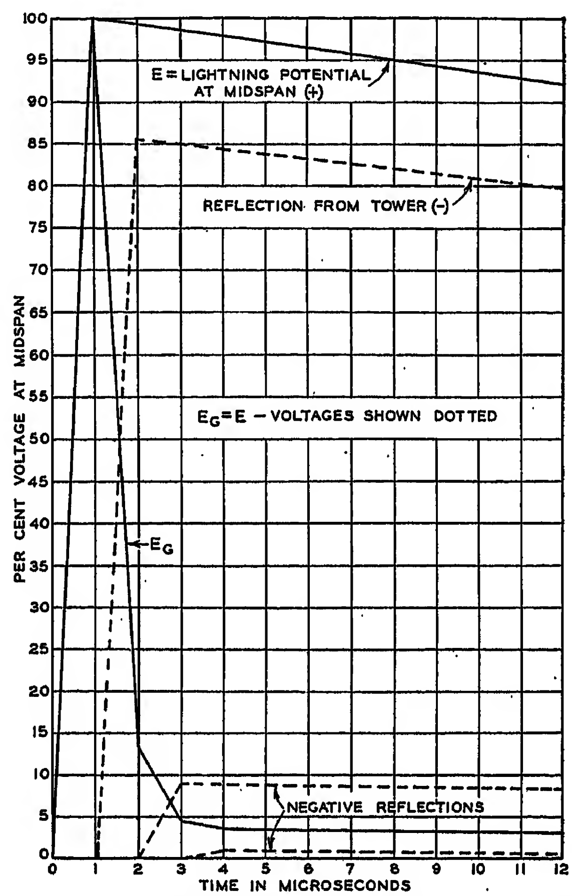

Fig. 11—Ground Wire Potential at Mid-Span Due to Stroke on Ground Wire at Mid-Span

flash over the insulator string, and will then be reduced to the potential of the tower. The resultant wave propagated beyond the region of disturbance will be very similar to that shown in Fig. 5.

The final consideration is that of a well-protected and well-insulated line. In this case the potential of the stroke to a tower, or to the ground wire in mid-span is quickly reduced by reflection from the adjacent towers and becomes of very small magnitude three spans distant from the point hit. The potential assumed by the tower when struck is shown in Fig. 15, while the potential of the adjacent towers is shown in Fig. 16. In these figures the voltage across the insulator string is shown also. Fig. 14 indicates that the potential of the towers with 10 ohms ground resistance is 4 per cent of the voltage at mid-span, the point struck.

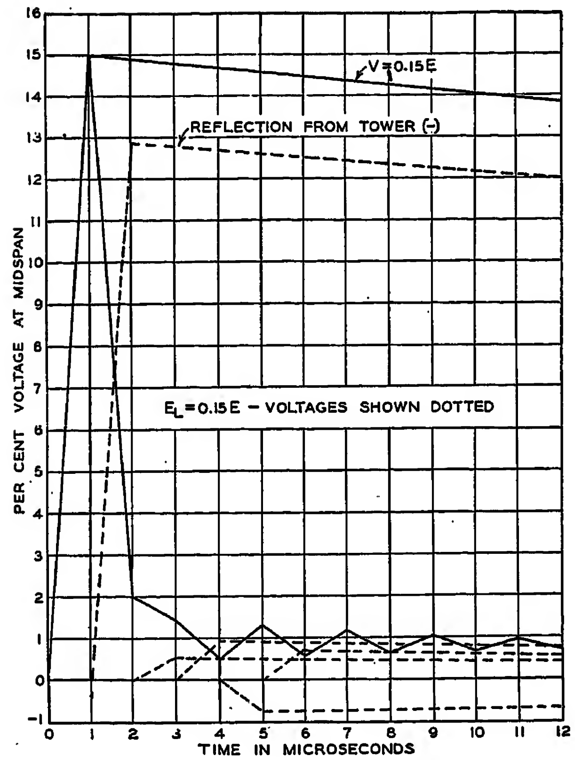

FIG. 12—LINE WIRE POTENTIAL AT MID-SPAN DUE TO STROKE ON GROUND WIRE AT MID-SPAN

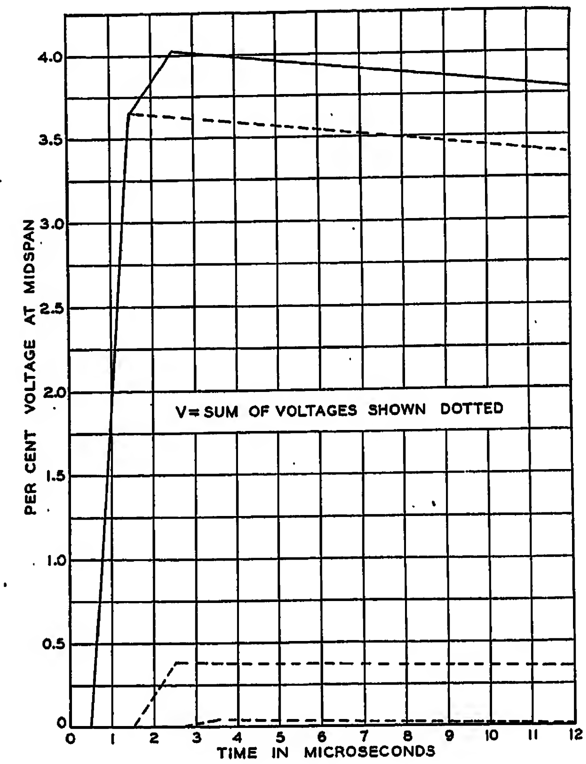

FIG. 14—POTENTIAL AT TOWER DUE TO STROKE ON GROUND WIRE AT MID-SPAN

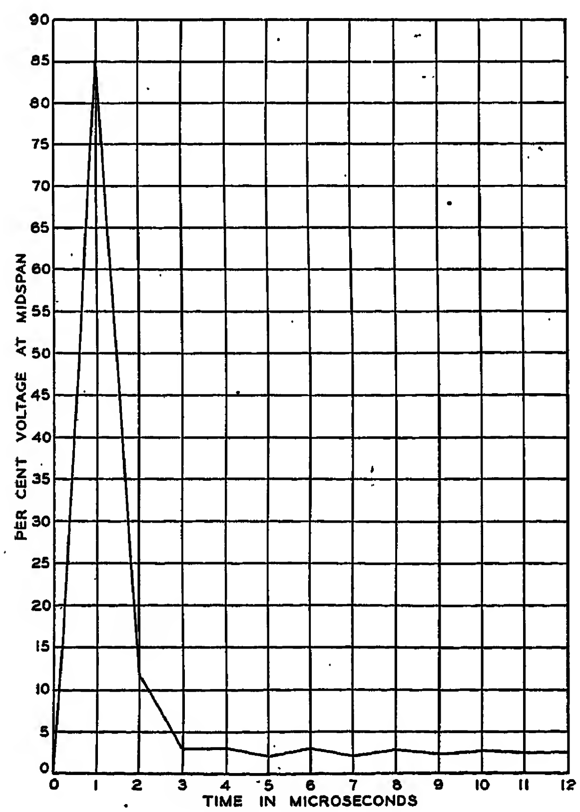

FIG. 13—VOLTAGE BETWEEN LINE WIRE AND GROUND WIRE DUE TO STROKE ON GROUND WIRE AT MID-SPAN

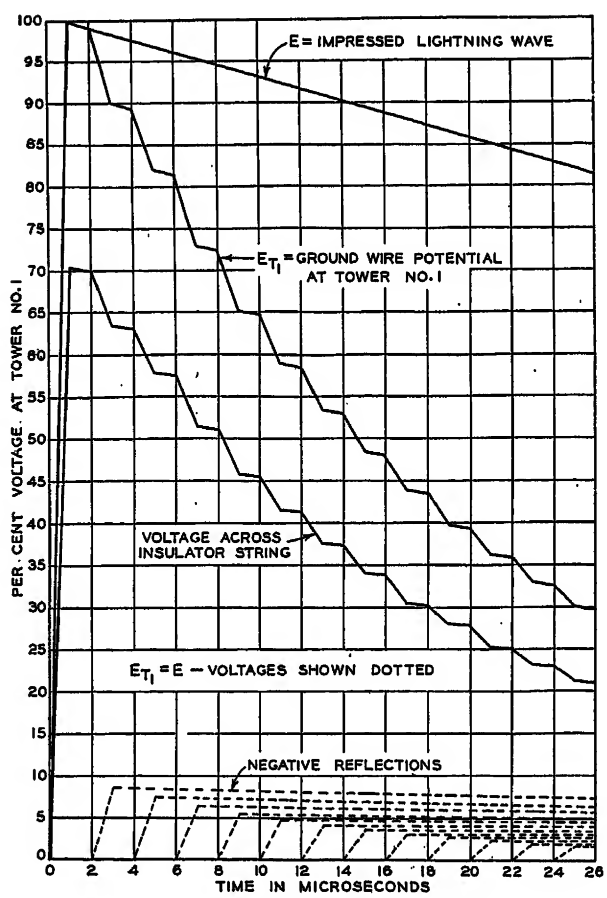

FIG. 15—POTENTIALS AT TOWER 1 DUE TO STROKE ON TOWER 1

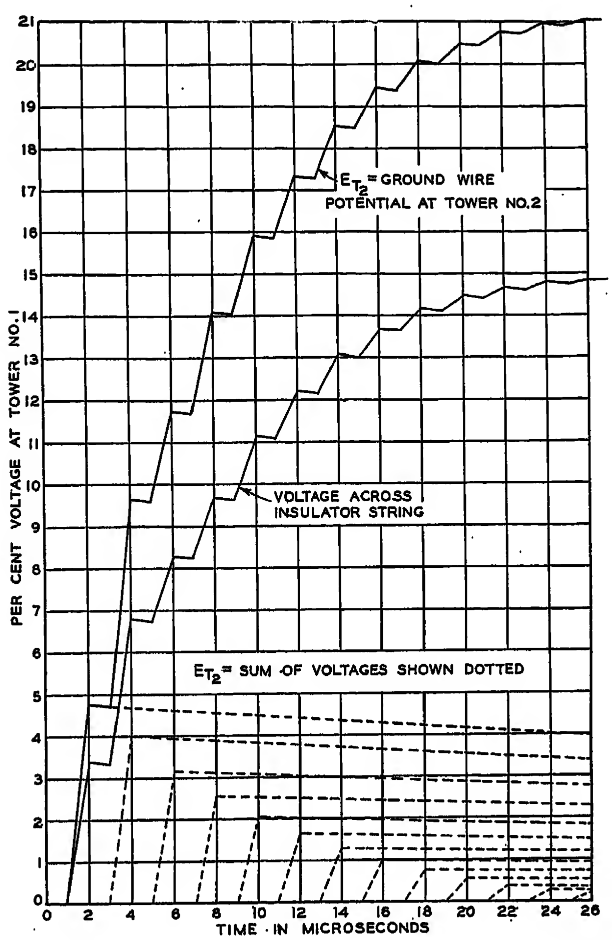

FIG. 16—POTENTIALS AT TOWER 2 DUE TO STROKE ON TOWER 1

SUMMARY OF 1930 LIGHTNING INVESTIGATIONS

The results herein reported were obtained on the Public Service Electric & Gas Company's 220-kv. lines during the lightning season of 1930.[1] During these investigations a portable cathode-ray oscillograph was located at the Roseland Switching Station, Roseland, New Jersey. The 220-kv. lines in use consisted of a six-mile section of the Roseland-Plymouth Meeting line and a six-mile section of the Siegfried-Roseland line. Surge tests with a portable million-volt lightning generator supplemented the lightning work. Phases 1 and 3 of the Siegfried-Roseland line were jumpered together at Tower 34. The method by which these line sections were connected is shown schematically in Fig. 17.

The 220-kv. lines were constructed with 16-unit standard suspension insulators, with arcing rings, and 18-unit strings in tension at the dead-end towers. Both sections of the line were dead during the investigation, construction work being not yet completed. Two ground wires were in place.

LIGHTNING CURRENT MEASUREMENTS

The installation of klydonographs for recording currents in towers was completed on June 26. These instruments were installed directly in parallel with the tower footings without voltage dividers, by locating the instruments on the tower a few feet from the ground and running light copper wire to an auxiliary ground 50 to 75 ft. from the tower base. A few weeks later the klydonographs were reconnected in order to utilize a

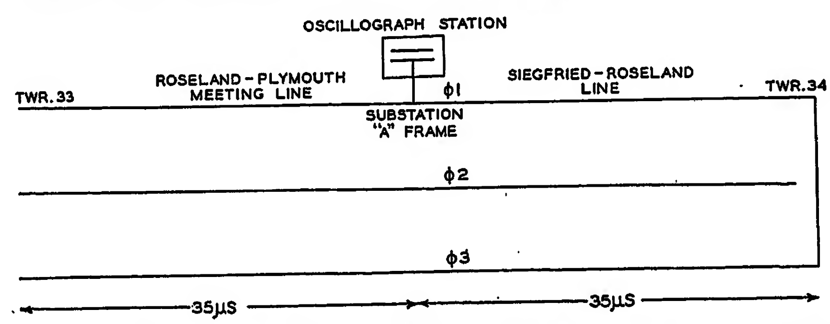

FIG. 17—SCHEMATIC DIAGRAM SHOWING METHOD USED FOR SURGE INVESTIGATIONS ON THE PUBLIC SERVICE ELECTRIC AND GAS COMPANY'S 220-KV. LINE

If the lightning voltage at mid-span is assumed to be 15,000 kv. the tower potential will be 600 kv. If the effect of the inductance of the tower is taken into consideration, this voltage may rise during the first microsecond to 1,200 kv. while the voltage across the insulator string would be 840 kv., a value insufficient to cause flashover on a 16-unit insulator string such as is used on 220-kv. transmission lines.

It has been pointed out in previous paragraphs that if no flashover takes place at mid-span or at the towers, the potentials induced by coupling on the conductors will become nil beyond the region of the disturbance. Thus no surge will be propagated over the line to terminal apparatus and the line will continue to operate unaffected by the lightning stroke.

resistance voltage divider. The voltage divider consisted of a high-resistance wire running from the tower to the auxiliary ground, the terminals of the klydonograph being connected across a small section of this resistance wire. This is shown schematically in Fig. 18. By considering the tower ground resistances, ratios were chosen to record 75,000 amperes ground current in the towers for positive surges. For negative figures this value was considerably greater.

Fig. 19 is an oscillogram taken during the course of a general storm over both sections of the line. The surge is positive and reaches a crest value of 30 kv. A second wave occurs at an interval of 55 microseconds. The second wave, which is a reflection of the first, shows that the surge probably originated on the Roseland-

Plymouth Meeting section about 7,000 ft. from the station. No klydonographs had been placed on this section of the line because the 220-kv. towers were overtopped throughout the entire distance by a parallel line of 132-kv. towers.

Fig. 20 is an oscillogram of a surge obtained during a storm on July 1 and shows a positive surge of 35-kv. crest value. A heavy stroke to ground was observed just east of Tower 1 on the Siegfried-Roseland section

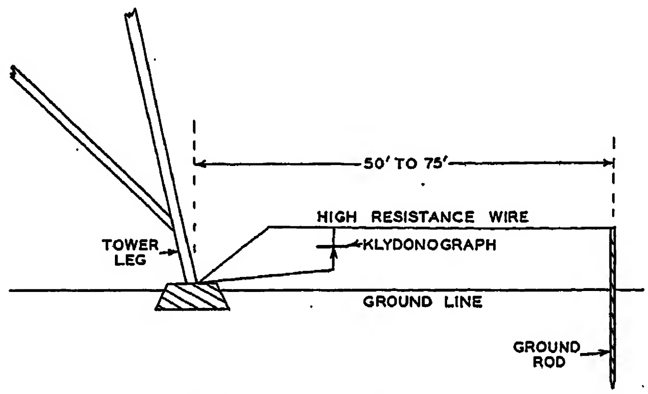

Fig. 18—Schematic Diagram of a Klydonograph Installation

of the line at the time the instrument tripped. Thunder and lightning seemed almost simultaneous, the interval being not greater than one second. Observers placed the stroke within 1,000 ft. from the station and not more than 300 ft. from the line. A second wave is noted on the oscillogram after an interval of 65 microseconds, approximately the time required for the wave to reflect from the open end of the line and return to the

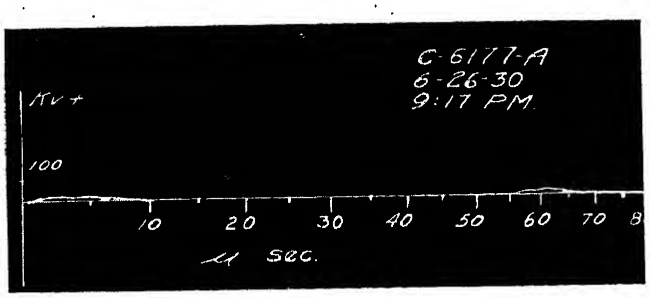

Fig. 19—Oscillogram of Lightning Surge on the Public Service Electric & Gas Company's 220-Kv. Line

station. Further confirmation of this theory is obtained by comparing the time of the second wave in this oscillogram with that of the oscillogram shown in Fig. 21 taken on the same time scale. Data from observations indicate that this surge was induced and originated close to the station.

Fig. 21 is an oscillogram taken during the same storm and shows a positive surge of 65-kv. crest. Analysis of the oscillogram indicates that the disturbance occurred on the Siegfried-Roseland section near Tower 23. The klydonograph films were collected shortly after the storms and they recorded ground current in the towers from Tower 20 to Tower 27 inclusive. The data obtained from these klydonographs are given in Table I.

The oscillogram shown in Fig. 22 records a positive surge of 85-kv. crest. A stroke to ground was observed at the time the oscillograph tripped and was located by a direction finder between Towers 11 and 20 on the Siegfried-Roseland section. The time interval between

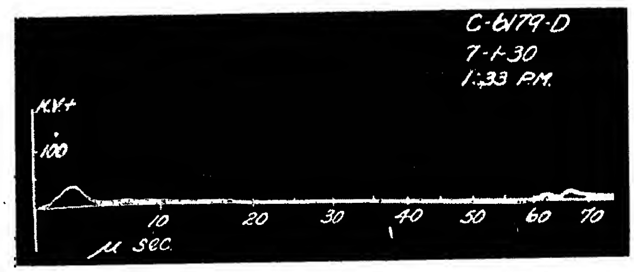

Fig. 20—Oscillogram of Lightning Surge on the Public Service Electric & Gas Company's 220-Kv. Line

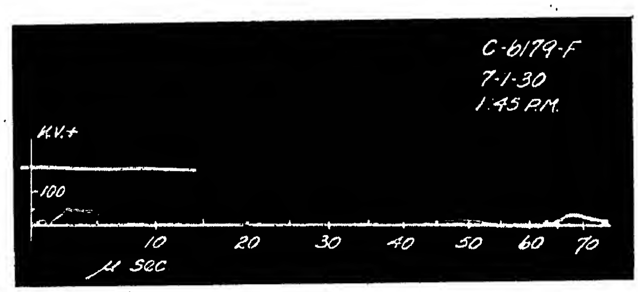

Fig. 21—Oscillogram of Lightning Surge on the Public Service Electric & Gas Company's 220-Kv. Line

Fig. 22—Oscillogram of Lightning Surge on the Public Service Electric & Gas Company's 220-Kv. Line

the lightning and thunder was about 15 seconds. In the oscillogram a second wave follows the first after an interval of 38 microseconds. By the aid of information given by the second and third reflections the disturbance was determined to have taken place about 14 microseconds from Roseland. Klydonograph films indicated that currents were present in Towers 15 and 16. Data from these films are given in Table II.

Another group of current records was obtained on Towers 4 to 9 inclusive. The surge resulting on the conductors was evidently so low in this instance that

TABLE I

Tower No.	Polarity	Radius of fig. (mm)	Instrument kv.	Instrument ratio	Meggered tower ground resistance	Tower amperes approximate
20	Pos.	10.0	7.2	1/1	1.2	6,000
21	Instrument broken			1/1	1.8	
22	Pos.	9.5	6.8	1/1	2.1	3,200
23	Pos.	3.5	3.4	1/1	2.4	1,400
24	Pos.	11.5	8.5	1/1	2.1	5,500
25	No. fig				2.7	
26	No. fig				5.0	
27	Pos.	13.0	9.7	1/1	20.0	500

TABLE II

Tower No.	Polarity	Radius of fig. (mm.)	Instrument kv.	Instrument ratio	Meggered tower ground resistance	Tower amperes approximate
14	No record		3 (max.)	1/7	5.1	4,000 (max.)
15	Pos.	8.0	5.8	1/7	4.2	9,700
16	Pos.	9.8	7.1	1/5	2.4	14,800
17	No record		3 (max.)	1/6	3.5	5,100 (max.)

TABLE III

Tower No.	Polarity	Radius of fig. (mm.)	Instrument kv.	Instrument ratio	Meggered tower ground resistance	Tower amperes approximate
4	Pos.	3.5	3.3	1/1	2.2	1,500
5	Pos.	12.0	8.8	1/1	2.5	3,500
6	Pos.	12.7	9.5	1/1	6.8	1,400
7	Pos.	Possible flashover	20.0(?)	1/1	7.5	2,700(?)
8	Pos.	13.0	9.7	1/1	13.1	750
9	Pos	5.0	4.0	1/1	3.2	1,200

it did not cause operation of the cathode-ray oscillograph. The tabulation of these current records is given in Table III.

On the basis of the induced surge theory, the currents indicated by the klydonograph measurements are not in agreement with the voltages correlated with these values and measured on the transmission line. Potentials of two or three million volts would have to appear on both the conductor and the ground wire if these currents were set up by induced voltages. The recorded voltages on the conductor were extremely low, showing quite conclusively that lightning struck the ground wire. The current disturbances on the tower and ground wire system were limited to a distance of approximately six spans due to low tower footing resistances. The potentials induced on the conductor arising from surges on the ground wires become nil beyond the region of the disturbance. Thus no surge on the conductor can be propagated beyond this region unless flashover takes place.

The voltages recorded on the oscillograph for these cases are probably due to the electrostatic disturbances resulting from the ionization of air during the formation of the lightning stroke. This ionization of the air draws energy from the transmission line, creating a positive impulse of short duration and fairly low magnitude.

To summarize, these data show that the voltages on the conductors with a stroke very close to the line or terminating on the tower or ground wire will be extremely low unless flashover takes place.

Effect of Line Construction on Potentials Impressed on Substations

The potentials appearing in a substation will be discussed with relation to the various forms of line construction: first, wood pole lines highly insulated against impulses but having no ground wires; second, lines with low impulse flashover such as steel towers having no ground wires; third, steel structures with inadequate ground wire protection; and fourth, a tower line having good ground wire protection and adequate insulation.

With wood pole structures using high insulation, and without ground wires, a stroke terminating on the conductor will impress upon it a surge of exceedingly high magnitude. This surge may travel for many miles and still be of a dangerously high potential. Such a surge entering a substation will be impressed across the substation insulation. Because of the lower insulation against impulses generally used in substations, flashover will occur within the substation causing a trip-out. To prevent such trip-outs or flashovers, exceptionally good lightning protection is required.

In the case of lines having low impulse insulation the conductor struck will flash over at the adjacent towers and the surge that is propagated over the conductor will be of a chopped nature. The tail of this wave, as previously described, will have a magnitude depending upon the tower footing resistance at the point where flashover took place. However, the wave, which will be triangular in form, will be of short duration and will attenuate very rapidly. With surges of this nature

entering into the substation, the stresses on the substation insulator will be fairly moderate and less lightning protection will be required to safeguard the substation equipment.

This analysis can be applied to the case of tower lines where inadequate ground wire protection is used, as the nature of the surge propagated over the conductor is very similar to the preceding case with the exception that in this case the tail of the wave will be of much shorter duration.

The final case to be considered is that in which adequate insulation and good ground wire protection are used on the steel tower line. It has been shown that under these conditions only surges of very low magnitude will be propagated over the transmission line and thus substations will not be subjected to dangerous surges. In the event of a stroke very close to the substation, thus involving the substation in the disturbed region, the voltages on the conductor may be moderately high. However, the station lightning arresters will limit the voltages appearing on the conductors to a value well below the substation insulation. Under this condition, the current carried by the lightning arrester is fairly low being only that required to charge up the conductor to about the potential of the substation ground.

SUMMARY

1. The charge in a thundercloud, being held by particles of moisture in suspension which are insulated from each other by the intervening air, can form a lightning stroke only if the air between these particles becomes conducting.

2. Energy is transferred from one section of a cloud to another through streamers. Laboratory experiments show their rate of development to be on the order of one-twentieth the velocity of light.

3. With the finite rates at which energy can be collected, a cloud volume one mile in diameter will require approximately 100 microseconds to discharge.

4. Flashover at mid-span will be followed by a flashover at the adjacent tower. Because of the shorter arc path at the tower, the 60-cycle arc is more apt to be maintained at the tower than in mid-span.

5. With the high rates of rise of potential measured on wires struck by lightning, large spacing between ground wires and conductors is required at mid-span to prevent flashover at that point.

6. The voltage to which a tower will rise when either it or the ground wire is struck is a function of the tower footing resistance.

7. When the tower or ground wire is struck, the voltage difference before flashover between the tower or ground wire and the conductor is equal to the potential of the tower or ground wire less the potential assumed by the conductor through mutual coupling with the ground wire.

8. When the ground wire or tower is struck and flashover takes place, a surge having the same potential as the tower is propagated over the conductor.

9. If a tower or ground wire is struck and no flashover results, severe electrical disturbance is limited to a few structures, and any surge propagated over the conductor will be of extremely low magnitude.

10. During one year's investigation on lines equipped with ground wires, no flashovers have occurred where tower footing resistances are below five ohms.

11. The ground wire protection adjacent to the station should be strengthened as much as possible to eliminate flashover and direct strokes to the conductor, thus preventing strokes close to the station from impressing dangerous surges upon the station equipment.

12. On well-protected lines, reduced insulation for structures adjacent to substations is not advisable as for strokes terminating at these points, the hazard of flashover is increased with the consequent greater possibility of transmitting a surge into the substation.

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Discussion

For discussion of this paper see page 1146.

Lightning Investigation
on the 220-Kv. System of the Pennsylvania Power & Light Company (1930)

BY EDGAR BELL* and A. L. PRICE†

Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—This paper describes the 1930 results (fifth consecutive year) of a lightning investigation conducted on 220-kv. lines located in eastern Pennsylvania and where lightning storms are prevalent. During 1930 the investigation was expanded; two new devices, the surge (insulator assembly flashover) indicator and the lightning (storm) severity meter being successfully applied.

Unique and comprehensive data were secured on the magnitudes and wave shapes of natural lightning surges both at and remote from the point of origin on the transmission line; numbers of phases faulted during trip-outs; numbers, magnitudes and effects of direct lightning strokes; some data on the relative importance of induced strokes; results of measurements of atmospheric voltage gradients; and the indicated effect of overhead ground wires and tower footing resistances.

General

The extensive investigation of lightning on the Pennsylvania Power and Light Company's 220-kv. system during 1930 is a continuation of research begun in 1926. Data obtained prior to 1930 have been presented in papers[1,2] before the A. I. E. E. in 1928 and 1930.

The field study included measurements of magnitude and wave shape of lightning surge voltages on 220-kv. transmission lines, location and current magnitudes of direct lightning strokes, atmospheric electric field intensities, induced voltage gradients, lightning storm severity, insulator flashovers (including flashovers which left no visible burns or marks on insulator assemblies), and careful and systematic collection of weather and operating data.

The results of the 1930 investigation have furnished a key to the mechanism of lightning influence on high-voltage (220-kv.) transmission lines, and the problems of evaluating the effects of overhead ground wires, tower footing resistance, and direct and induced strokes are in process of solution.

Line Data

The 220-kv. system of the Pennsylvania Power & Light Company consists of the Wallenpaupack Tap, the Siegfried-Delaware River section of the Siegfried-Roseland line, and the greater portion of the Plymouth-Siegfried line. The sections of line between Wallenpaupack and Siegfried (formerly known as the Wallenpaupack-Siegfried line) are now called the Wallenpaupack Tap-Siegfried Roseland line. Practically all measurements were made on this line, and unless otherwise specified, references in this paper will be to this line.

General characteristics of the above mentioned lines including a description of the so-called "counterpoise" installed along a 2½-mile section of line on which flashed insulators had been very numerous have been described[2] in a previous paper. These lines, together with present and future interconnections, are shown in Fig. 1.

During 1930 two radical changes were made to sections of the Wallenpaupack Tap. The tower footing resistances of all towers equipped with overhead ground wires (exclusive of the "counterpoise" section) were reduced by connection of tower footing grounding cables to the tower footings. Each cable consisted of a 50-ft. length of 00 stranded copper cable trenched in the ground to a depth of about a foot, and connected to one corner of the tower. Four cables extending radially

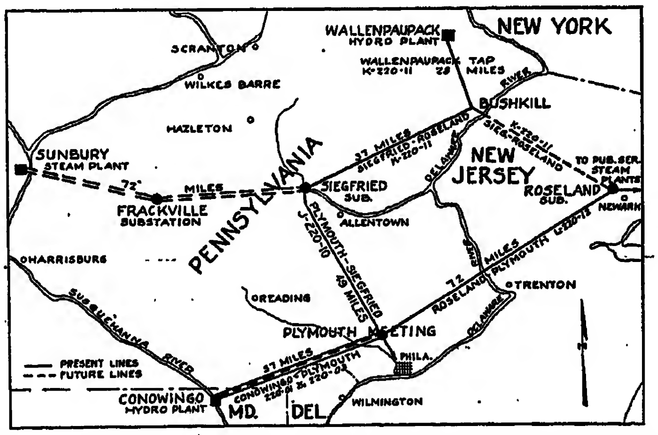

Fig. 1—Pennsylvania-New Jersey 220-Kv. Interconnection

outward were used at each of the 80 towers. The immediate effect of these tower footing grounding cables was to reduce tower footing resistances to about one-half their former values.

A 3½-mile section of this line outside the ground wire section, and which was particularly subject to insulator flashovers, was equipped with lightning stroke diverting cables,[3] or overhead ground wires of unusual design.

Facilities and Measuring Devices

The investigation was conducted and facilities were made available cooperatively by the Pennsylvania Power and Light Company and the General Electric

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1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

Company, the Electric Bond and Share Company acting as consultant for the former. The Public Service Electric and Gas Company of New Jersey and the Philadelphia Electric Company also cooperated by supplying lightning storm and weather data from their respective operating territories, the latter company also furnishing operating data from its 220-kv. lines which are interconnected with those of the Pennsylvania Power and Light Company. All five companies participated in analysis of data.

Measuring devices and facilities used are listed in Table I which also indicates trends in scope of the investigation by years. Locations of devices and facilities with respect to transmission lines are shown in Fig. 2.

TABLE I—MEASURING DEVICES AND FACILITIES

	Number of instruments or facilities				
	1926	1927	1928	1920	1930
A—Instruments used on 220-kv. lines					
Surge-voltage recorders	9	26	46	28	21
Lightning-stroke recorders	0	0	0	284	314
Surge (flashover) indicators	0	0	0	0	1,041
Cathode-ray oscillographs	0	0	1	1	2
Magnetic oscillograph elements or high-speed graphic ammeters	2	2	2	5	12 to 22
B—Other instruments (not coupled to 220-kv. lines)					
Surge-voltage recorders	0	0	6	12	13
Cathode-ray oscillographs	0	0	0	1	0
Field-intensity recorders	0	0	0	1	3
Rate of change of field recorder	0	0	0	1	0
Storm-severity meters	0	0	0	0	4
C—Special installations					
Antennas	0	0	5	10	10
Towers with counterpoise	0	0	0	14	14
Towers with tower footing grounding cables	0	0	0	0	80
Towers with lightning stroke diverting cables	0	0	0	0	18
Lightning arrester installations	0	0	0	0	2
D—Weather data					
Weather observations (operating stations)	7	26	27	142	138
E—Operating data					
Overhead (tower climbing) patrols	1	1	7	23	5
Operating records, from 220-kv. operating stations	2	2	2	3	3

The surge-voltage recorder,[4] the lightning stroke recorder,[5] the cathode-ray oscillograph[6] and the field intensity recorder[5] have been described in other papers before the Institute.

The surge (insulator assembly flashover) indicator[7] is a small device applied to each insulator assembly of a tower, and designed to cause a target, visible to a ground patrolman, to show upon occurrence of flashover of that assembly. This instrument was developed during the winter of 1929-30 by the General Engineering Laboratory of the General Electric Company. As used during 1930 towers having one (or more) targets showing were climbed, all insulator assemblies carefully inspected for marks of flashover and a new indicator link installed. Broken pieces of the old link were saved, and inspected by experienced engineers. At the end of the season a tower climbing inspection was made of the entire Wallenpaupack Tap-Siegfried Roseland line for the purpose of checking the performance of the surge indicators and to secure the best possible insulator assembly flashover data. Fig. 3 shows surge indicator installations on a typical steel tower.

The lightning (storm) severity meter[7] is a small device of simple construction, consisting of a roll film box camera containing a special glow tube. This tube is connected between a short vertical antenna (30 ft. long) and ground. Collapse of electric charges collecting on the antenna during lightning storms causes the tube to glow, and record a spot on the film. The intensity or

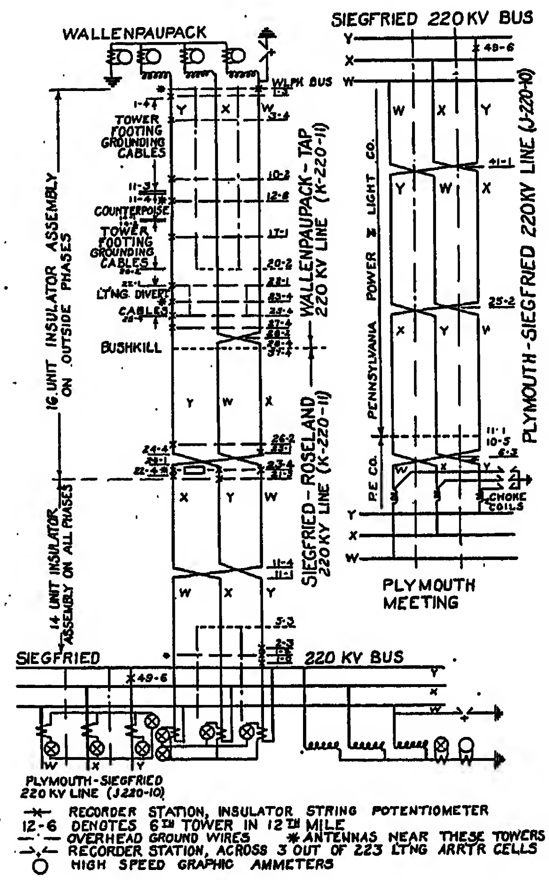


FIG. 2—DIAGRAM OF WALLENPAUPACK TAP-SIEGFRIED ROSELAND AND PLYMOUTH-SIEGFRIED LINES

Showing locations of recording devices and facilities

degree of darkness of this spot is an integrated function of the number of charges on the antenna and their magnitudes. As used during 1930, films were advanced by hand once a week to a new position, and all records secured were on a weekly basis. Installations were made in open fields as free as possible from trees, buildings, and electric power lines.

The lightning severity meter was developed by the General Engineering Laboratory of the General Electric Company.

During 1930 the Wallenpaupack Lightning Laboratory was abandoned, and a new one established near

tower 23-4 of the Siegfried Roseland line. Fig. 4 is an illustration of the new laboratory and its site. At this location, known as Cherry Valley, the line is not equipped with overhead ground wires, and flashed insulators had been particularly numerous. During the 1930 season a direct stroke of lightning contacted the line conductor at not more than 500 ft. from where the cathode-ray oscillographs were coupled. The oscillogram of this voltage surge is described later.

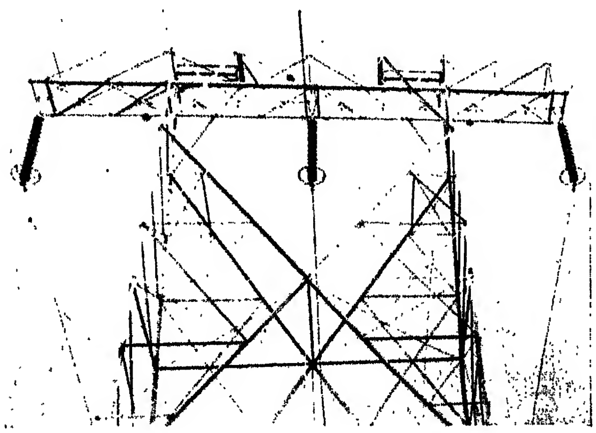


Fig. 3—Surge-Indicator Installation on Typical 220-Kv. Steel Tower

Two cathode-ray oscillographs were employed, both being coupled to the same line conductor. One instrument, called the "fast sweep" had a time axis of 50 microseconds duration; the other or "slow sweep" oscillograph had a time axis of 2,000 microseconds duration. The latter instrument was particularly useful in determining the approximate locations of the line faults associated with oscillograph records. This result was possible because of voltage or current reflections from faults, line terminals, and line junctions.

A transmission line voltage indicator located at the laboratory and controlled by the electrostatic field from the line conductors was utilized for exact and positive time correlation of oscillograms with line trip-outs.

Auxiliary oscillograph circuits[2] were substantially as used in 1929. Sensitivity of the trip circuit of the fast sweep oscillograph was increased by changes in circuit constants and by exposure of trip gaps to ultra-violet light. The slow-sweep oscillograph circuit was initiated by the cathode voltage of the fast-sweep oscillograph.

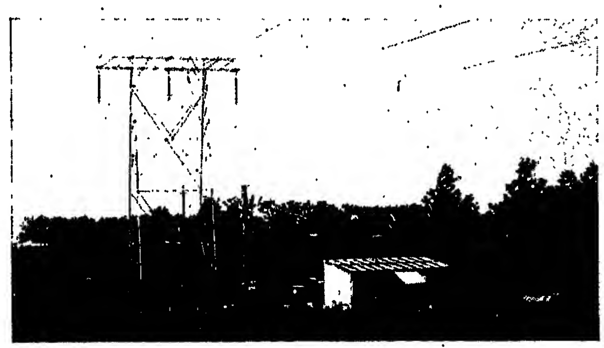


Fig. 4—Cherry Valley Lightning Laboratory

Tower *SR* 23-4 immediately behind laboratory

For over 90 per cent of line faults associated with

TABLE II—TRIP-OUT DATA—1930—WALLENPAUPACK TAP-SIEGFRIED ROSELAND LINE

Trip-out number	Phases faulted	Insulator assembly flashovers: No. of towers involved	Insulator assembly flashovers: No. of assemblies involved	Measured direct lightning strokes	Overhead ground wires installed	Location of fault (tower and phases where flashover occurred)	Cathode-ray oscillogram obtained
1	X, W	2	4	Instruments	No	SR16-2X, W; 16-3X, W	
2	W			not	?	?	
3	Y, X, W	2	4	installed	No	SR19-4X; 20-1Y, X, W	
4	W				?	?	
5	W			"	?	?	
6	W			"	?	?	
7	Y, W*			"	?	?	Yes
8	X	2	2	"	No	SR11-4X; 11-5X	Yes
9	Y			?	?	?	
10	Y			?	?	?	Yes
11	X, W	1	2	Yes	No	WT 22-4X, W	Yes
12	X	2	2	Yes	No	SR34-1X; 34-2X	Yes
(13)	W	1	1		Yes	Plymouth-Siegfried Line	No
14	Y	1	1	No	Yes	WT15-5Y	
15	W			?	?	?	Yes
16	W	2	2	Yes	No	SR11-1W; 11-2W	Yes
17	Y	2	2	Yes	No	SR31-2Y; 31-3Y	
18	W*			?	?	?	Yes
19	Y	2	2	Yes	No	SR8-4Y; 9-1Y	Yes
(20)	W	1	1		Yes	Plymouth-Siegfried Line	Yes
21	W	4	4	Yes	No	SR16-3W; 16-4W; 16-5W; 17-1W	
22†	X	2	2	No	No	SR23-3X; 23-4X	Yes
23	Y	2	2	Yes	No	SR24-5Y; 25-1Y	Yes
24	X	2	2	No	No	SR14-5X; 15-1X	Yes
25	Y	2	2	Yes	No	SR9-5Y; 10-1Y	Yes
26	W	3	3	Yes	No	SR13-5W; 13-6W; 14-1W	Yes

*No. of phases faulted not known with certainty. Possibly one other phase involved.
†Trip-out due to stroke near lightning laboratory. See oscillogram 902379.
Total number of lightning trip-outs 24 (excluding numbers 13 and 20).

trip-outs the number of phases involved was successfully determined by means of high-speed graphic recording ammeters at Wallenpaupack and magnetic oscillographs at Siegfried.

Results

Line Trip-outs. Of the 24 lightning trip-outs during 1930, the location of the fault, the number of phases and insulator assemblies involved, and whether or not a direct stroke exceeding approximately 50,000 amperes occurred, were successfully determined in fifteen cases. In the remaining nine cases, complete data are either lacking or correlation is uncertain. Table II presents summarized data for each trip-out.

The Plymouth-Siegfried line tripped twice, each case being due to a single-phase fault.

Insulator Assembly Flashovers. Table III summarizes 1930 insulator assembly flashover data with respect to overhead ground wires. Of the 65 line insulator assemblies flashed on this line during 1930, 71 per cent showed glaze burns characteristic of dynamic arcs. Of 38 surge (flashover) indicator operations 29 (76 per cent) were associated with marked insulator assemblies. Possibly 24 per cent of the flashovers during 1930, therefore, left no discernible marks on the insulators. As in previous years, most flashovers occurred on phase conductors occupying the outer positions in the towers. The numbers are 36, 6, and 23 respectively for the west, middle, and east conductors.

TABLE III—SUMMARIZED INSULATOR FLASHOVER DATA 1930

Line Name	Under ground wires		No ground wires		Total for line	
	No.	No./mi.	No.	No./mi.	No.	No./mi.
Wallenpaupack Tap	5*	0.26	8	0.94	13†	0.46
Siegfried-Roseland	3	0.67	49	1.51	52	1.41
Plymouth-Siegfried	2	0.04			2	0.04

*Two of these flashovers occurred before installation of tower footing grounding cables.
†Excludes four flashovers at Wallenpaupack protective gaps.

Table IV shows that the great majority of insulator assembly flashovers involve only one phase (32 groups out of 36) but that in 15 of these 32 cases more than one assembly was involved (single-phase flashover on two or more adjacent towers).

TABLE IV—GROUPING OF FLASHED INSULATOR ASSEMBLIES

Number of phases involved in group	Number of assemblies composing group				Total
	(1)	(2)	(3)	(4)	
1	17	12	2	1	32
2		1	0	2	3
3			0	1	1
Total	17	13	2	4	36

Fig. 5 shows the flashed insulator assemblies by line miles for 1930 and for the five-year period 1926 to 1930 inclusive.

Lightning Strokes. All towers of the Wallenpaupack Tap-Siegfried Roseland line were equipped with lightning stroke recorders. These devices were serviced after each trip-out or group of trip-outs. Table V lists detailed data.

Table VI summarizes important results, and indicates the beneficial effect of overhead ground wires.

Antennas. Data from these installations are meager. The following results appear to be well established, however.

a. Simultaneous records from antennas of varying heights are approximately proportional to height above the earth's surface.

b. The fifteen records obtained during 1929 and 1930 although varying in magnitude from about 300 to 2,700 kv. have in no case been accompanied by more than a low voltage on the adjacent transmission line conductors.

c. Voltages of flashover value occurring on the transmission line adjacent to antennas have not been accompanied by records on the latter.

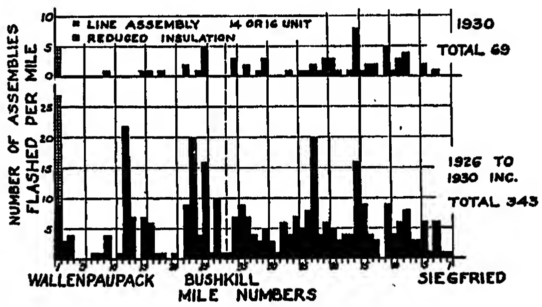


Fig. 5—Insulator Assembly Flashovers

Wallenpaupack Tap-Siegfried Roseland Line

d. A 60-ft. high-grounded antenna erected over a 50-ft. high antenna indicates a protective ratio of about 0.5 to 0.6.

Cathode-Ray Oscillograms

During 1930 oscillograms of 22 lightning surges which exceeded 100 kv. at the point of measurement were obtained. These have been classified into three groups, as follows:

Group I. Surges accompanied by line trip-out caused by a fault involving the conductor phase to which the oscillographs were coupled (phase X).

Group II. Surges accompanied by line trip-out caused by a fault not involving the conductor phase to which the oscillographs were coupled.

Group III. Surges unaccompanied by line trip-out and line fault (nature and location of origin unknown).

Table VII presents pertinent data of these 22 oscillograms.

Fig. 6 shows a typical pair of oscillograms of Group I. Both oscillograms are records of the same surge. The latter record (902388) began at about $-$ 400 kv. and is about one microsecond later in starting than the corresponding fast-sweep oscillogram or 902387.

These records were obtained coincident with trip-out

TABLE V—LIGHTNING STROKES RECORDED ON WALLENPAUPACK TAP-SIEGFRIED ROSELAND LINE

Stroke No.	Structure at which record was obtained	Magnitude of structure current amperes¶	Polarity of structure current	Overhead ground wires	Structure footing resistance ohms	Insulator flashovers	Conductor position	No. of phases affected	Correlating trip-outs
(*A*) *Lightning strokes which caused trip-outs* (This list includes one doubtful case, No. 2)									
1	WT22-4	80,000	Osc	No	91	WT22-4W	E	2	11
			Neg			WT22-4X	M		
2	SR17-4	110,000	Neg	No	23	None discovered	?	?	Probably none
3	SR34-1	90,000	Neg	No	33	SR34-1X	E	1	12
	SR34-2	110,000	Neg	No	37	SR34-2X	E		
4	SR36-2	110,000	Neg	No	10	SR36-2Y	W	1	10
					13	SR36-1Y	W		
5	SR8-1	100,000 90,000	Neg	No	34	SR8-1W	W	1	15 or 18
	SR8-2	100,000	Neg	No	23	SR8-2W	W		
		60,000		No	28	SR8-3W	W		
6	SR11-1	90,000	Neg	No	27	SR11-1W	W	1	16
	SR11-2	100,000	Neg	No	17	SR11-2W	M		
7	SR16-5	80,000	Neg	No	85	SR16-5W	E	1	21
	SR17-1	Less than 60,000	Neg	No	43	SR17-1W	E		
				No	30	SR16-3W	E		
				No	22	SR16-4W	E		
8	SR31-3	110,000	Neg	No	34	SR31-3Y	W	1	17
				No	12	SR31-2Y	W		
*	SR23-3	None recorded		No	51	SR23-3X	W	1	22
	SR23-4			No	23	SR23-4X	W		
9	SR24-5	100,000	Neg	No	32	SR24-5Y	W	1	23
†	SR25-1	100,000	Neg	No	16	SR25-1Y	W		
10	SR9-5	150,000	Neg	No	34	SR9-5Y	E	1	25
	SR10-1	120,000	Neg	No	15	SR10-1Y	E		
11	SR13-6	110,000	Neg	No	100	SR13-6W	E	1	26
	SR14-1	100,000	Neg	No	21	SR14-1W	E		
				No	42	SR13-5W	E		

Stroke No.	Structure at which record was obtained	Magnitude of structure current amperes¶	Polarity of structure current	Overhead ground wires	Structure footing resistance ohms
(*B*) *Lightning strokes which did not cause trip-outs or flashovers*					
12	SR4-4	90,000	Neg	Yes	20
13	WT3-2	60,000	Neg	Yes	27
14	WT6-4	130,000 120,000	Neg	Yes	23
	WT6-5	60,000	Neg	Yes	47
15	WT16-3	260,000 220,000	Neg	Yes	79
16	WT17-4	70,000	Neg	Yes	52
	WT17-5	160,000	Neg	Yes	18
17 ‡	WT22-1 (Guy)	Less than 60,000 by L. S. R. Approx. 120,000 by S. V. R. in shunt	Neg	Diverting Cables	32
	WT22-2 (Guy)	Less than 40,000 by L. S. R.	Neg	Diverting Cables	Approx. 30
18	WT23-1 (Tower)	60,000	Neg	Diverting Cables	Approx. 30
	WT23-1 (Guy)	112,000	Osc Neg	Diverting Cables	
	WT22-4 (Guy)	Less than 40,000	Osc Neg	Diverting Cables	Approx. 30

*Surge-voltage recorders coupled to all three-phase conductors at tower SR23-4 indicated that the maximum possible voltages on phases Y and W (22½ and 45 ft. from phase X which flashed over) were −1,620 kv. and − 710 kv. respectively. On phase X conductor a voltage in excess of − 2,400 kv. was recorded. Also see oscillogram 902379.

†A surge-voltage recorder coupled to phase X conductor at tower SR25-1 (45 ft. from phase Y which flashed over) indicated that the maximum possible voltages on phase X were −1,190 kv. and + 1,280 kv.

‡A surge-voltage recorder coupled to Y phase conductor at tower WT22-1 indicated + 2,400 kv. coincident with this stroke. No insulator flashover or trip-out occurred.

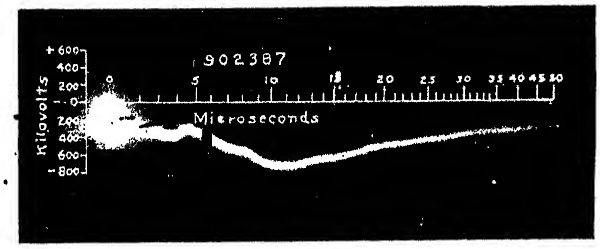
¶According to present calibrations, subject to revision.

TABLE VI—SUMMARIZED LIGHTNING STROKE RECORDER DATA WALLENPAUPACK TAP-SIEGFRIED ROSELAND LINE

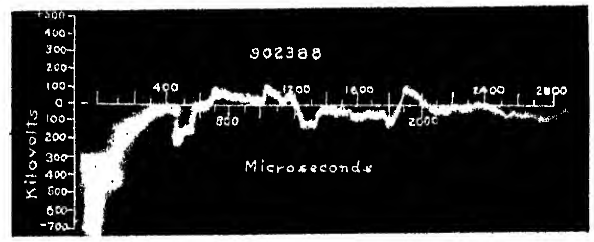
No. of towers in line	314
No. of towers equipped with recorders	314
No. of current records obtained	35
Probable No. of strokes causing these records	18
Maximum current recorded, amperes	260,000
Minimum current recorded, amperes	40,000
No. of negative strokes	16
No. of oscillatory strokes (negative predominating)	2
No. of recorded strokes which definitely caused trip-outs (no ground wires in any case)	10
No. of recorded strokes which did not cause trip-outs (ground wires in every case)	7
No. of recorded strokes which may have caused trip-outs	1

24. A single-phase fault developed on phase X at towers SR 14-5 and 15-1, about 8.7 miles south of the lightning laboratory. Flashed insulators were found on phase X at these towers, and the reflections recorded on oscillogram 902388 show that the location of the fault was approximately 9 miles south of the laboratory.

Similarly Fig. 7 shows a typical pair of oscillograms of Group II., 902365 and 902366 are the short and long duration oscillograms, respectively. These records were obtained coincident with trip-out 16. A single-phase fault developed on phase W at towers SR 11-1 and 11-2, about 12.5 miles south of the lightning laboratory. Flashed insulators were found on phase W at these towers, and the reflections recorded on oscillogram 902366 show that the location of the fault was about 13 miles south of the lightning laboratory. In this case,



A



B

FIG. 6—OSCILLOGRAMS OF LIGHTNING SURGE—TYPICAL GROUP I SURGE

Cathode-ray oscillograms 902387 and 902388, obtained July 24, 1930 at 1:50.5 p. m. coincident with trip-out No. 24. Ammeters at Wallenpaupack and magnetic oscillographs at Siegfried indicated a fault on phase X. Surge indicator targets showed on phase X at towers 14-5 and 15-1 (Siegfried-Roseland line) and the insulators were found flashed. Reflections on the slow-sweep oscillogram indicate origin of the surge as about 9 miles south of the laboratory or at about this location

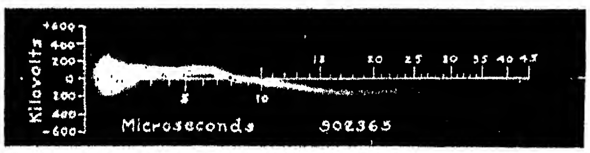
TABLE VII—CATHODE-RAY OSCILLOGRAMS OBTAINED AT CHERRY VALLEY, PA. LIGHTNING LABORATORY

Oscillogram numbers: Fast sweep 50 μ sec.	Oscillogram numbers: Slow sweep 2,000 μ sec.	Corresponding trip-out number	Phases faulted	Maximum recorded kv.: Fast sweep	Maximum recorded kv.: Slow sweep	Nature of wave	Polarity characteristics: First loop	Polarity characteristics: Second loop	Approx. distance from oscillograph in miles‡	Time in microseconds to reach: 75% of max. (front)	Max. voltage	50% of max. (tail)	Zero of first loop: Fast sweep	Zero of first loop: Slow sweep
Group I—Fault on phase X (to which oscillographs were coupled)														
902315	902316	8	X	− 500	−480	Uni.	Neg.		12 S	0.8	2.0	6.5	42. +	140
902329	902330	11	X, W	+ 100	−525	Osc.	Pos.	Neg.	20 N	0.1	0.1	2.0	2.5	..
902338	902339	12	X	− 380	−375	Uni.	Neg.		10 N	1.5	15.	50. +	50. +	250
902379	No S. S.	22	X	−2760		Uni.	Neg.		0.1 or less	?	?	?	5.9	..
902387	902388	24	X	− 750	−725	Uni.	Neg.		9 S	8.0	11.0	31.	53. +	460 +
Group II—Fault on phase Y or W or both, but not on phase X														
902306A	902308A	7	Y, W*	+ 380	−320	Osc.	Pos.	Neg.	18 N	1.5	2.5	11.0	13.5	
902325	902326	10	Y	+ 140	+162	Osc.	Pos.	Neg.	11 N	0.1	0.1	3.0	6.0	
902344	No S. S.	See Note†	W	+ 150		Uni.	Pos.		100 S	1.0	5.0	30.	47. +	
902363	902364	15	W	+ 200	+155	Uni.	Pos.		14 S	4.5	6.0	9.0	13.0	
902365	902366	16	W	+ 100	−175	Osc.	Pos.	Neg.	13 S	5.0	6.0	6.5	8.0	
No F. S.	902367	18	W*		−175	Osc.	Pos.	Neg.	15 S		...	..		35
No F. S.	902368	19	Y		−150	Osc.	Pos.	Neg.	15 S		...	..		25
No F. S.	902370	20†	W		+285	Osc.	Pos.	Neg.	60 S		...	..		40
902383	902384	23	Y	− 250	−300	Uni.	Neg.		1 N	0.3	0.3	1.0	2.0	
902409	902410	25	Y	+ 150	−200	Osc.	Pos.	Neg.	13 S	2.0	2.5	4.2	10.0	
902411	902412	26	W	+ 310	+300	Osc.	Pos.	Neg.	10 S	1.0	2.0	7.0	9.0	
Group III—No corresponding trip-out (location of fault, if any, unknown)														
902307B	902308B	None	Unknown	?	−230	Osc.	Neg.	Pos.	3.5 N?	?	?	?	5.0	
902307C	902308C	"	"	+ 120	−430	Osc.	Pos.	Neg.	42.5 N?	2.5	3.5	3.8	4.0	
902313	902314	"	"	+ 180	−450	Osc.	Pos.	Neg.	? S	0.1	0.1	9.0	12.	
902335	902336	"	"	+ 230	+240	Uni.	Pos.		Near lab.?	1.5	2.0	6.0	50. +	400
902337	No S. S.	"	"	+ 350.		Uni.	Pos.			2.0	3.5	25.	50. +	
No F. S.	902369	"	"		−150	Osc.	Neg.	Pos.			40.			170

*No. of phases faulted not known with certainty. Possibly one other phase involved.
†Trip-out of interconnected 220-kv. line. No fault on Wlpk. Tap.-Sieg. Rose. Line.
‡Judging from reflections on slow-sweep oscillograms. N (North) and S (South).

lightning-stroke recorder records were obtained at both towers, indicating that a direct negative stroke occurred to phase *W* conductor between towers 11-1 and 11-2, and that the magnitude of current was sufficient to produce records at both towers.

Fig. 8 showing oscillogram 902379 is a Group I record, unusual in that the direct stroke causing the line fault and trip-out occurred not more than 500 ft. away from the lightning laboratory. No lightning-stroke recorder records were obtained, (minimum sensitivity being about 60,000 amperes). The fault was confined to phase *X* on towers SR 23-3 and 23-4, and caused trip-out 22. The record shows that the rise of voltage was rather gradual for the first 3.5 microseconds, but quite rapid during the next 2 microseconds or until insulator flashover occurred on the crest of the wave at over − 2,760 kv. After flashover the voltage collapsed



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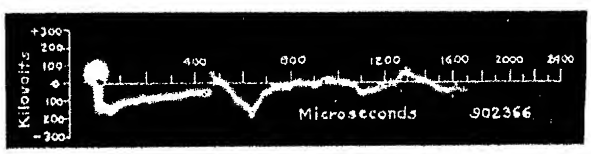

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Fig. 7—Oscillograms of Lightning Surge—Typical Group II Surge

Cathode-ray oscillograms 902365 and 902366 obtained July 9 at 5:27.3 p. m. coincident with trip-out No. 16. Short-circuit 60-cycle currents indicated a fault on phase *W*. Surge-indicator targets showed on phase *W* at towers 11-1 and 11-2 (Siegfried-Roseland line). Insulators were found flashed and lightning stroke recorders indicated at those structures, currents of 90,000 and 100,000 amperes, respectively. Reflections on the slow-sweep oscillogram indicate origin of the surge as about 13 miles south of the laboratory or at about this location

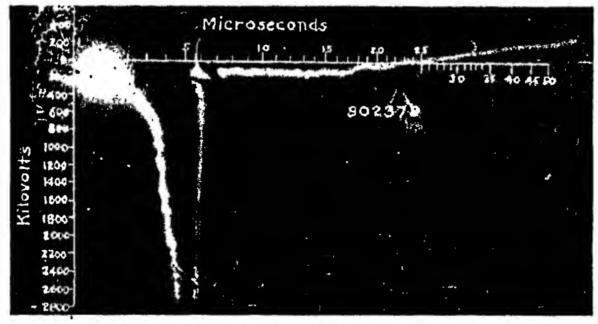
very quickly in about 0.2 microsecond to zero, later increasing to about − 150 kv. and gradually diminishing to zero again at 23 microseconds.

The wave front of this surge resembles that of the voltage across a capacitor when being charged through an inductance from a constant voltage source.

Based on wave-shape characteristics before reflections occur, the records of Group I appear, in general, to be unidirectional and negative. Those of Group II appear to be oscillatory, first loop positive, second loop (longer time duration) negative. Composite wave shapes based on these records are shown in Fig. 9.

Most of the records in these two groups have been associated with specific groups of insulator flashovers, and a number was also accompanied by measured direct strokes.

As shown in Table VII the times to reach crest voltage ranged from less than 0.1 to 15 microseconds: Composite wave shapes representative of Groups I and II are summarized in Table VIII.



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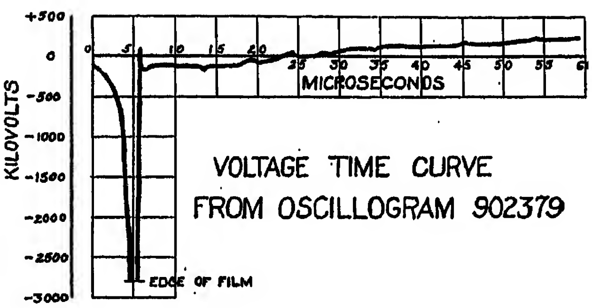

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Fig. 8—Oscillogram of Lightning Surge Voltage Measured at Point of Origin

Cathode-ray oscillogram 902379 obtained at the Cherry Valley lightning laboratory—July 24, 1930 at 1:41 p. m. and coincident with trip-out No. 22. Flashover occurred across phase *X* insulator assembly at tower 23-4, a point about 125 ft. south of where the oscillographs were coupled. The flashover was checked by (1) visual observation, (2) surge indicator operation, and (3) flash marks on insulators. Flashover on phase *X* at tower 23-3 (adjacent tower) was also indicated. This record begins at − 100 kv., increases to − 720 kv. in 3.5 microseconds and is off scale (− 2,760 kv.) in 4.8 microseconds. Record reaches zero due to the accompanying insulator flashover in 5.9 microseconds

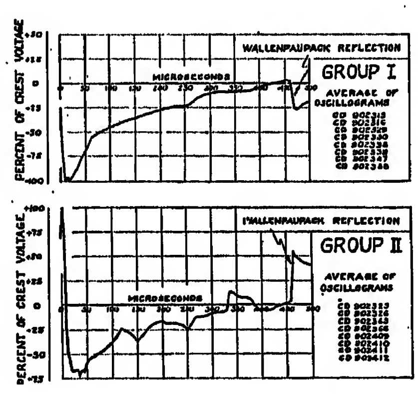

Fig. 9—Composite Lightning Surge Wave Shapes

Surge-Voltage Recorders

As in previous years surge-voltage recorders located at rather widely separated intervals along the line yielded records of all magnitudes up to about 2,700 kv. (breakdown value of line insulation) and of bothpositive

and negative polarities. Polarity characteristics and magnitudes of surge voltages measured during 1930 are indicated in Table IX.

Data for five consecutive years classified with respect to trip-outs and overhead ground wires are presented in Table X. It appears that surge-voltage recorder records are largely dependent upon the distance between the originating disturbance and the point of measurement, whether or not overhead ground wires are present, and whether or not the measurements are made on the same phase as that involved in the original disturbance. In the limited number of cases in which correlation can be made with a reasonable degree of confidence, high negative voltages have been associated with flashed insulators on the phase to which the surge-voltage recorder is coupled, either at the same tower or a very few spans away. The high positive voltages do not always correlate with the insulator flashovers. When they do, the flashover occurred at the surge-voltage recorder tower but not on the conductor phase coupled to the recorder. Table X shows that all voltages of high magnitude which are not coincident with line trip-outs, are positive in polarity and are measured on sections of line protected by ground wires.

TABLE VIII—SUMMARIZED CATHODE-RAY OSCILLOGRAM CONSTANTS

Time in microseconds to reach	First loop	Second loop
Group I		
75 per cent crest	5.0	
Crest	11.0	
50 per cent crest on wave tail	80.	
Zero	420.	
Group II		
75 per cent crest	2.1	17
Crest	2.5	25
50 per cent crest on wave tail	5.0	92
Zero	8.0	335

TABLE IX—POLARITY CHARACTERISTICS OF LIGHTNING SURGES RECORDED ON THE WALLENPAUPACK TAP-SIEGFRIED ROSELAND AND PLYMOUTH-SIEGFRIED LINES DURING 1930

	WT-SR line		PS line	
	Number	Max. voltage	Number	Max. voltage
Unidirectional				
Positive	1	450	0	
Negative	0		8	550
Oscillatory				
Highest crest value positive	6	2400	0	
Highest crest value negative	21	2640-3080	0	
Positive and negative crest values equal	1	tr	2	tr
Total	29		10	

TABLE X—NUMBER, LOCATION AND POLARITY OF VOLTAGE SURGES EXCEEDING VARIOUS TIMES NORMAL, AS MEASURED BY SURGE-VOLTAGE RECORDERS OVER THE FIVE-YEAR PERIOD 1926 TO 1930, INCLUSIVE

Location and polarity		No. exceeding times normal (180-kv.) shown (1)	(5)	(10)
Surges coincident with trip-outs				
On open line	positive	8	8	6
On open line	negative	29	21	8
Under ground wire	positive	4	2	1
Under ground wire	negative	26	11	2
Surges not coincident with trip-outs				
On open line	positive	5	1	0
On open line	negative	6	2	0
Under ground wire	positive	16	8	7
Under ground wire	negative	5	0	0

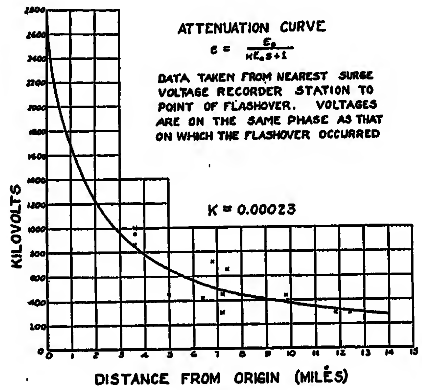

FIG. 10—ATTENUATION OF LIGHTNING SURGE VOLTAGES

Fig. 10 shows the magnitude of surge voltages at various distances from their source. All of these voltages were measured on sections of line unprotected by ground wire and on a phase which flashed over at the point of surge origin.

Electric Field Intensity and Lightning Severity

Electric-field intensity recorders installed at three locations from two to three miles away from the Cherry Valley lightning laboratory recorded voltage gradients up to − 37 kv. per ft. and + 35 kv. per ft. During any storm period, voltages at a given location are usually preponderantly of one polarity, which may be either positive or negative.

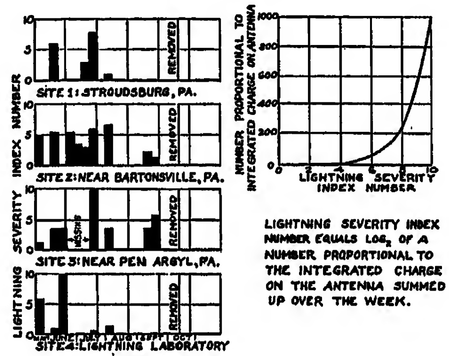

FIG. 11—LIGHTNING (STORM) SEVERITY METER DATA

Lightning severity meters were installed at each of the three sites mentioned above, and also near the lightning laboratory. Fig. 11 shows graphically the records obtained from these four locations. The lightning severity index number is proportional to the

logarithm to the base 2 of the charge on the severity meter antenna summed up for a week and is an arbitrary rather than an exact index of lightning storm severity.

Records from both types of instruments indicate that lightning at one location does not create sufficient field intensity to cause records on similar recorders at points 3 to 4½ miles away.

Overhead Ground Wires and Tower-Footing Resistance

The most reliable data available for evaluating the effect of overhead ground wires are those of flashed insulators. Data from 1929 and 1930 are shown in Fig. 12. It will be remembered that only about 37 per cent of the line is equipped with overhead ground wires.

Although many flashed insulator data were secured prior to 1929, these are not wholly reliable, with respect to time of occurrence and total numbers per season.

Fig. 12 shows the effect of overhead ground wires and tower footing resistance. The effect of the former (bearing in mind that these data are applicable only to

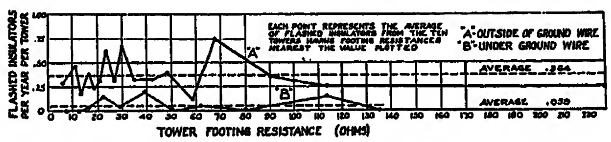

Fig. 12—Indicated Effect of Tower Footing Resistances on Insulator Assembly Flashovers (With and Without Overhead Ground Wires)

the construction used on this line) is to reduce the number of flashovers to about 15 per cent of those which would otherwise occur. The effect of tower footing resistance, at least above 17 ohms, seems to be negligible.

Summary and Conclusions

1. At least a large proportion of the lightning trip-outs of the Pennsylvania Power & Light Company's 220-kv. transmission lines are caused by direct strokes to line structures or wires.

2. Voltages induced in the power conductors by the collapse of cloud fields, coincident with lightning discharges to other objects, usually do not appear to be of sufficient magnitude to endanger service over circuits as highly insulated as these lines. For antenna voltages ranging up to 2,500 kv., the highest recorded corresponding transmission line voltage was 350 kv. Usually, no measurable induced voltages existed on the transmission line at times when appreciable voltages were impressed on the antennas.

3. With the insulation and clearances employed on these lines most lightning strokes contact only one object (ground wire, power conductor, or tower top) without forking to other objects (power conductors) in the same vicinity, and usually do not give rise to sufficient potential difference between the contacted object and surrounding non-contacted objects to cause local side-flashing.

4. Lightning strokes apparently vary through a considerable range of current intensity. Structure currents interpreted as ranging from approximately 40,000 amperes to approximately 260,000 amperes have been recorded.

5. Most lightning strokes are of negative polarity (earth positive and cloud negative) as indicated by measured structure currents. A smaller proportion of structure currents appear to be oscillatory with negative polarity predominating.

6. Positive-surge voltages between conductor and tower are usually indicated at locations where a ground wire, or a conductor not coupled to the surge-voltage recorder, is contacted by a direct stroke of negative polarity. They are not, therefore, indications of induced surges due to cloud fields.

7. The wave form of lightning surge voltages as recorded at the Cherry Valley Laboratory at the time of transmission line trip-outs, appears to be dependent, among other things, upon whether the surge originated and created fault on the phase to which the oscillographs were coupled, or on either or both of the other two phases. Coincident with the reported direct lightning stroke in the immediate vicinity of the laboratory which caused flashover at a tower 125 ft. from the laboratory and on the phase to which the oscillographs were coupled, an oscillogram was obtained which indicated that the line voltage rose to something in excess of − 2,760 kv. before flashover occurred. An average voltage change of 1,540 kv. per microsecond between the range of − 750 kv. and − 2,760 kv. was recorded.

8. The location of surge origin can be determined approximately from reflections shown on the oscillogram from the slow-sweep (2,000 microsecond time scale) oscillograph. The nature and time of these reflections supported by the records of the recording ammeters at Wallenpaupack and magnetic oscillographs at Siegfried, and the indications of the surge (flashover) indicators enable identification of fault locations in many cases, even when several trip-outs occur in a single day.

9. Most trip-outs of these 220-kv. lines are occasioned by single-phase faults.

10. The number of insulator assemblies involved in a given disturbance causing line trip-out seems, from the 1930 data for the Wallenpaupack Tap-Siegfried Roseland line, to range from one to four, with two (both on the same phase and at adjacent towers) as a rather usual condition.

11. A protective ratio of the order of 0.5 to 0.6 is indicated for the ground wire installed over the 50-ft. antenna near Wallenpaupack Tap Tower 1-3. The voltages recorded on the voltage measuring antennas appear to be approximately proportional to antenna height above ground. These voltages as recorded display no pronounced trend toward any given polarity characteristic.

12. The absence, during the past two summers, of insulator flashovers in the High Knob section of the

Wallenpaupack Tap leads to the conclusion that the buried "counterpoise," installed there in the spring of 1929, increases the effectiveness of the overhead ground wires. During the years 1926 to 1928 insulator assembly flashovers in this section of line were numerous.

13. Conventional overhead ground wires, as now applied, and especially when grounded through towers of comparatively low-footing resistance, seem to provide protection against a majority of the direct strokes encountered. Of the twenty-four lightning trip-outs of the Wallenpaupack Tap-Siegfried Roseland line (37 per cent of which is equipped with conventional overhead ground wires), one (or possibly four) resulted from faults which developed in the ground wire sections. The Plymouth-Siegfried line (which is equipped with conventional overhead ground wires throughout) tripped twice.

Acknowledgments

Acknowledgment is made to Messrs. J. R. Fisher, G. J. Gross, H. B. Newsom, H. M. Poller, N. Rohats, J. E. Treweek, A. F. Werner and others, including the personnel of the Pennsylvania Power and Light Company and the General Electric Company for field work, analysis, and correlation of data, and to Messrs. C. M. Foust, C. A. Jordan, E. S. Lee, W. W. Lewis, W. L. Lloyd, Jr., F. W. Peek, Jr., H. S. Phelps, H. K. Sels, A. E. Silver, J. A. Snyder and N. Stahl for general direction of this investigation.

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Discussion

For discussion of this paper see page 1146.

1930 Lightning Investigations

On the Transmission System of the American Gas and Electric Company

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Fellow, A. I. E. E. Member, A. I. E. E.

INTRODUCTION

THIS paper covers two field investigations carried on in 1930 on the lightning problem. One of these was made on the 132-kv. transmission system of The Ohio Power Company and was really a continuation of the investigation carried on in 1929 and previously described.[1] The Philo-Canton Transmission line and the system of which it is a part have both been recently and fully described.[2] The second investigation was made on the 132-kv. transmission line of the Atlantic City Electric Company between Deepwater, New Jersey, and Atlantic City, New Jersey. A map showing the location of the two termini and the route of this line is shown in Fig. 1. The entire double circuit line is 63.5 miles long, 57 miles being steel tower 132-kv. construction with a 397,500 A. C. S. R. conductor, and the balance, between Pleasantville and Atlantic City, being wood-pole 66-kv. construction. The steel tower portion, except for a short distance from Deepwater is insulated with ten and twelve 4¾-in. disks, using two ground wires in a vertical plane through the center of the tower, and plain, approximately 15 in. by 30 in. rings, both top and bottom. The entire line was operated throughout the investigation period at 66 kv.

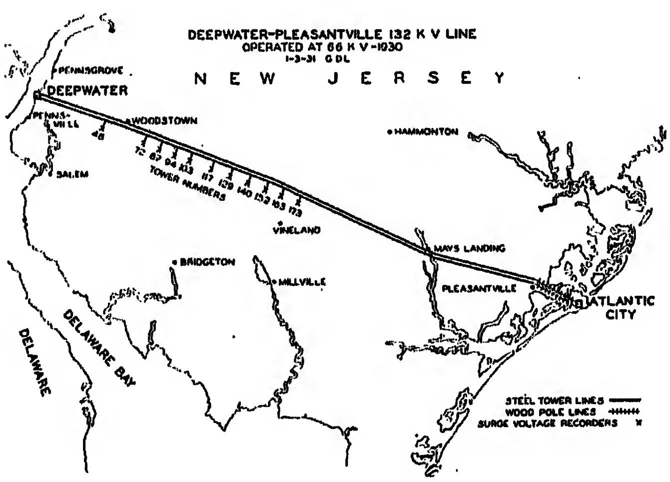

FIG. 1—ROUTE OF DEEPWATER-ATLANTIC CITY LINE OF ATLANTIC CITY ELECTRIC COMPANY

This paper is, of necessity, limited to as brief a summary of the salient data as possible; if at times the resulting impression is somewhat hazy, it should not in fairness be ascribed to a similar condition in the data.

INSTRUMENTS EMPLOYED AND THEIR LOCATION

The instruments used in these investigations were the surge-voltage recorder, the cathode-ray oscillograph, the direct-stroke recorder and the flashover indicator. The first three instruments have been described elsewhere.[3,4,1] Surge-voltage recorders were installed at every tower indicated in Fig. 1, on the Deepwater-Atlantic City line; on the Philo-Canton line instruments were placed at towers 200, 190, 181, 172, 164, 159, 156, 152, 148, 143, 134, 129, 123, 118, 113, thus giving a greater continuous stretch of line over which to obtain data than was available in 1929. Surge-voltage recorders were also installed on 132-kv. lightning arresters at Zanesville* and Newcomerstown.† The cathode-ray oscillograph station at Newcomerstown was operated throughout the entire lightning year; no oscillograph was in service on the Deepwater line.

Direct, or lightning, stroke recorders of the same type and connected in the same manner as previously described[1] were installed on both the Philo-Canton and Deepwater-Atlantic City lines. On the first line instruments were installed on towers 72 to 200 inclusive; on the second line 200 instruments were installed, one on each tower, from towers 20 to 219 inclusive.

The flashover indicator, or surge indicator, is an instrument used for the first time this year. When properly connected the instrument gives a visual indication that an insulator string has flashed over as a result of the passing or flowing across it to ground of a lightning surge current following the breakdown of its insulation by a lightning or surge voltage. A more detailed description of this instrument has been given elsewhere. A view showing such an instrument installed is shown in Fig. 2. A total of 200 of these was in service on the Philo-Canton line. On the Deepwater-Atlantic City line 300 of these instruments were installed on towers 41 to 140 inclusive.

These instruments were connected across a portion of the steel tower arm. Laboratory tests were made on the instrument before it was installed in the field. These consisted of surge voltages of known magnitude and wave shape applied to the line end of the insulator

*Thyrite.

†Oxide Film.

*American Gas and Electric Company, New York.

†General Electric Company, Pittsfield, Mass.

1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

assembly by means of an artificial lightning generator. The tests were made at a voltage of 450 kv., and the insulator flashover current was limited by resistance to a value of approximately 2,500 amperes; this was less than the minimum value of lightning current that would be carried by the tower arms when the line string flashed over. Using this value of flashover current, considerable time and effort were spent in finding the

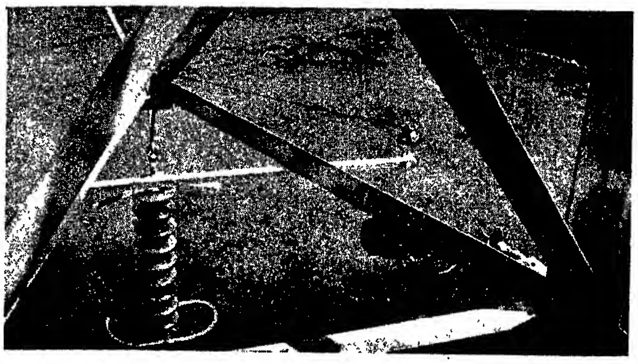

Fig. 2—Flashover- or Surge-Indicator Installation on Philo-Canton Line

proper lead length, location and instrument position to give the best flashover indicator performance without at the same time offering hazard to normal line operation. With the setup finally adopted 100 per cent operation was obtained in the laboratory.

The 1930 circuit arrangement of the cathode-ray oscillograph is shown in Fig. 3. This circuit is a modi-

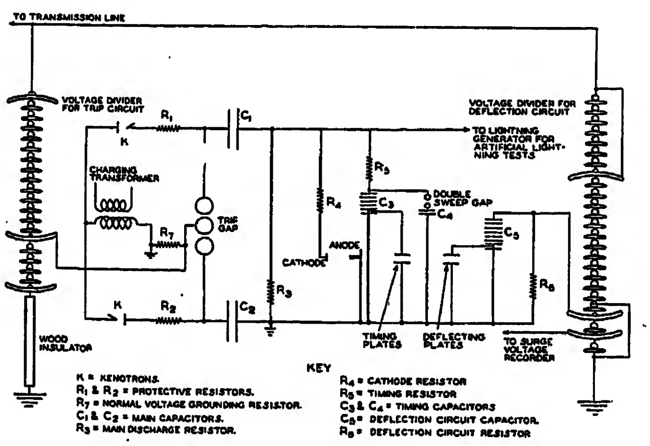

Fig. 3—1930 Circuit Arrangement of Cathode-Ray Oscillograph at Newcomerstown

fication of the 1929 circuit. Its principal changes consist of the following:

1. A change in the method of sweeping the cathode beam across the time axis. This method consisted of the use of a rather small capacity C_3 for first giving a rapid sweep of beam across the time axis, thus making it possible to obtain the front of a wave on a fairly long time scale. After the voltage across condenser C_3 has reached a value sufficiently high the gap in series with condenser C_4 (whose value of capacity is approximately from 20 to 40 times that of C_3) breaks down, the voltage during the process of breakdown being for an instant reduced to zero and a new sweep rate established on a much slower basis owing to the larger capacity of C_4. It was thus possible to obtain the tail of a long wave on the more compressed scale. An oscillogram of a long tail wave showing the double scale obtained by the double sweep gap is shown in Fig. 4. It will be noted that this represents a wave having reached a crest of 170 kv. in 9 microseconds and attenuating to 50 per cent crest in 30 microseconds; the total length of the wave is 160 microseconds.

2. Changes were made in the initiating of the trip gap to eliminate previously found erratic behavior. It had been found, for example, that under certain conditions the gap would be initiated apparently by air currents without any increase in voltage. To eliminate this difficulty the gap was entirely enclosed. Further, it was subjected continuously to the action of a mercury-vapor quartz-glass lamp so as to keep the space between

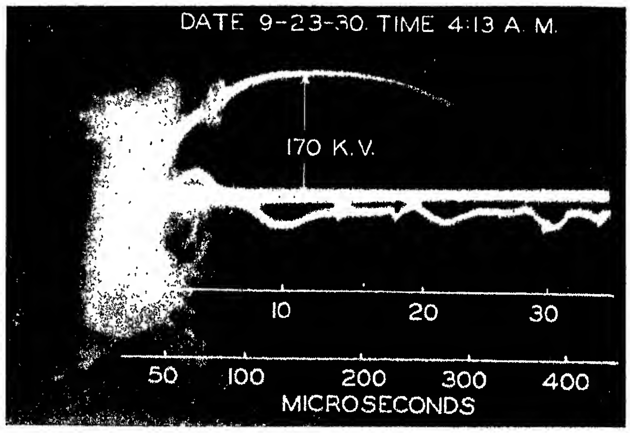

Fig. 4—Oscillogram of Long Tail Wave Showing Double Scale Obtained by Double Sweep

the gap in a highly ionized state and thus reduce any initial time lag owing to the necessity for ionization.

This is believed to be the first installation of a single cathode-ray oscillograph in the field to do the work of two. Heretofore, two oscillographs have been required to get oscillograms of a single lightning wave at two different sweeps. The method described above permits these two oscillograms to be taken with the same oscillograph at a saving in equipment and maintenance. Positive correlation is obtained.

Scope of Investigation

In undertaking the investigation it was hoped to obtain data on the following:

Magnitude of Lightning Voltage. The installation of surge-voltage recorders on both the Philo-Canton and the Deepwater-Atlantic City lines was expected to yield further information on this point. On the Philo-Canton line approximately 22 miles were quite

thoroughly covered by 15 instruments as previously outlined. These instruments were of further value for furnishing correlating data in connection with the information yielded by the direct lightning stroke recorders. On the Deepwater-Atlantic City line approximately 23 miles of line were covered by 11 surge-voltage recorders located as indicated in Fig. 1; at each one of these points, too, direct-stroke recorders were installed so that the surge-voltage recorders were available for furnishing correlating data in connection with any records obtained from such direct-stroke recorders. It was also expected that the data obtained by means of the surge-voltage recorders would furnish an approximate measure as to the intensity and severity of lightning during this year's investigation as compared with other years.

Characteristics of Lightning Voltages. It was expected to obtain further oscillograms of natural waves through the cathode-ray oscillograph installed at Newcomerstown. The reason for the location of an oscillograph at this point as well as physical details in connection with the installation have previously[1] been described.

Measurement of Current in Lightning Strokes. Last year an instrument was placed upon the circuit, designed to differentiate between induced voltages and direct strokes. This was accomplished by adjusting the instrument so that it would not record until the current was well above the maximum possible current due to induction. Since the instrument appeared to record approximate current its use was extended to that purpose. While it is realized that the current readings are approximate and subject to correction as calibrations are changed, it appears worth while to include them in this report.

Relation between Direct and Induced Strokes and between Direct Strokes and Flashover. It was expected that the large number of lightning-stroke recorders installed on both the Philo-Canton and Deepwater-Atlantic City lines, combined with the surge-voltage recorders, would yield data on the relationship between direct and induced strokes. The installation of flashover indicators, it was hoped, would also result in disclosing information as to the relationship between flashovers at towers and direct strokes of lightning terminating at the same towers.

Lightning Arrester Performance. In discussing the present status of rationalization of transmission system insulation[6] it was pointed out how important a *knowledge* of lightning arrester performance is in any attempted solution of the problem and further, how small an amount of actual data were available on that phase of the problem. It was hoped, that the surge-voltage recorders installed at Zanesville and Newcomerstown, would yield some very definite data on that point. The surge recorder, for current measurement, in each case was connected across a resistance which was in series with a lightning arrester at the grounded end. By placing the resistor inside the surge-recorder housing, disturbance from stray fields was reduced to a minimum and the connecting leads were shortened from a few feet to a few inches.

Analysis of Data Obtained

Surge-Voltage Data. No records were obtained from the surge-voltage recorders on the Deepwater-Atlantic City line. However, it should be pointed out, the surge-voltage recorders were not installed until all lightning trip-outs on the line had taken place.

On the Philo-Canton line 107 surges were obtained, divided as follows:

Lightning	35
Lightning and switching	3
Switching	64
Unknown	1
Testing	4
Total	107

In Table I there is shown a classification of these 107 surges according to five groupings, namely, positive, negative, predominantly positive, predominantly negative, and oscillatory.

TABLE I—PREDOMINATING NATURE OF SURGES
(Number Recorded)

Cause of surge	Positive: Pure	Positive: Predom.	Negative: Pure	Negative: Predom.	Oscillatory	Total
Lightning	23	7	3	0	2	35
Lightning and switching	0	0	0	1	2	3
Switching—energizing	0	0	0	0	1	1
Switching—deenergizing	0	0	1	0	0	1
Switching—mixed energizing and deenergizing	14	5	14	3	26	62
Unknown	0	1	0	0	0	1
Testing	4	0	0	0	0	4
						107

It will be seen that of the lightning surges 74.3 per cent were unidirectional, 20 per cent were predominantly unidirectional and 5.7 per cent were oscillatory; further, 65 per cent were purely positive, 20 per cent predominantly positive, 9 per cent purely negative, 0 per cent predominantly negative and 6 per cent oscillatory.

In the light of the observed data that the polarity of direct strokes as recorded by the lightning-stroke recorder was in every case negative, it is of particular interest that only 9 per cent were either negative or predominantly negative. It suggests very definitely that 91 per cent of these surges were of an induced origin.

In Table II is shown the voltage amplitude in times of normal of the highest recorder surge in each polarity classification. It will be noted that the highest surge voltage recorder record was 6½ times normal. Only

TABLE II—PREDOMINATING NATURE OF SURGES WITH MAXIMUM RECORDED VOLTAGES
(Times Normal to Ground)

Cause of surge	Positive Pos.	Positive Neg.	Negative Pos.	Negative Neg.	Oscillatory Pos.	Oscillatory Neg.
Lightning	5.6	0	0	3.1	6.4	6.4
Lightning and switching	....	....	2.6	4.0	6.5	Trace
Switching—energizing circuit No. 1	....	....		...	3.5	3.5
Switching—deenergizing circuit No. 1	....	....	0	2.3	..	...
Switching—mixed energizing and deenergizing circuit No. 1	....	....		...	2.6	2.3
Unknown	2.7	0		...	..	...

two line trip-outs occurred during the investigation indicating that the lightning storm severity was unusually light and accounted to a large extent for the absence of high-voltage surge measurements.

Cathode-Ray Oscillograph Data. Sixty-two lightning

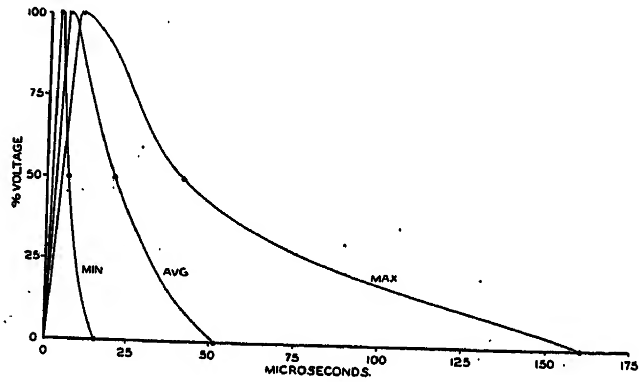

FIG. 5—THREE WAVE SHAPES REPRESENTING ENVELOPES OF MINIMUM, AVERAGE, AND MAXIMUM TIME ELEMENTS TO CREST, 50 PER CENT VALUE ON TAIL AND ZERO VOLTAGE OF 29 OSCILLOGRAMS 50 KV. CREST OR ABOVE

surges were recorded by the cathode-ray oscillograph. Of these 61 or 98.5 per cent were of positive polarity. The time to crest value of these surges varied from 2 to 13 microseconds; to half crest value on the tail from 5 to 85 microseconds and the time to zero voltage from 9 to 160 microseconds. The lowest crest value recorded was 20 kv. and the highest 180 kv.

In Fig. 5 there has been shown graphically three wave shapes which represent minimum, average, and maximum time elements to crest, 50 per cent value on the tail, and zero voltage. These curves are based on a selected group of 29 oscillograms recording 50 kv. on crest or above. It is particularly striking that the highest crest value recorded was only 1.67 times normal (180 kv.); this wave was a 9 by 16 microsecond wave.

Direct-Stroke Data. In Table III there are shown the lightning stroke currents obtained by direct addition of measured tower currents for 13 strokes recorded on the Philo-Canton line and for 12 strokes recorded on the Deepwater-Atlantic City line. A typical distribution of current on each side of a tower struck by lightning is shown in Fig. 6. This represents case No. 16, in Table III, in which tower 72 on the Atlantic City line was struck. It will be noticed that the lightning current traveled a distance of approximately 3,000 ft. in one direction, covering three spans, and approximately 2,000 ft. in the other direction, covering approximately two spans. The distribution of current on each side of

TABLE III—SUMMATION OF INDICATED TOWER CURRENTS

Reference	Date—1930	Tower No.	Lightning stroke recorder readings*
1	6–12 to 6–19	128	40,000
2	6–11 to 6–17	176	540,000
3	6–13 to 6–20	87	40,000
4	6–19 to 6–27	126	470,000
5	6–24 to 7–1	196	475,000
6	7–8 to 7–15	181	60,000
7	7–11 to 7–18	99	155,000
8	8–26 to 9–9	194	110,000
9	8–27 to 9–10	156	355,000
10	9–11 to 9–25	122	155,000
11	9–11 to 9–25	131	130,000
12	9–9 to 9–23	173	515,000
13	9–25 to 10–2	113	515,000
14	6–12 to 7–8	50	410,000
15	6–16 to 7–8	61	70,000
16	6–16 to 7–7	72	670,000
17	6–16 to 7–7	89	90,000
18	7–8 to 7–25	49	60,000
19	7–4 to 7–16	143	200,000
20	7–4 to 7–15	180	120,000
21	7–4 to 7–14	192	600,000
22	7–2 to 7–13	202	90,000
23	7–4 to 7–15	164	130,000
24	7–4 to 7–15	168	230,000
25	7–4 to 7–15	175	170,000

From 1 to 13 Philo-Canton.
From 14 to 25 Deepwater-Atlantic City.
*Approximate amperes—subject to correction.

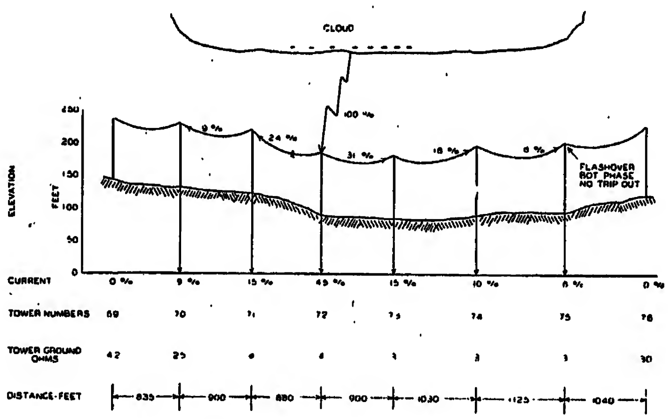

FIG. 6—TYPICAL CURRENT DISTRIBUTION ON EACH SIDE OF A TOWER STRUCK BY LIGHTNING

the towers struck for all 25 cases recorded is shown in Table IV.

An examination of Tables III and IV shows first, several cases where the approximate currents measured in individual towers may be expressed in terms of hundred thousands of amperes; second, if currents in adjacent towers due, apparently, to the same stroke are directly added, much larger currents are indicated. It is realized, of course, that the time-phase relation of the currents existing in the various parts of the circuit

TABLE IV—DIRECT-STROKE RECORDS APPROXIMATE KILOAMPERES—SUBJECT TO CORRECTION

Ref.	Adjacent towers				Tower struck	Adjacent towers			S. V. R. readings kv. at feet	
1					40				0	900
2		50	20	90	300	80			+600	6,110
3					40				0	31,680
4				85	310	75			+300	3,710
5				85	200	100			+360	7,270
6					60				+280	0
7					100	55			+360	103,000
8				55	55				+600	6,610
9				85	270				0	0
10				70	85				+280	716
11				65	65				−520	8,870
12			50	70	260	70	65		+260	0
13				90	300	75	50		+260	0
14				60	200*	60	40	50		
15					70					
16			60	100	300	100	70	40*	No surge-voltage	
17					90*				recorders as yet	
18					60				installed when	
19					200†				these direct	
20					120				strokes took	
21	60	90		70	280	100			place.	
22					90†					
23				60	70†					
24					160	70				
25				70	100					

* = Flashover of insulator string (total 3 cases).
† = Flashover of insulator string and trip-out (total 3 cases).
1 to 13 = Philo-Canton.
14 to 25 = Deepwater-Atlantic City.

are not definitely known and that an accurate analysis of traveling wave phenomena involved might give values of current less than the totals indicated in Table III. On the other hand, preliminary analysis has indicated that a rigidly correct addition of currents would increase the value obtained in the tower directly struck to at least 50 per cent of the total obtained by adding directly all the currents in the other towers. The results indicate that lightning strokes of several hundred thousand amperes do occur.

It is significant that in the case of the Philo-Canton line there was only one automatic line trip-out where there existed a possible correlation with a direct-stroke record. On September 23, 1930 line No. 1 tripped automatically at Newcomerstown and Philo and line No. 2 at Canton due to lightning. During this period (September 9 to 25) three lightning strokes were indicated by lightning-stroke recorders (cases 10, 11, and 12 in Tables III and IV) showing a possible correlation with the single trip-out. The remainder of the lightning-stroke records could not be correlated with any known trip-outs, although flashovers may possibly have occurred. On the Deepwater-Atlantic City line there is a correlation between line trip-out and line insulator assembly flashover in three of the twelve cases. These are reference Nos. 19, 22, and 23 in Table III. In all the other nine cases no correlation with any known trip-outs could be obtained, although inspection of the line completed on December 2nd indicated that insulator flashover without line trip-out had apparently occurred in three other cases. Twenty-five per cent of the direct strokes to the Deepwater-Atlantic City line therefore produced a trip-out with the possibility that another 25 per cent produced flashover not followed by trip-out.

Flashover (Surge) Indicator Records and Other Data Bearing on Question of Direct vs. Induced Strokes. The flashover or surge indicators on the Deepwater line were not placed in service until late in the lightning season and after all line trip-outs due to lightning had taken place.

No insulator assembly flashovers were indicated by the flashover indicators on the Philo-Canton line during the period of the investigation. The three principal reasons for this are: first, the fact that the instruments were installed rather late in the lightning season, second, they were not placed on every insulator assembly, and third, lightning conditions were not as severe as in the past years. Records of damage to the line insulators were rather few and in every case were at towers where there were no flashover indicators installed.

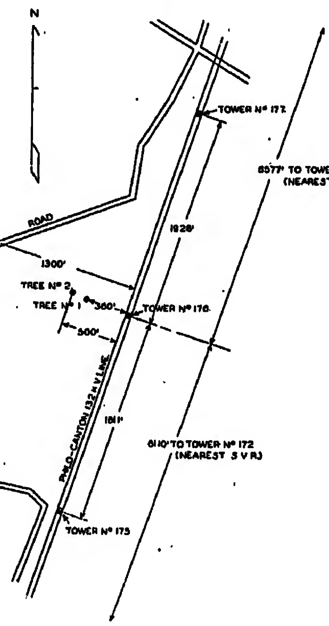

FIG. 7—LOCATION OF TREES STRUCK BY LIGHTNING IN VICINITY OF TOWER NO. 176 ON PHILO-CANTON LINE

It will be seen from the above that little was contributed to the knowledge of the relationship between direct and induced strokes by the flashover or surge indicators. Other data, however, were obtained which shed some light on this question. During the lightning season two trees were struck by lightning in the vicinity of tower 176. The location of these trees with respect to the line is shown in Fig. 7. Both trees Nos. 2 and 3 were struck apparently at the time of a storm which occurred on September 23. The nearest surge-voltage recorder (located at tower No. 172 more than a mile away) indicated a highly damped voltage surge of − 4.8 and + 2.4 times normal at 4.13 A. M. on September 23. It has not been found possible, however, to correlate these lightning strokes to trees Nos. 2 and 3 with any trip-out. They are apparently cases of direct strokes 500 ft. and 1,300 ft. from the line which, although possibly producing a flashover, produced no line trip-out or high-voltage record on any instrument connected to the line (the nearest instrument, of course, being more than

a mile away). Both trees had the bark split and torn off around the periphery of the tree, the lightning apparently striking the top of the tree and extending down to the ground in four or five parallel paths. In each path the bark torn off was 2 to 3 in. wide. The trees were oak, approximately 18 in. in diameter at the base, tree No. 2 being about 75 ft. high and No. 3 about 50 ft. high. Fig. 8 is from a photograph of tree No. 3 after it had been struck.

FIG. 8—TREE NO. 3 (FIG. 7) AFTER BEING STRUCK BY LIGHTNING

In the same general location a tree marked No. 1 in Fig. 7 was struck in 1928, and that time a surge recorder was located at tower No. 176 only 360 ft. away. This instrument recorded at that time maximum voltage 2,100 kv. positive; trip-out occurred. It seems evident from the 1928 record that a direct stroke of lightning striking tree No. 1 had caused an induced voltage on the line of 2,100 kv., flashover, and trip-out. The 1930 record indicates that one or more direct strokes of lightning striking trees Nos. 2 and 3 resulted in relatively low, if any, induced voltages on the line, no outages, and therefore just the opposite of the 1928 experience.

Lightning Arrester Data. Surges obtained on lightning arresters showing both current discharges and voltages are given in Table V.

The arresters at Newcomerstown are three legged oxide film type, 400 cells per leg. The series gaps on these arresters were set at 3.5 in. throughout the entire lightning season. Thyrite arresters are installed at Zanesville. Referring to Table V it will be seen that the values of voltages obtained across lightning arresters are at or below 300 kv. in all cases except on surge No. 14 B where a voltage of 500 kv. was recorded. A surge-voltage registration equivalent to 300 kv. is obtained under conditions where the reliability and accuracy of the registration is comparatively poor; the same thing holds true for currents of 100 amperes or less. Under these conditions no great reliability can be placed in the precision of measurements obtained. The best that can be gathered from the data is to state that it seems fairly certain that neither of the arresters at Newcomerstown or Zanesville was called upon to discharge surges of high voltage or current of any appreciable magnitude.

From the summary of the data obtained it will be seen that while everything was set for the gathering of a

TABLE V—LIGHTNING ARRESTER SURGE DATA

Surge No.	Location			Amperes		Kilovolts		Cause of surge
	Station	Circuit	Phase	+	−	+	−	
5	Zans. sub.	2	2	*N*	tr	*N*	*N*	Switching
6	Zans. sub.	2	2	*N*	115	*N*	*N*	Switching
7	Zans. sub.	1	1	*N*	tr	(2)	(2)	Switching
7	Zans. sub.	1	3	*N*	130	*N*	300	Switching
8	News. sub.	bus	2	100	*N*	(2)	(2)	Lightning
14	Zans. sub.	1	3	(1)	110	(2)	(2)	Switching
14 B	Zans. sub.	2	2	340	*N*	*N*	*N*	Lightning
14 B	Zans. sub.	1	1	270	*N*	510	230	Lightning
15	News. sub.	bus	2	110	*N*	280	*N*	Lightning
16	News. sub	bus	2	(1)	tr	(2)	(2)	Lightning
17	News. sub.	bus	2	(1)	tr	(2)	(2)	Lightning
38	Zans. sub.	2	1	*N*	100	*N*	tr	Switching
38	Zans. sub.	2	2	*N*	tr	*N*	*N*	Switching
41	Zans. sub.	1	3	110	(1)	300	370	Switching
48	Zans. sub.	2	2	*N*	tr	*N*	*N*	Switching
49	Zans. sub.	2	2	*N*	tr	*N*	*N*	Switching
61	Zans. sub.	1	3	tr	*N*	(2)	(2)	Switching
62	Zans. sub.	1	1	105	(1)	230	230	Switching
64	News. sub.	bus	1	90	(1)	230	300	Lightning
76	Zans. sub.	1	1	110	(1)	280	tr	Switching
82	News. sub.	bus	2	(1)	120	(2)	(2)	Lightning
93	Zans. sub.	2	2	210	*N*	250	*N*	Lightning
95	Zans. sub.	1	1	*N*	tr	(2)	(2)	Switching
95	Zans. sub.	1	2	*N*	105	tr	*N*	Switching
95	Zans. sub.	1	3	*N*	160	(2)	(2)	Switching

tr = trace.
(1) 70 amperes or less.
(2) 225 kv. or less.
(*N*) No record above normal line voltage.

rather extensive set of data, the amount of actual information obtained was not as great as had been expected. Nevertheless some very important new data were obtained and confirmation obtained of other previously gathered observations. The results obtained, it is believed, fully justify the conclusions below.

CONCLUSIONS

1. The highest lightning voltage obtained by surge-voltage recorder was 6.5 times normal, positive; the highest switching surge recorded was 3.5 times normal, oscillatory and highly damped; the highest cathode-ray oscillograph voltage recorded was 1.67 times normal, positive. The low severity of lightning storms during the year accounts to a great extent for the absence of high-voltage surge measurements.

2. Practically all lightning voltage surges on the transmission line of appreciable magnitude are unidirectional. The investigation disclosed:

74.3 per cent wholly unidirectional
20.0 per cent predominantly unidirectional
5.7 per cent oscillatory but highly damped.

The oscillatory figures may be due to superposed surges, which the timing of the surge recorder is unable to separate.

3. Positive lightning surges on the conductors predominate. The surge records show:

23 pure positive
23 predominantly positive
3 pure negative
0 predominantly negative
2 oscillatory

Out of 62 cathode-ray oscillograms, only one negative surge was recorded; the balance being positive.

4. From 29 oscillograms of natural lightning voltages above 50 kv. obtained on the line, wave fronts ranging from 2 to 9 microseconds were recorded. Tail time values to 50 per cent crest ranged from 6 to 40 microseconds; and total length to zero from 10 to 160 microseconds.

It should be noted in this connection that these waves were not measured at the point of origin where steeper fronts and possibly shorter tails may have existed.

5. The lightning-stroke recorders indicate that direct strokes are negative in polarity.

6. Direct lightning strokes to the overhead ground system of the transmission line take place frequently, 13 having been recorded on the Philo-Canton line and 12 on the Deepwater-Atlantic City line. Considering the fact that out of these 25 cases only 4 trip-outs resulted, it appears that the induced surge voltages on the conductors under these conditions are in the great majority (84 per cent) of cases insufficient in amplitude to cause line trip-out.

7. Direct strokes to the tower or ground wire also occurred without apparent flashover of the insulator assembly. Out of 13 direct-stroke records on the Philo-Canton line 8 were obtained under conditions where flashover indicator operation might have taken place with top or bottom insulator assembly flashover. Not a single case of such operation was recorded.

8. The study of direct-stroke records shows frequent cases where currents of the order of several hundred thousand amperes or more are indicated in a single lightning stroke.

9. Direct lightning strokes to the overhead ground system produced discharge currents through as many as 6 adjacent towers of a transmission line.

10. Repeated discharges (possibly up to 4 or 5) in the lightning stroke are indicated by the presence of superimposed Lichtenberg figures on several of the lightning-stroke recorder films. Another possibility, however, is that these are individual strokes some short time apart.

11. The presence of a large number of surges of positive polarity indicates that many induced surge voltages are present on the line conductors. In at least one known case a tree was struck approximately 360 ft. away from the line, inducing a positive voltage of 2,100 kv. on a conductor at a tower that distance away from the tree; this caused a flashover of the insulator string and a trip-out of the line.

12. The lightning arresters at Newcomerstown and Zanesville were not called upon during the period of active data taking to discharge surges of high voltage or current of any appreciable magnitude.

Acknowledgment is due to the executives of the General Electric Company, The Ohio Power Company, the Atlantic City Electric Company and the American Gas and Electric Company, who made this investigation possible; to the various members of the above companies for their cooperation and help in the design, installation, and operation of the apparatus and equipment, and for their help in supervising, correlating, and working up the data; in particular to Messrs. H. L. Rorden, J. C. Dowell, S. D. Day, G. D. Lippert, E. W. Whitmer and S. S. Smith, who actually carried out the work and were in charge of the various tests and field investigations.

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Discussion

For discussion of this paper see page 1146.

Lightning Investigation

On the Appalachian Electric Power Company's Transmission System

BY I. W. GROSS* and J. H. COX†

Associate, A. I. E. E. Associate, A. I. E. E.

INTRODUCTION

Plan of Investigation. Upon the organization of the Lightning Subcommittee of the Power Transmission and Distribution Committee of the A. I. E. E., in the fall of 1926, it was decided that a comprehensive investigation should be started, in an effort to obtain some definite information regarding lightning and its effects, which were causing havoc with electric power service and which, at that time, were relatively unknown. For this purpose, the American Gas and Electric Company placed its 132-kv. system at the disposal of the Committee, and in the following spring an investigation was started using the klydonograph, at that time the only instrument available for the purpose. This investigation was conducted cooperatively by the American Gas and Electric Company and the two principal electrical manufacturers, and most of the equipment was pooled for the purpose.

The following year, 1928, the activity was considerably expanded, and the efforts of the different participating groups were concentrated on two different parts of the system. The tests in which the Westinghouse Electric & Manufacturing Company cooperated with the American Gas and Electric Company, the results of which are being reported here, were performed on the Southern System, or that of the Appalachian Electric Power Company. Since its initiation the work has been continued up to the present, and use was made of all additional equipment, such as the cathode-ray oscillograph, and the application of the klydonograph to the recording of occurrence and current in direct strokes, as these applications became available.

During the four years in which this investigation has been conducted, it has been paralleled by similar work in other localities. A number of papers and articles has appeared, and these, of course, have been influenced by all the information currently existing. Also, all the results of this investigation have been continuously at the disposal of the Lightning and Insulator Subcommittee. However, a wealth of data has been accumulated and it is felt that a complete analysis and summarization of these data would constitute a worthy contribution on the subject. Not only does it, because of its volume, lend weight of confirmation by smoothing out the curves of previous papers, but several records of special interest have been obtained which throw light on various phenomena not completely understood. Furthermore, because of different settings of apparatus, previous curves have been extended.

OBJECTIVE

The information needed in connection with lightning was listed at the beginning of the investigation and these items have, in general, remained the objectives throughout the work. On many points the information is now fairly definite while on others it is still uncertain. The factors to be determined may be grouped under three headings, as follows:

1. *Nature of Lightning, that is:*
   a. Its magnitude, voltage, and current.
   b. Its polarity, unidirectional or oscillatory.
   c. Its wave shape, front, duration, and tail.
   d. Its effect on transmission lines, direct or induced.
2. *Behavior of Surges on Systems, that is:*
   a. Attenuation.
   b. Reflections at junctions and ends.
   c. Ability to flashover line structures.
3. *Influence of System Characteristics, such as:*
   a. Configuration of conductors, including height.
   b. Right-of-way terrain.
   c. Ground wires.
   d. Protective apparatus, lightning arresters, gaps, etc.
   e. Condition of system neutral ground.
   f. Tower and footing impedance.

SCOPE OF INVESTIGATION

System Investigated. The parts of the transmission system,[1] on which the investigation was concentrated comprised three sections of line and seven substations. The sections of line involved the first 9 miles out of Turner on the Turner-Logan line, the first 26 miles out of Glen Lyn on the Glen Lyn-Roanoke line, and the entire 24 miles of the Turner-Cabin Creek line. The substations where the investigation was carried on were Turner, South Point, Cabin Creek, Saltville, Kingsport, Switchback and Glen Lyn.

The transmission lines are of two-circuit steel tower construction, with conductors in vertical configuration and in reverse phase order on each side of the tower.

One ground wire is employed, being located at the

*American Gas and Electric Co., New York, N. Y.

†Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

1. For references see Bibliography.

center peak of the tower except on the Turner-Logan line. On this line no ground wire was in service in 1927 and 1928, the first two years of the investigation, but the conventional one ground wire was added late in 1928, so that on this line field data have been collected on the line both with and without ground wires.

Grading shields were in service on all lines where tests were made, these shields consisting of a 15 in. by 30 in. oblong ring at the line side and either a 30-in. horn or oblong strap ring at the ground end of the string.

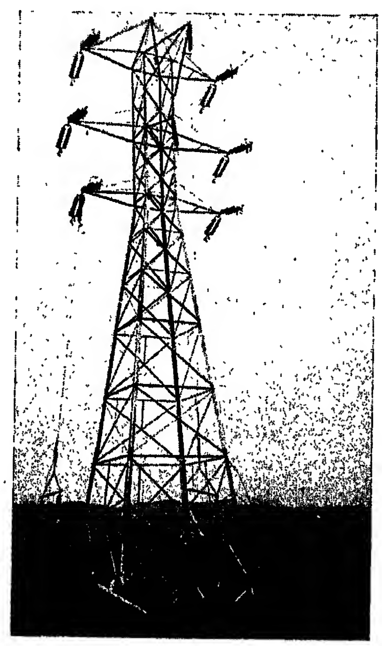

FIG. 1—FIELD INSTALLATION OF KLYDONOGRAPHS, TOWER 5· Turner-Logan line showing potentiometers and ground wire connection

The line insulation is nominally ten 5⅛-in. disks in suspension strings, and two strings of twelve 5⅛-in. insulators each at dead ends.

The line wires are 336,400 cir. mils. A. C. S. R. on the Turner-Logan and Turner-Cabin Creek lines, and 397,500 cir. mils A. C. S. R. on the Glen Lyn-Roanoke line. The ground wire is A. C. S. R., being 177,000 cir. mils, 159,000 cir. mils, and 397,500 cir. mils, respectively, on the three lines.

The nature of the terrain over which the transmission lines run is very rough and mountainous, ranging in elevation from 600 to 3,000 feet. The soil is mostly shale or a mixture of shale and clay, and even solid rock is encountered at some locations, where blasting is necessary in placing the tower footings. As might be expected, the tower footing resistances of these lines are relatively high, in a number of instances exceeding 250 ohms. The elevation, footing resistance, and locations where flashovers were found are shown in Figs. 18, A, B, and C.

Instruments Used in the Investigation. In studying lightning phenomena in the field three instruments have been used, namely, the klydonograph, the cathode-ray oscillograph, and the direct-stroke recorder.

The klydonograph[2] used was of the three or four electrode type and was coupled to the line by pipe type potentiometers having a ratio of about 65 to 1. A typical field installation of the klydonograph at a tower, as used in this investigation, is shown in Fig. 1. A close-up view of the potentiometers, together with the circuit arrangement as used in investigating lightning arrester operation, is shown in Figs. 9A and B.

Wherever klydonograph installations were made at towers in the field the instrument ground was connected to a separate ground system consisting of the six potentiometer metal anchors and one or more 8 ft. ground rods, all located at least 10 ft. from the base of the tower. This was done in an attempt to measure the maximum voltage from line to ground and not the line to tower potential. Actually, this may not have been completely achieved in some cases as the separate grounds were only of the order of 25 ft. distant from the tower legs. At installations in the substations, the instrument ground was made to the station ground bus which had a resistance of only a fraction of an ohm.

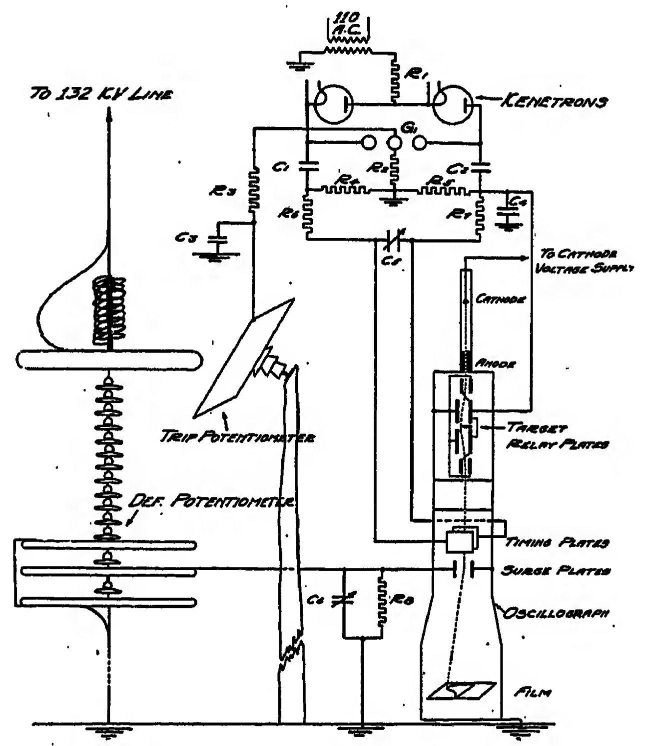

FIG. 2—SCHEMATIC WIRING DIAGRAM OF CATHODE-RAY OSCILLOGRAPH AND POTENTIOMETERS AT TOWER 20, TURNER-LOGAN LINE

The cathode-ray oscillograph used was of the cold cathode type, using the Norinder relay, which allows of continuously exciting the cathode, thus insuring that the electronic beam is ready for recording the transient at the instant the transient arrives. The schematic diagram of the oscillograph wiring and the coupling to the 132-kv. line is shown in Fig. 2. When the surge

to be measured arrives at the oscillograph, the excess potential on the trip potentiometer causes the gap $G\,1$ to break down imposing a voltage from the condenser $C\,2$ on the oscillograph relay plates. This allows the beam to focus on the film. The charging of condenser $C\,5$ sweeps the beam across the film, providing the timing circuit; and the main surge from the deflection potentiometer excites the surge plates causing the beam to reproduce the voltage magnitude of the surge.

The field installation of the lightning laboratory

FIG. 3—FIELD INSTALLATION OF LABORATORY AND POTENTIOMETERS AT TOWER 20, TURNER-LOGAN LINE

housing the oscillograph, together with the trip and deflection potentiometers at Tower 20 of the Turner-Logan line where the oscillograph was in operation the past two lightning seasons, is shown in Fig. 3. An interior view of the laboratory with the oscillograph as employed in 1929, is shown in Fig. 4.

The direct-stroke recorder used, and designed by Mr.

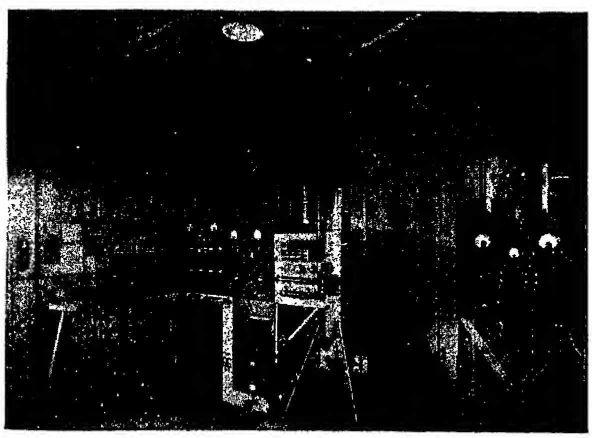

FIG. 4—INTERIOR OF LIGHTNING LABORATORY, TOWER 20, TURNER-LOGAN LINE, SHOWING OSCILLOGRAPH AUXILIARY EQUIPMENT

J. J. Torok of the Westinghouse Company, is essentially a small klydonograph with timing accessories omitted. The instrument consists of a regular telephone receiver shell having metal electrodes extending from both ends. The telephone diaphram is replaced with a photographic film on a micarta disk which rests on a metal plate. The disassembled instrument is shown in Fig. 5 and the field installation in Fig. 6. The potentiometer method of using this direct-stroke recorder at a tower, where the resistance wire is in parallel with the tower footing resistance, is shown in Fig. 7. The point at which the direct-stroke recorder was tapped to the resistance wire

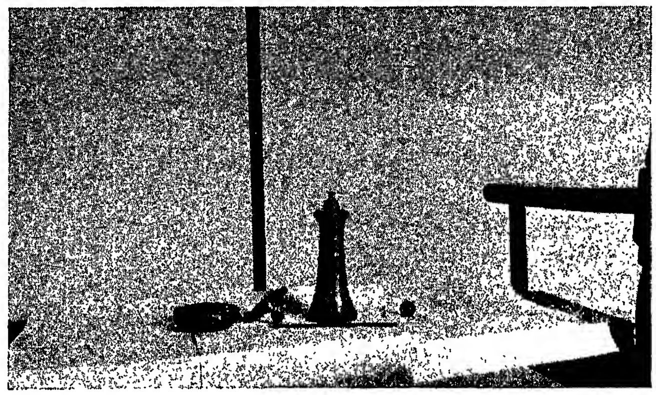

FIG. 5—DISASSEMBLY VIEW OF DIRECT-STROKE RECORDER

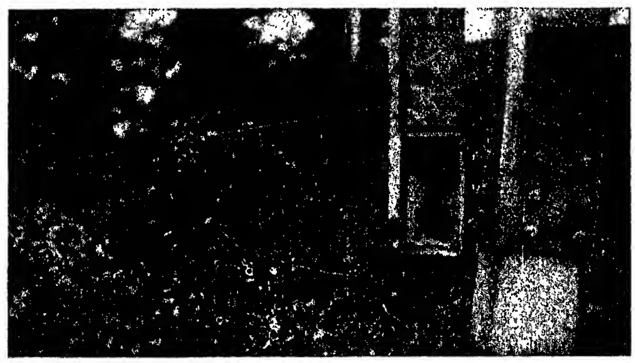

FIG. 6—FIELD INSTALLATION OF DIRECT-STROKE RECORDER AT THE BASE OF TOWER

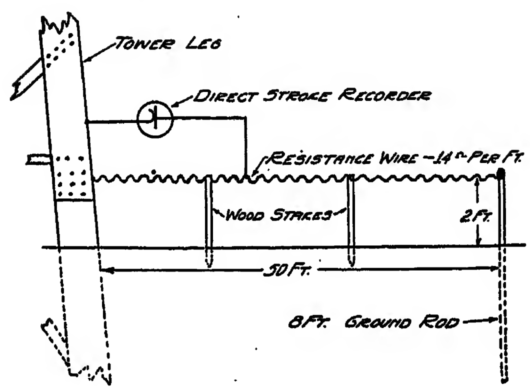

FIG. 7—SCHEMATIC WIRING DIAGRAM OF DIRECT-STROKE RECORDER

was so chosen that the instrument would not flash over with 100,000 amperes in the tower, and yet would be able to record currents as low as 10,000 amperes.

Location of Instruments. The locations of klydonographs, cathode-ray oscillograph and direct-stroke recorders are shown for all line installations in Fig. 8.

In addition, klydonographs were also installed at the South Point, Switchback, Saltville, and Kingsport 132-kv. substations. The direct-stroke recorders were placed at each tower on the Turner-Logan line for the first 36 towers out from the Turner substation.

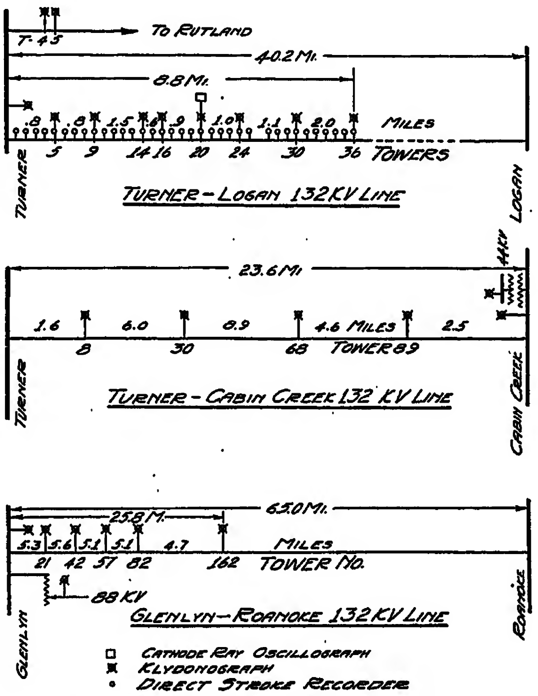

FIG. 8—LOCATION OF LIGHTNING MEASURING INSTRUMENTS ON THE LINE DURING FOUR YEARS' INVESTIGATION

During lightning seasons the cathode-ray oscillograph was in service in 1929 and 1930, the direct-stroke recorders in 1930, and the klydonographs in 1927 to 1930, inclusive. The locations of klydonographs during these four years are shown in Table I.

TABLE I—KLYDONOGRAPHS IN SERVICE

Location	1927	1928	1929	1930
Glenlyn–Roanoke Line		5	5	
Turner–Cabin Creek Line		4	4	
Turner–Logan Line	2	3	8	8
Turner–Rutland Line	1	1		
Cabin Creek substation		2(1)	2(1)	2(1)
Glenlyn substation	3(2)	3(2)	2(3)	2(3)
Kingsport substation		2(4)	2(4)	2(4)
Saltville substation		3(5)	3(5)	3(5)
South Point substation			1	1
Switchback substation		2(3)	2(3)	2(3)
Turner substation	1	1	1	1
Total	7	26	30	21

(1) One instrument on 44-kv. bus.
(2) One instrument on 13.2-kv. transformer tertiary.
One instrument on 88-kv. auto-transformer.
(3) One instrument on 88-kv. auto-transformer.
(4) One instrument on 132-kv. lightning arrester.
(5) One instrument on 88-kv. auto-transformer.
One instrument on 132-kv. lightning arrester.

In studying lightning arrester performance, the klydonograph was used to measure both the voltage on the line and the discharge currents in the arrester. Zircon resistors were placed in the legs of the arrester and in the ground connection common to the three arrester legs. The resistances used were approximately thirty and ten ohms, respectively. Zircon has the property of having a constant resistance under impulse conditions, and, as proved by laboratory tests after one year's operation, appears to be free from aging after repeated use.

The schematic wiring diagram of the klydonograph

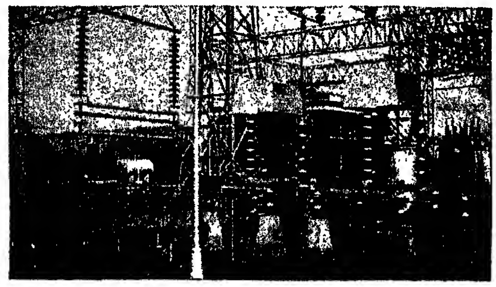

FIG. 9A—POTENTIOMETER INSTALLATION AT KINGSPORT FOR MEASURING VOLTAGE AT LIGHTNING ARRESTER

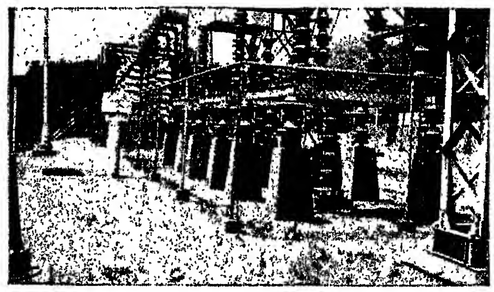

FIG. 9B—KLYDONOGRAPH INSTALLATION FOR MEASURING LIGHTNING ARRESTER DISCHARGE CURRENTS

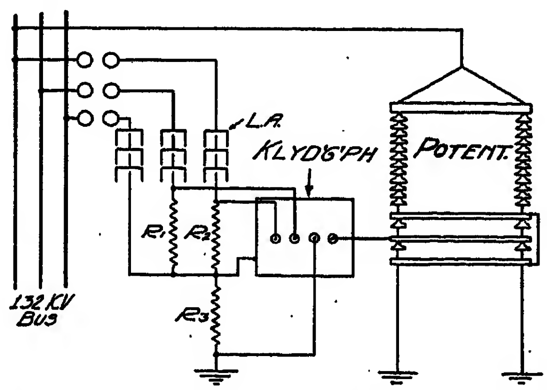

FIG. 9C—SCHEMATIC WIRING DIAGRAM OF KLYDONOGRAPH INSTALLATION ON LIGHTNING ARRESTER

as coupled to the circuit for the lightning arrester tests is shown in Fig. 9c, where one instrument measures two arrester leg currents, the total current and one phase potential. This instrument was backed up by a separate klydonograph on the bus measuring all phase voltages. This separate klydonograph was located about 150 circuit ft. away from the arrester connections, due to space limitations at the station.

During the winters of 1927 and 1928, klydonographs were kept in operation at substations, where they could

be conveniently serviced, to observe if any troublesome voltages existed on the system under normal operation, at times when lightning was not prevalent.

Analysis of Data

During the four years' investigation there were recorded some 900 voltage surges yielding over 3,000 Lichtenberg figures, 21 cathode-ray oscillograms of lightning voltages on the line, and two direct-hit records.

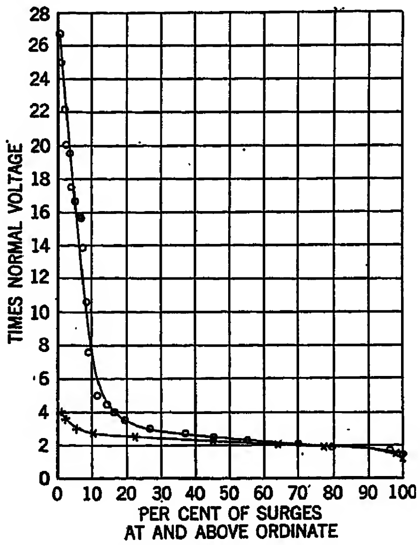

Fig. 10—Voltage Surges, 1927, 1928, 1929, 1930. Transmission Records

oo = 143 surges—lightning plus lightning and switching
xx = 98 surges—switching

Klydonograph Records on Line and Substations. The voltages recorded on the system by the klydonograph have, for purposes of analysis, been classified as to cause into three groups: first, those due to lightning, or lightning and switching (the condition obtaining when a lightning flashover of the line causes line trip-out); second, those due to switching operations on the line itself; and third, those of unknown origin, or caused by disturbances on parts of the lower voltage system not under investigation. The first two groups have been further separated, as to physical location; one, those occurring on the line; and two, those occurring at substations. This segregation was made to observe the relative magnitude and frequency of transient voltages on the line itself, and on the substation apparatus separately.

The record of transient voltages on the transmission line is given in Fig. 10, where it is shown that switching surges reach a maximum of four times normal (normal being taken as the crest voltage of the 60-cycle voltage from line wire to ground—108 kv. for the 132-kv. lines). The lightning voltages reach a maximum value of 26 times normal; and 10 per cent of the surges were above seven times normal.

To determine the relative frequency of these high lightning voltages on lines in the territory investigated, the total lightning year miles for the four years of the investigation were computed (using total lengths of Turner-Cabin Creek and Glen Lyn-Roanoke and one and one-half miles beyond the last klydonograph on the Turner-Logan Line) and by using Fig. 10 the following Table II was obtained:

TABLE II—LIGHTNING SURGES PER 100 MILES OF LINE PER YEAR

Times normal voltage	Crest kv.	No. of surges
7	757	10.9
10	1,060	8.3
15	1,620	4.6
20	2,160	3.5
25	2,700	1.0

By comparison with previously published data, it is apparent that the lines investigated are in a very severe lightning territory. For example, they are subject to 4.6 lightning voltages of 15 times normal per 100 miles of line per year.

Lightning voltages appearing at the substations are not as high as those which occur on the line. The substation records are shown in Fig. 11 where the highest lightning voltage is 12 times normal. Only 4.5 per cent of the surges are above 7 times normal, or 9.2 in number against 14.3 on the line (Fig. 10). This would be expected as in every case the substations were protected by lightning arresters and furthermore, there are in general divided paths at substations into which an incoming surge can be dissipated. Totaling the substation lightning years (22 for the four-year period) it appears that a station is subjected in a ten-year period, on an average basis, to five surges above seven times normal (757 kv.) to two surges above ten times normal (1,060 kv.), and to one-half surge of twelve times normal (1,300 kv.).

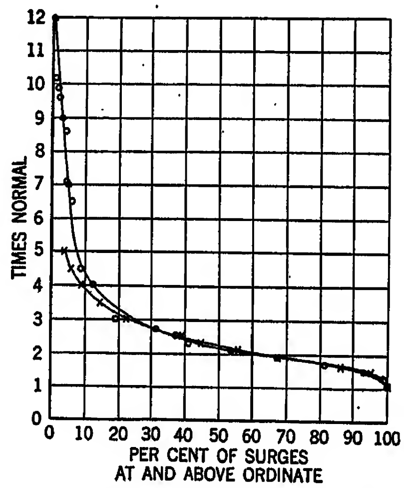

Fig. 11—Voltage Surges, 1927, 1928, 1929, 1930. Substation Records

o = 205 lightning plus lightning and switching
x = 170 switching

In addition to the surges indicated on the curve for

switching surges, five records were obtained which indicated from 6.2 to 9.0 times normal voltage. These were all recorded at Turner on the middle phase potentiometer where close physical spacing had raised the ratio. Therefore, these values were discounted in the curve, as it is believed that they were actually not greater than the maximum, in the order of six times normal, as measured on other parts of the system and in other investigations.

The magnitude of switching surges at the substation is of the same general order as observed on the line, being slightly higher, however, as would be expected.

The voltage surges of unknown origin, or due to disturbances foreign to the lines investigated, were relatively unimportant reaching a maximum of 3.8 times normal, except in two cases where 4.5 and 4.0 times normal were recorded at the end of the South Point 132-kv. line.

During the two winters (1927-1928) when klydonographs were in operation at substations, the voltage surges recorded were very few reaching a maximum of 3.5 times normal, and occurring at the time of switching operations.

Effect of the Ground Wire—Klydonograph Records. The beneficial effects of the use of a ground wire are clearly indicated by the records obtained on the Turner-Logan line, as shown in Table III.

TABLE III—LIGHTNING VOLTAGES ABOVE 700 KV. RECORDED ON TURNER-LOGAN 132-KV. LINE WIRES

1927–1928 No ground wire	1929–1930 One ground wire
− 700	−2,240
−1,700	−2,400
−1,800	
+1,900	
2,100	
−2,900	

While the numerical order of the highest voltages observed are the same, as might be expected due to the line insulation limiting this voltage, there are three times as many surges of a magnitude which might be expected to give trouble on the line or station apparatus when no ground wire exists. It should also be pointed out that in the two years of operation with a ground wire about twice the extent of line was investigated with the klydonograph as was the case during the previous two years of operation without a ground wire which further accentuates the indicated advantage of the ground wire.

Lightning Arrester Records—Klydonograph. Lightning voltages at the two substations where investigation was made on lightning arresters are given in Table IV. In only two cases was a discharge current recorded in the arrester, one of these where the maximum voltage on any phase was −970 kv., and the other where the voltage was − 1,300 kv. The corresponding discharge currents were − 300 and − 500 amperes. In another instance, at Saltville, a voltage of + 1,100 kv. was recorded without a current discharge measured on the arrester. The minimum current in the arrester which could be measured was about 80 amperes, which, with the 30-ohm Zircon resistor used, produces 2,400 volts on the klydonograph, which is near the low limit of recording.

The arresters used are of such a rating that they are expected to begin discharge at a voltage of about 700 kv. Some of the records listed in Table IV indicate voltages considerably above 700 kv., both with and without discharges.

It should however be pointed out that three factors were present which might account for voltage records higher than those which actually existed:

1. The voltages were recorded some 150 circuit ft. from the terminals of the arrester.
2. Tests have shown that stray fields which exist in the vicinity of substation structures have, under certain conditions, a marked influence on the klydonograph records.
3. No flashover or apparatus failure occurred in the substation at the time.

Ground Resistance and Line Flashover. An attempt to relate line flashover to tower footing resistance proved futile. Reference to Figs. 18A, B, and C indicates some tendency for flashovers to concentrate near the points of high elevation on the line, a fact which correlates with the reasonable expectation that lightning will strike to the most exposed points in the storm path; and these points are those of highest elevation.

Wave Fronts of Lightning Surges—Klydonograph Records. The klydonograph records of 21 lightning

TABLE IV—132-KV. LIGHTNING ARRESTER PERFORMANCE 1929–1930

Location	Line voltage–kv.			Arrester current–amperes			
	Phase 1	Phase 2	Phase 3	Phase 1	Phase 2	Phase 3	Total
Kingsport S. S	− 800	± 240	−970	*	±270	−340 +280	+ 625
Kingsport S. S	+ 288	+ 524	+322	*	None	None	None
Kingsport S. S	+ 288	+ 465	+322	*	None	None	None
Kingsport S. S	−1000	−1,300	−900 +270	*	−500	−650	−1180
Kingsport S. S	+ 325	− 512	+247	None	None	None	*
Saltville S. S	+ 640	+1,100	+825	None	None	None	*

*Not measured.

surges of 600 kv. and above obtained during the four years were analyzed to determine the wave fronts. The analysis is given in Table IX. It is not contended that this method of determining wave fronts is very accurate, but it is believed the record may show the predominating characteristics of the wave fronts. Fast fronts of one microsecond or less were observed in nine of the 21 surges, and wave fronts of the order of five microseconds were observed for 11 surges. Only

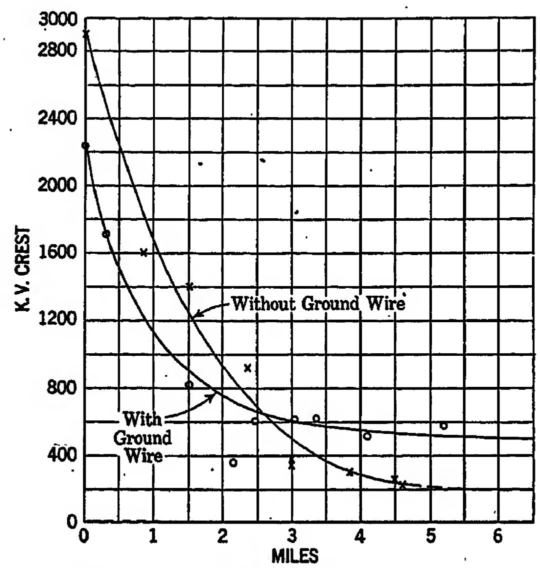

FIG. 12—ATTENUATION OF NEGATIVE LIGHTNING SURGES ON LINE, WITH AND WITHOUT GROUND WIRE

x—No. 233—line trip-out
o—No. 424—line trip-out

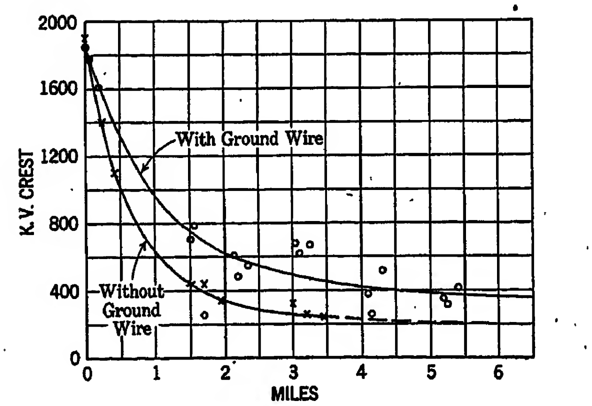

FIG. 13—ATTENUATION OF POSITIVE LIGHTNING SURGES ON LINE, WITH AND WITHOUT GROUND WIRE

x—No. 169—no line trip-out
o—No. 424—line trip-out

one slow front surge was observed, this being a positive surge of 685 kv. No positive surge with a fast front (one microsecond or less) was found.

Since the records analyzed were of surges above 600 kv., and the 60-cycle crest flashover of the line is of the same order, it is believed these surges were recorded near the point where the surge originated. The conclusion is therefore drawn that at the point of inception some lightning voltage surges have fast fronts, in the order of one microsecond or less, the corresponding rates of voltage rise being as high as 2,500 kv. per microsecond.

Voltages on Top, Middle, and Bottom Conductors. At the start of the investigation it was expected to find some relation between the voltages observed on the top, middle, and bottom conductors on a transmission line. That no fixed relation exists on the line under service lightning conditions is apparent from Tables VIII, VI-A and VI-B. An average of all voltages in Table VI-A and VI-B (the highest being taken where the surge was oscillatory) shows 1,165 kv. on the top, 1,060 kv. on the middle, and 965 kv. on the bottom conductor. These are 100 per cent, 91 per cent, and 82.6 per cent in terms of relative voltage. There are individual cases, however, where the voltages on the bottom and middle conductors are equal and considerably higher than on the top conductor.

A B

FIGS. 14A, 14B—DIRECT-STROKE RECORDER RECORDS AT TOWERS 9 AND 10

The conclusion drawn from these data is that over a period of time the top conductor will be subjected to an average voltage of 10 per cent higher than the middle or bottom conductor, and the middle conductor will be subjected to a voltage 10 per cent higher than the bottom conductor. Individual surges may give values just the reverse of this.

In the event of induced strokes the magnitudes of the voltages on the various conductors in vertical configuration would be expected to be almost equal with a ground wire since the amount of protection afforded by a ground wire against induced strokes practically equalizes the difference in the heights above ground. Referring to Table VIII, the first 13 surges were recorded before the ground wire was installed and the last four after the ground wire was installed. These records show that no particular change in the distribution of voltages occurred as a result of the ground wire. Actually the differences to be expected would not be easily detected with the klydonograph. In general the differences between the voltages on the three conductors for the lower valued surges were not great, which might be

taken either as an indication of induced strokes or a direct stroke to the ground wire which did not flash over. In the event of direct strokes to tower or ground wire, which cause flashover, a random distribution of voltages would be expected depending upon the number of conductors which flashed over. If a flashover occurred on only one conductor it would be just as likely to do so on the bottom conductor since the coupling between the various conductors and ground wire would be least

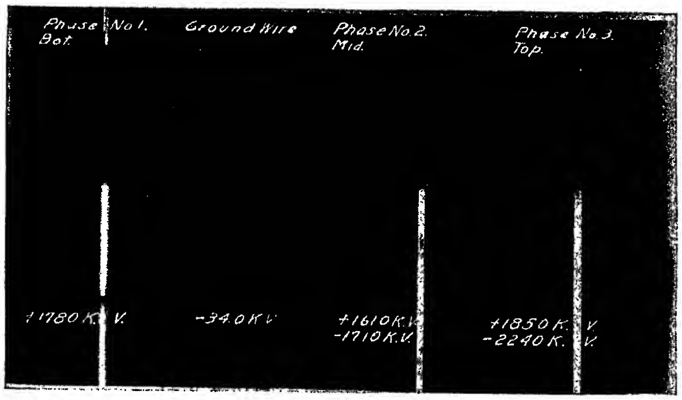

Fig. 15—Klydonograph Record at Tower 9 at the Time Records in Figs. 14a and 14b were Recorded

on the bottom conductor, due to its greater distance of separation, and therefore the potential difference between this conductor and the tower would be greatest. In case of a stroke to a conductor it might be expected to take place on the top conductor most often, as it is most exposed, and with no ground wires the top conductor should be subjected to the highest surges. The

TABLE V—SURGE CHARACTERISTICS OF NATURAL LIGHTNING
1929–1930

No.	Polarity	Crest kv.	Wave characteristics Front (1)	Tail (2)	Length (3)	Lightning
1(5)	+	185	2.0	6.5	10.5	Moderate
2(5)	+	180	5.0	15.0	35.0	Slight
3(5)	−	160	2.5	8.0	15.0	Moderate
4	−	70	2.0	5.0	7.0	Severe
5	−	75	Indefinite			Moderate
6	−	75	1.0	5.0	10.0	Moderate
7	+	75	Indefinite			Severe
8	−	100	Indefinite			Slight
9	+	100	15.0	55.0 +	70.0 +	Slight
10	+	110	2.0	80.0 +	80.0 +	Moderate
11	−	120	Indefinite			Slight
12	+	125	2.0	15.0	25.0	Severe
13	+	125	2.0	20.0	50.0	Severe
14	+	140	2.0	25.0	50.0	Severe
15	+	150	2.0	20.0	55.0	Slight
16	+	175	12.0	35.0	40.0	Severe
17	+	230	2.0	52.0	60.0	Severe
18	+	240	2.0	45.0	50.0	Severe
19	−	250	35.0	80.0 +	80.0 +	Slight
20	+	350(4)	2.0	8.0	80.0	Severe
21	±	1,000(4)	2.0	3.0	4.0	Severe

(1) Microseconds to crest.
(2) Microseconds to 50 per cent crest on tail.
(3) Microseconds to first zero.
(4) Line trip-out.
(5) 1929 reccrds.

records show that in a large percentage of the higher valued surges there was a wide variation between the voltages on the different conductors.

Cathode-Ray Oscillograms. During the two years the oscillograph was in operation, 21 oscillograms were obtained of natural lightning. The characteristics of these surges are shown in Table V. Excluding waves below 125 kv. in magnitude, also No. 20 and No. 21, due to line flashovers occurring at the time, this table may be summarized as follows:

NATURAL LIGHTNING VOLTAGE CHARACTERISTICS MICROSECONDS

	Front	to 50% tail	Length
Minimum	2.0	6.5	10.5
Maximum	35.0	80.0 +	80.0 +
Average	6.2	30.1	43.1

The oscillograms of lightning voltage obtained at Tower 20 on the top phase of the Turner-Logan circuit under test when line trip-outs occurred are shown in Figs. 16 and 17. Fig. 16 shows the largest surge recorded, which was of the order of 1,000 kv. It is oscillatory in nature, having a period of the order of ten microseconds, with higher frequency oscillations superimposed. This surge was accompanied by a flashover. Although it was not possible to fit these oscillations to readily apparent circuit constants, it is possible for reflection phenomena to be quite complicated, and the oscillations are believed to be due to reflections resulting from the line flashover rather than to oscillations in the

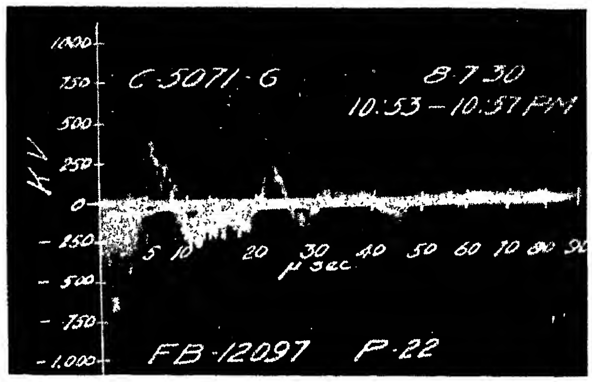

Fig. 16—Oscillogram of Lightning Surge at Tower 20 at the Time Figs. 14a and 14b and Fig. 15 were Recorded

lightning stroke itself. The superimposed high frequency oscillations are probably due to reflections of the wave between tower top and ground.

The klydonograph record at Tower 20 correlating with this surge indicated voltages of + 685 and − 625 kv. on the top phase, + 685 and − 625 kv. on the middle phase, and + 610 kv. on the bottom phase. Taking into consideration the 60-cycle potentials which influence klydonograph records, but were eliminated in the oscillograph circuit, the voltages from the cathode-

ray oscillograph record (+ 600 and − 1,000 kv.) check reasonably well the klydonograph data.

The correlating klydonograph record at Tower 9, Fig. 15, which is 3.07 miles from Tower 20, is interesting in indicating negative impulse voltages with a very fast front on the ground wire and top conductor, with a slower front negative on the middle wire, and no appreciable negative on the bottom conductor, and also high positive voltages on all line conductors.

The polarity analysis of the surges recorded by the oscillograph shows 13 positives, 7 negatives, and one oscillatory.

Records of Special Interest

a. Arcing Grounds. All of the voltages recorded by the klydonograph were either unidirectional or highly damped oscillations with the exception of one record which showed a sustained oscillation similar to those

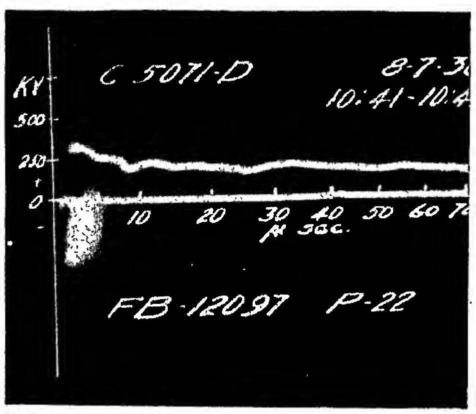

Fig. 17—Lightning Surge Recorded at Tower 20

recorded on isolated neutral systems. Upon investigation it was found that at the time this record was obtained, the system connections were such that the line to which the klydonograph was connected was operating momentarily ungrounded. The voltages recorded at the time were 315 kv. (3 times normal) on two phases, and 215 kv. (2 times normal) on the other phase on which the arc was later located.

b. Bushing Flashover-Klydonograph Record. Klydonographs registered a potential of 860 kv. across the high- and low-tension bushings of a transformer during a lightning storm at Glen Lyn in 1927, which checked with the air line distance at which breakdown would be expected to take place. The occurrence was attributed to a direct stroke of lightning to the 88-kv. system near a point where the 88-kv. and 132-kv. systems crossed. It is believed a negative direct stroke to the 88-kv. line transmitted a voltage to the transformer which was registered as − 650 kv. on the 88 kv. bushing; and the induced voltage on the 132-kv. line entered the station at the same time where it was recorded at + 210 kv., giving a total voltage of 860 kv.

c. Lightning Voltages and Trip-Outs. Surges recorded above 600 kv., which caused line trip-outs are listed in Table VI-A; those above 600 kv., which did not cause line trip-outs are listed in Table VI-B. As would be expected, many of the surges in Table VI-A are considerably above the 60-cycle crest flashover of the line insulation (about 600 kv. crest), indicating that some flashovers occurred on the front of the wave. The voltages listed which are near the 60-cycle crest flashover of the line are in some cases, at least, attenuated values of surges originating at some distance from the point of measurement.

TABLE VI-A—LIGHTNING SURGES ABOVE 600 KV. ON 132-KV. LINES RECORDED ON LINE WIRE AND CAUSING LINE TRIP-OUT

	Kv.		
Ref.	Top	Mid.	Bot.
8	+ 635	+ 575	+ 600
13	− 1800	− 1500 + 980	+ 650 − 620
14	− 1080	− 1020	− 580 + 375
15	− 735	− 625	− 390 + 190
16	− 2900	− 2900	−1,600 + 680
23	− 890	± 240	− 970
28	+ 236	− 1010	−1,010
29	−2,500	± 300	± 300
36	+1,850 −2,240	+1,610 −1,710	+1,780
37	+ 320 − 525	+ 470 − 308	+ 750 −1,470
44	+ 705 − 820	+ 256	+ 770 − 525
46	+ 610 − 350	+ 550 − 615	+ 480
47	+ 685 − 625	+ 685 − 625	+ 610

TABLE VI-B—LIGHTNING VOLTAGES ABOVE 600 KV. NOT CAUSING LINE TRIP-OUT ON 132-KV. SYSTEM

Ref.	Top ϕ	Mid. ϕ	Bot. ϕ	Grd. wire	Where recorded
2(1)	− 710	− 212	− 238		Substa.
10	− 700	± 300	+ 300		Substa.
12	+1400	+1900	+1100		Line
20	±1800	±1290	(2)	−25	Line
22	+1710	+1710	+1710	+30	Line
24	−1800	−2400	−2400	−30	Line
40	+ 640	−1100	+ 825		Substa.
41(1)	± 345		± 640		Subtsa.
42	− 805	− 745	− 925		Substa.
19(3)	−1190	−1695	−1695	−35	Line

Notes: (1) On 88-kv. system.
(2) Not recorded.
(3) Line not energized.

A factor which might account, in some cases, for recorded surge voltages considerably above line flashover voltages without a corresponding trip-out is that when a direct hit to the tower occurs, the potential of the tower is altered with respect to ground due to the surge impedance of the tower and tower footing resistance. The voltage across the insulator strings, being the difference between the potential of the line conductor and tower top, will not, therefore, be the same as the potential of the conductor to ground.

Applying this consideration to Table VI-B fails, however, to explain an absence of line trip-out for all cases listed, as flashover in all probability occurred for the higher valued records such as surges, references 12, 20 and 24.

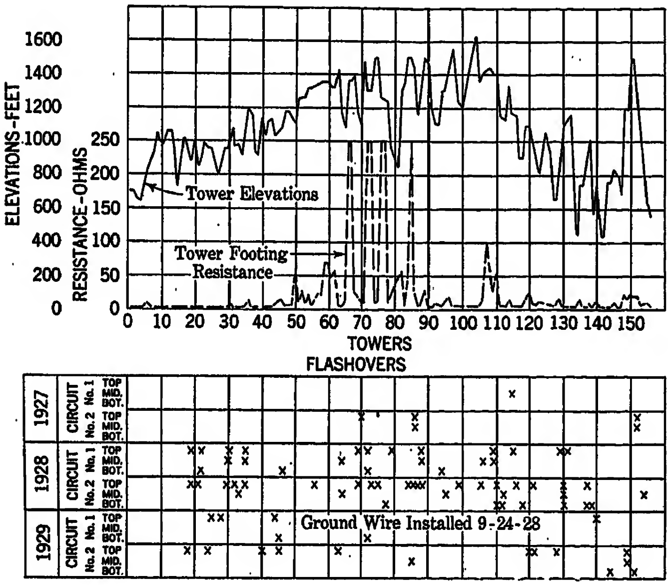

FIG. 18A—LINE FLASHOVER RECORD OF TURNER-LOGAN LINE

of natural lightning waves on the line gave a limited amount of data. These data were obtained on the Turner-Logan line under two conditions, first, before the ground wire was installed; and second, after it was installed. Figs. 12 and 13 show the attenuation of the crest values of the lightning voltages over about five miles of line.

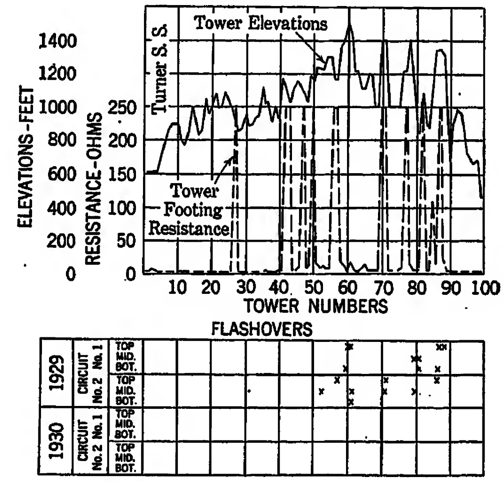

FIG. 18C—LINE FLASHOVER RECORD OF TURNER-CABIN CREEK LINE

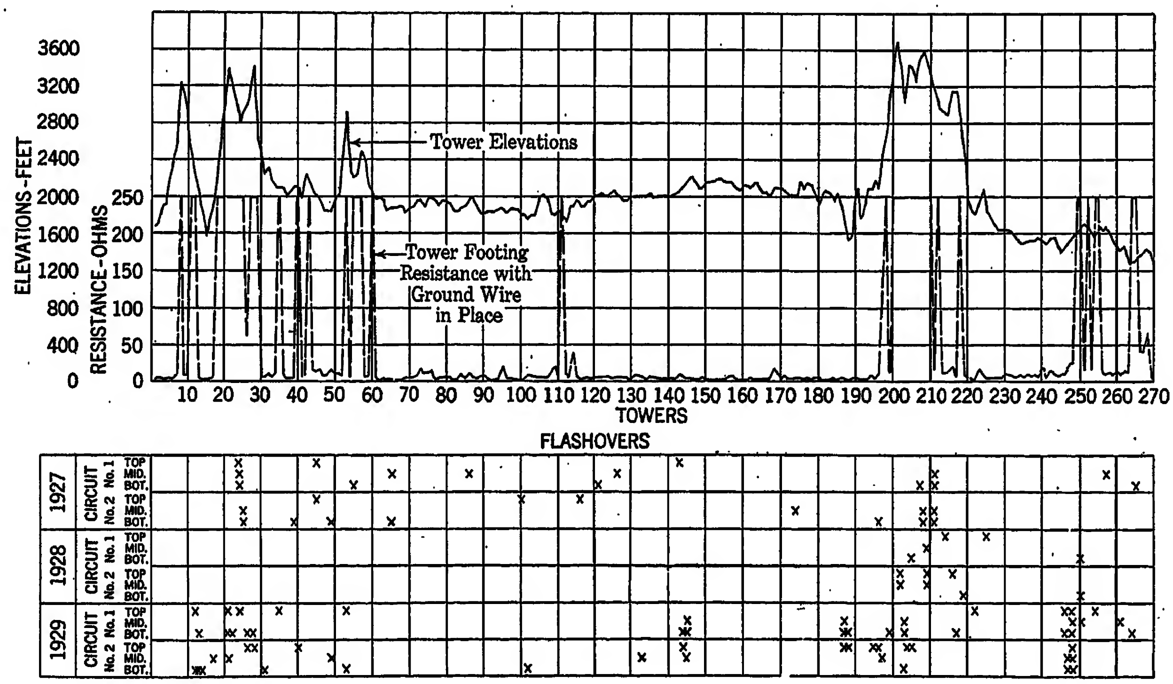

FIG. 18B—LINE FLASHOVER RECORD OF GLENLYN-ROANOKE LINE

An important fact brought out by these data is that voltage surges well above the flashover value of the line occur without causing line outages and the corollary follows that lightning impulse flashover on insulators is not necessarily accompanied by power follow-up.

d. Attenuation of Lightning Surges—Klydonograph Records. The attenuation study with the klydonograph

Two characteristics of attenuation are brought out by these data, so far as the accuracy of the klydonograph in this type of analysis permits. One is that the attenuation is greater for positive lightning voltage surges than for negative surges; and the other is that the attenuation is greater when there is no ground wire. It shows definitely that the attenuation for high-voltage

surges is very rapid, the maximum crest values in the order of 2,000 kv., being reduced to one-half value within about a mile. Considering the influence which the wave steepness and length has on attenuation and the indefiniteness of these values of the waves here recorded, these curves agree quite closely with those obtained in artificial surge investigations. Where the line was not equipped with a ground wire, and the surge was negative (surge 233), the attenuation appears less rapid at the start. The data from one surge only, however, are too meager to justify great confidence in the particular shape of the curve.

The data obtained on attenuation of natural lightning voltage surges show that klydonographs must be placed at frequency intervals along the line (one mile or so apart) if any quantity of information is to be obtained. It is also clear from the curves in Figs. 12 and 13 that the klydonograph data can only record the general order of attenuation and are accurate enough to indicate the trend, but not to study the effects produced by the presence of ground wires, positive and negative surges, etc., where the relative voltages to be measured may vary some 20 per cent or less.

TABLE VII—SURGES ON GROUND WIRE

Ref.	Date	Time	Tower location	Surge kv. Ground wire	Top	Mid.	Bot.	Cause	Line T. O.
1	7/16/27	6:27 P. M.	4 TR	+ 2.3		± 285	± 240	Ltn.	No
2	8/ 1/27	10:20 "	4 TR	+ 9.6	∓ 520	± 285	± 230	"	Yes
3	8/ 1/27	10:20 "	4 TR	+ 9.0	+ 285	+ 220	+ 270	"	Yes
4	8/ 1/27	10:31 "	4 TR	+ 5.0	+ 190	+ 230		"	No
5	8/ 6/27	3:30 "	4 TR	− 4.7	+ 180	+ 255	+ 210	"	No
6	8/ 8/27	5:30 "	4 TR	+14.8	+ 340	+ 515	+ 240	"	No
7	9/18/27	10:52 "	4 TR	+20.0	± 585	± 285	+ 285	"	Yes
8	6/ 9/28	2:55 "	14 TL	+ 4.0*	+ 435	+ 435	+ 340	"	Yes
9	6/16/28	5:30 "	14 TL	− 6.2*	−1080	− 920	− 490 + 270	"	Yes
10	6/19/28	9:30 "	14 TL	+ 3.3*	± 180	+ 340	+ 230	"	No
11	6/20/28	3:50 "	14 TL	+ 3.8*	+ 270	+ 300	+ 180	"	No
12	8/21/28	6:10 "	5 TR	+ 6.8	+ 230	+ 160	+ 160	"	No
†13	8/24/28	1:00 "	14 TL	−35 *	−1190	−1695	−1695	"	No
14	6/18/29	1:10 "	5 TL	+20		+ 245	+ 190	"	Yes
15	6/18/29	1:10 "	9 TL	+10	+ 245	+ 250	+ 200	"	Yes
16	6/22/29	4:35 "	5 TL	+ 7.5		+ 250	+ 300	"	No
17	6/22/29	4:35 "	9 TL	+ 5.7	+ 260	+ 275	+ 215	"	No
18	7/ 1/29	4:30 "	20 TL	−20	+ 250	+ 340	+ 250	"	No
19	7/ 1/29	4:30 "	30 TL	+ 3.0	+ 230	+ 250	+ 230	"	No
20	7/ 1/29	4:30 "	36 TL	+ 2.7				"	No
21	7/ 1/29	4:50 "	36 TL	+ 3.0		+ 295		"	Yes
22	7/ 9/29	4:15 "	30 TL	−30	−1800	−2400	−2400	"	No
23	7/ 9/29	4:15 "	36 TL	+ 3.0	− 360 + 250	− 360 + 270	− 220 + 180	"	No
24	8/27/29	7:30 "	24 TL	+ 4.5	+ 275	+ 190	+ 225	"	Yes
25	8/27/29	7:30 "	30 TL	+ 4.	+ 255	+ 250	+ 185	"	Yes
26	8/27/29	7:40 P. M	14 TL	+ 4.	+ 240	+ 300	+ 250	Ltn.	Yes
27	8/27/29	7:40 "	16 TL	± 4	+ 110	+ 110	+ 110	"	Yes
28	8/27/29	7:40 "	24 TL	+ 4.0	+ 250	+ 190	+ 225	"	Yes
29	8/27/29	7:40 "	30 TL	+ 3.6	+ 230	+ 225	+ 210	"	Yes
30	8/27/29	7:40 "	36 TL	+ 4.5	+ 255	+ 355	+ 240	"	Yes
31	8/ 7/30	11:00 "	5 TL	+ 6.2	±1030	± 486	+ 352	"	Yes
32	8/ 7/30	11:00 "	9 TL	−35	±2175	±1665	+1920	"	Yes
33	8/ 7/30	11:00 "	14 TL	±10	± 985	+ 265	± 770	"	Yes
34	8/ 7/30	11:00 "	20 TL	± 4	+ 600	± 775	+ 520	"	Yes
35	8/ 7/30	11:00 "	30 TL	+ 6.2	+ 250	+ 375	+ 270	"	Yes
36	5/30/29	4:00 "	21 GL-R	25	1420	1420	1420	"	Yes
37	5/31/29	3:30 "	82 GL-R	± 5.4	+ 490	+ 245	+ 220	"	Yes
38	6/18/29	10:30 "	21 GLR	+30	+1710	+1710	+1710	"	No
39	7/10/29	5:00 "	82 GLR	−35	+ 255	+ 325		"	No
40	8/22/29	6:20 "	42 GLR	+14.8	+ 220	+ 215	+ 215	"	Yes
41	8/22/29	6:20 "	162 GLR	−30	± 236	±1010	±1010	"	Yes
42	8/21/28	4:40 "	8 TCC	+ 9	+ 265	+ 315	+ 215	"	No
43	8/29/28	5:00 "	‡68 TCC	−25	±1290	±1800		"	No
44	6/22/29	4:35 "	8 TCC	+ 7.3	+ 250	+ 350	+ 295	"	No
45	6/22/29	8:30 "	89 TCC	+ 3.7				"	No
46	8/23/29	4:55 A. M.	‡68 TCC	− 3.3	± 525	± 525	± 485	"	Yes
47	8/23/29	4:55 "	89 TCC	+15	−2520	± 300	± 300	"	Yes
48	9/ 7/29	12:30 "	30 TCC	+ 8.2	+ 275	+ 300	+ 230	"	No
49	9/24/30	5:20 P. M.	9 TL	−12.1	+ 450	+ 455		"	No

†Line not alive at this time, being out of service to install ground wire.
*Measured at peak of tower—no ground wire.
‡Horizontal tower.

TR—Turner-Rutland Line.
TL—Turner-Logan Line.
GLR—Glenlyn-Roanoke Line.
TCC—Turner-Cabin Creek Line

e. Surges on the Ground Wire. Voltages recorded on the ground wire by the fourth electrode of the klydonograph with the corresponding voltages recorded on the line conductors are listed in Table VII. It is not believed these records are accurate as to voltage magni-

tude, as the klydonograph which was located near the ground was connected to the ground wire by a vertical wire about 100 ft. long extending up the side of the tower. However, they are interesting in three respects. First, they are recorded only at times of lightning storms. Second, of the 47 surges, 33 are positive, 10 negative, and 4 oscillatory. Third, although there is a tendency for the surge on the ground wire to have the same polarity as the surges on the conductors, there is a number of exceptions where the conductor surges are oscillatory with a unidirectional surge on the ground wire, and again where the ground wire and conductor surges are of opposite polarity. This condition obtains both when a line did or did not trip out.

f. Pure Lightning Surges. When the Turner-Logan line was out of service in August and September 1928, for installation of the ground wire, the klydonographs recorded 17 pure lightning surges as recorded in Table VIII. Only one of these, recording − 1,190 kv., − 1,695 kv., and − 1,695 kv. on top, middle, and bottom conductors, respectively, and − 35 kv. at the peak of the tower, was relatively high in value. This surge was negative; and all others were positive, the highest positive being 385 kv.

TABLE VIII—PURE LIGHTNING SURGES ON TURNER-LOGAN LINE
8-1-28 to 9-24-28
(Ground wire being installed)

Ground Wire	Ref.	Date	Time	Location	Surge kv. Top	Mid.	Bot.	Cause
Not Installed	1	8- 1-28	7:20 P. M.	9	+ 240	+ 220	+ 200	Lightning
	2	8- 1-28	7:20 "	14	+ 200	+ 220	+ 200	"
	3	8- 1-28	7:20 "	20	+ 150	+ 200		"
	4	8- 1-28	7:40 "	9	+ 300	+ 300	+ 270	"
	5	8- 1-28	7:40 "	14	+ 220	+ 270	+ 240	"
	6	8- 1-28	8:00 "	9	+ 270	+ 240	+ 220	"
	7	8- 1-28	8:00 "	14	+ 180	+ 200	+ 200	"
	8	8- 4-28	12:0[illegible] "	9	+ 240	+ 240	+ 215	"
	9	8- 4-28	12:20 "	14	+ 255	+ 265	+ 240	"
	10	8- 4-28	12:15 "	20	+ 200	+ 220	+ 200	"
	11	8- 4-28	5:45 "	9	+ 180	+ 180	+ 180	"
	12	8- 4-28	5:45 "	14	+ 155	+ 205	+ 155	"
	13	8- 4-28	5:45 "	20	+ 110	+ 110	+ 110	"
Installed	14	8-21-28	5:10 "	9	+ 385	+ 300	+ 255	"
	15	8-21-28	4:45 "	14	+ 225	+ 265	+ 245	"
	16	8-21-28	4:45 "	20	+ 215	+ 215	+ 200	"
	17	8-24-28	1:00 "	14	−1.190	−1 695	−1,695	"
Average					277	314	302	
Per cent					100	113	109	

Except for the high negative voltage surge recorded, it is remarkable to note that the voltages on top, middle, and bottom conductors are nearly equal on all three conductors, there being no tendency for higher voltages to be recorded on the top phase. The average for all 17 surges shows 17 per cent higher voltage on the middle conductor than on the top, and 9 per cent higher on the bottom conductor.

Assuming, since they were of relatively low magnitude, that the positive surges might have been caused by induced lightning strokes, it appears that the induced voltages are approximately equal in magnitude on all three phases when in vertical configuration, whether the ground wire is installed or not.

g. Direct-Stroke Recorder Records. Two records were obtained on direct-stroke recorders, both being associated with one storm, and believed to be due to one lightning stroke on the Turner-Logan line. Simultaneous records were obtained on the klydonographs between Turner substation and Tower 36. The klydonogram recorded at Tower 9 is shown in Fig. 15. Also, a simultaneous record was obtained on the cathode-ray oscillograph at Tower 20, three miles distant. This record is reproduced in Fig. 16A.

The maximum voltages recorded on the line conductors of this circuit at Tower 9 were − 2,200 and + 1,850 kv., − 1,700 and + 1,610 kv., and + 1,780 kv. on the top, middle, and bottom conductors, respectively. The voltage recorded on the ground wire was − 34 kv. This low voltage was discussed previously under "Surges on the Ground Wire."

The direct-stroke records, Figs. 14A and B, indicate currents of − 46,000 amperes in Tower 9 and − 97,000 in Tower 10. No current was recorded in Tower 8, and the recorders were not in operation at Towers 7 and 11. The circuit tripped out three times within twenty minutes during this storm, the relay targets of the line under test indicating on the middle and bottom conductor in the first case; on the top conductor in the second case; and on the top and bottom conductors in the third case. The only flashover which could be located on this section of the line, by an inspection after the storm, was on the top phase at Tower 8 of the circuit under test. It should be pointed out that the above currents were those measured in the tower footing. Since the current in a lightning stroke on hitting a tower

TABLE IX—WAVE-FRONT DETERMINATION OF LIGHTNING SURGES FROM KLYDONOGRAPH RECORDS. 21 SELECTED SURGES OF 600 KV. UP

Ref.	Location	Polarity and microsecond fronts: Top	Mid.	Bot.	Gr. wire
2	Glenlyn 88 kv	−Fast*			
4	Glenlyn 88 kv	−Fast*			
8	Tower 4-T-R	+Slow	+Slow	+Slow	
10	Turner bus	−5	+Slow	+Slow	
12	Tower 9-T-L	+5	+5 −5	+5	
13	Tower 20-T-L	−Fast	+Slow	+Slow	
14	Tower 14-T-L	−5	−5	±5	−5
15	Tower 9-T-L	−5	−5	±5	
16	Tower 20-T-L	−Fast*	−Fast	±5	
19	Tower 14-T-L	−Fast*	−Fast	−Fast	−Fast
20	Tower 68-T-CC	−Fast*	−5		−Fast
23	Kingsport 132 kv	−5		−5	
24	Tower 30-T-L	−Fast*	−Fast	−Fast	−Fast
26	Kingsport 132 kv	−Fast*	−Fast	−Fast	
28	Tower 162-GLR	−5	−Fast	−Fast	−Fast
36	Tower 9-T-L	±Fast*	±5	+5	−Fast
37	Tower 5-T-L	±5	±5	±5	±5
38	Switchback 132 kv	+5	+5	+5	
39	Glenlyn 88 kv	+5	+5	+5	
40	Saltville 132 kv	+5 +	+5 +	+5 +	
42	Switchback	−5	−5	−5	
	Of 21 surges	9 Fast* 11-5 μ sec. 1 Slow	5 Fast* 11-5 μ sec. 3 Slow	4 Fast* 11-5 μ sec. 3 Slow	5 Fast* 2-5 μ sec.

*Less than one microsecond.

divides into multiple paths consisting of the tower, the ground wire in both directions, and such line conductors as become involved, and since the stroke current is further modified by the reflected wave from the ground when it reaches the tower top, the maximum current in the stroke probably is not the same as the ground current. Calculations for typical cases indicate that the latter is of the order of 120 per cent of the maximum stroke current.

Conclusions

1. In general the conclusions to be drawn from these tests are similar to those in previously published publications but contain quite a number of additional points of considerable interest.

2. The lightning voltages recorded on the line reached magnitudes of 2,500 kv. Ten per cent of the total recorded were above 750 kv. Reduced to a basis of surges per 100 miles of line per year there were nine surges above 1,060 kv.

3. At substations lightning voltages reached a magnitude of 1,200 kv. Reduced to a yearly basis there were 0.2 surges per substation per year above 1,060 kv. Comparing this with the above paragraph it seems that a substation is equivalent to 2.2 miles of line in being subjected to lightning surges of this magnitude.

4. Lightning surges of the order of 2,500 kv. have been found on a line without trip-out, which, with the insulation values used, indicates clearly impulse flash-over of the line without power follow-up.

5. Switching surges as high as six times normal voltage from line to neutral at substations and four times normal on the line were recorded, which confirms past data.

6. Except at times of lightning occurrence impulse voltages on these transmission lines throughout the year are of minor importance, not exceeding 3.5 times normal.

7. Attenuation of lightning surges is rapid for high voltages, attenuating from 2,000 kv. to 1,000 kv. in one mile. Keeping in mind the limited accuracy of the klydonograph the records indicate that the attenuation of lightning surges is more rapid with positive than with negative voltages and also it is more rapid where the line is not equipped with a ground wire.

8. Lightning arresters must be placed close to the equipment they are intended to protect. Voltages some 60 to 80 per cent higher than the value to which the arrester should hold the voltage were recorded some 150 ft. distant from the arrester.

9. The beneficial effect of the ground wire in reducing the lightning voltages is indicated by the fact that six surges above 700 kv. were recorded during a period with no ground wire and only two above 700 kv. during a similar period when one ground wire is used.

10. The maximum current recorded in a tower struck by lightning was in the order of 100,000 amperes which calculations indicate to be of the order of 120 per cent of that in the lightning stroke. The records indicate a current of 50,000 amperes in one adjacent tower and calculations indicate that the voltage may have been sufficient to flash over the line conductors on a total of five towers. Calculations also indicate that the voltage of the tower top struck was of the order of 4,000 kv. The voltage measured at the oscillograph station three and one-half miles distant was one million kv.

11. The characteristics of natural lightning on transmission line investigated with the cathode-ray oscillograph show minimum and average fronts of 2.0 and 6.2 microsecond fronts; 6.5 and 30.1 microseconds to 50 per cent on the tail; and 10.5 and 43.1 microseconds total lengths. A maximum front of 35.0 and total length of 80.0 microseconds were observed. None of these surges so far as is known was recorded near the point of origin. The majority of the waves were positive but the highest recorded was negative.

12. An arcing ground producing oscillatory voltages of three times normal to ground was observed in one case at a time when the system was operating ungrounded. This shows a type of voltage that is dangerous; but it was not as high as theoretical analysis shows as the maximum.

13. Wave fronts of natural lightning recorded by the klydonograph apparently near the point of origin have indicated that negative surges have wave fronts of one microsecond or less. Positive lightning surge records indicated that the fastest fronts were in the order of five microseconds. This indicates that nega-

tive lightning voltage surges initially appear on transmission lines with steeper fronts than positive waves.

14. The relative lightning voltages on top, middle, and bottom conductors of a vertical configuration circuit for voltages over 600 kv. (approximately 60-cycle crest flashover) appear as a matter of chance. Cases were observed (Tables VI-A and B) where the highest voltage appears on top, middle, or bottom conductor and also on combinations of any two conductors. In general there was less variation between the voltages on different conductors in the case of lower voltage surges. It seems reasonable to assume that where a surge produced approximately equal voltages on the three conductors, and where no line trip-out occurred, that the surge was caused by an induced stroke and that where the relative voltages on the three conductors vary widely the surge was due to a direct stroke.

15. The records of high voltages on the line at times of line trip-out (Table VI-A) indicate 60 per cent of these are predominantly negative and 40 per cent positive. From a study of flashover locations and tower footing resistance no relation could be found between tower footing resistance and line flashover on this line.

16. Surge voltages were observed on the ground wire only at times of lightning storms. Of these records 70 per cent were positive, 21.5 per cent negative and 8.5 per cent oscillatory. The oscillatory records are believed to be due to reflections on the line rather than oscillations in the lightning stroke.

17. Lightning voltage on a deenergized line (Table VIII) indicates positive voltages of approximately equal magnitude on all three phases regardless of whether the ground wire is installed or not. The approximate equal and low magnitude of the voltages on the three conductors seems to indicate that these were caused by induced strokes. It should also be pointed out that all of these voltages were positive. No high or negative surges were recorded during the period when the ground wire was not installed.

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Discussion

For discussion of this paper see page 1146.

Experimental Studies in the Propagation of Lightning Surges on Transmission Lines

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Synopsis.—*A discussion is given of cathode-ray oscillograms showing the attenuation and distortion of artificial lightning surges on a power transmission line, the voltages induced by such surges in parallel wires and the effect on tower footing impedance of earth wires or counterpoises.*

* * * * *

Introduction

The study of the laws of propagation of electromagnetic waves along parallel conductors has become of increasing interest to the power engineer, since it was realized that these waves play an important part in lightning disturbances on power transmission systems.

Electromagnetic wave propagation has received considerable theoretical treatment at the hands of the telephone engineer[1] and exact formulas for attenuation and distortion have been derived under certain simplifying conditions. The conditions existing in lightning surges however, differ in several important respects from those encountered in electrical communication in that the circuit parameters (inductance, resistance, capacitance, and leakance) cannot be treated as constants, but must be considered as variable. As mathematical analysis consequently becomes very complicated, the experimental approach to the problem seems particularly desirable.

The present paper presents some of the results of experimental work which has been carried out during the last two summers on the S.19 line of the Consumers Power Company between Croton Dam and Grand Rapids, Michigan. The work was done by the Research Section of the Lightning Arrester Engineering Department, General Electric Company, Pittsfield, Mass., and the Production and Transmission Departments of the Consumers Power Company of Michigan.

Previous papers have reported on different aspects of this work and described the equipment used.[7,9,10] High-voltage surges were applied to the transmission line by means of an impulse generator generating one and one-half million volts on open circuit. The surges were measured at any desired point on the line by means of a portable cathode-ray oscillograph which was connected to the line through a resistance potentiometer.

I. Attenuation and Distortion

Choice of Wave Shapes. It had been shown by sphere-gap tests[7] that the rate of attenuation of a surge was affected by its voltage, polarity, wave-shape, and its distribution over one or more conductors. Accordingly, certain test waves were chosen to bring out these features in particular.

A selection of the oscillograms obtained is shown in Fig. 1. For more ready comparison, these have been replotted to a uniform time and voltage scale in Fig. 2.

Corona. The first thing which strikes one in examining these oscillograms is the clearly defined point on the front of the wave at which the effect of corona becomes evident on the high-voltage surges; it is also evident that the wave is attenuated much more rapidly above corona voltage than below it. (The calculated value of critical corona voltage is 85 kv. for a surge on one conductor, 125 kv. for a surge on three conductors in parallel.)

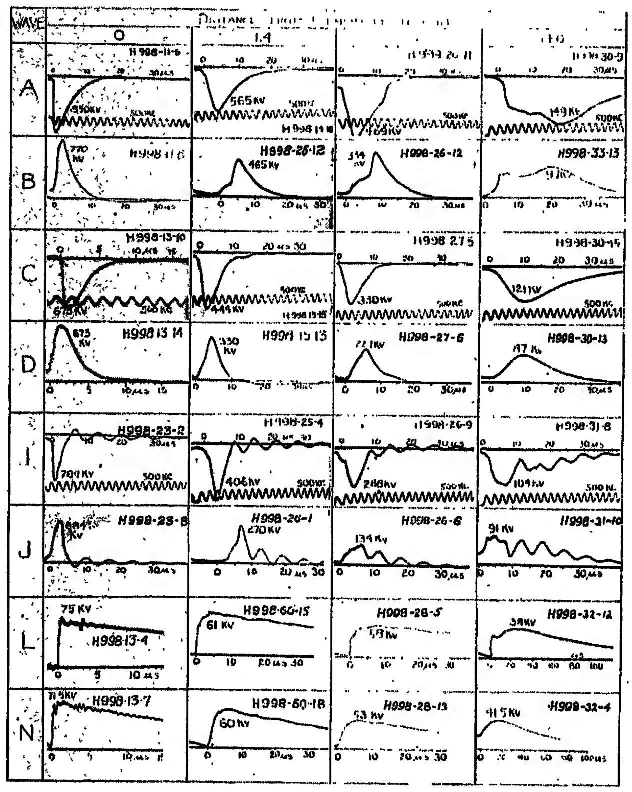

Fig. 1—Selected Attenuation Oscillograms

Resistance. It can also be seen however, that there

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1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

are other influences besides corona affecting the surge. This is most clearly brought out by the oscillograms of low-voltage surges in which the voltage is too low for corona to play any appreciable part (there will always be a little corona on sharp points of insulator fittings, etc.).

On these surges we see the type of change to be expected from series resistance in the line. A physical picture of this type of attenuation and distortion can be obtained in the following way:

Consider a rectangular wave of finite length. This is associated with a fixed amount of electric charge on the line. Assuming no leakance in the line, the amount of this charge remains constant although its distribution on the line may change. Then imagine that the resistance instead of being distributed is lumped at regular intervals. At every point at which the surge strikes such a lumped resistance, one part of the wave will be transmitted as a rectangular wave, while the remaining part will be reflected. The front of the wave will therefore undergo a continuous attenuation. The reflected portion will however, again be reflected on its way back so that ultimately it contributes an increase of voltage in the tail of the forward-going wave. If the resistance is distributed instead of lumped, there will not be separate waves but the distortion of a single wave shown in Fig. 3a. The very front of the wave will suffer the greatest attenuation; succeeding parts will be attenuated less and less and the whole wave will be lengthened in such a way that although the voltage of the wave is decreased, the total charge still remains constant. Such a change may be noticed on the oscillograms of the low-voltage waves.

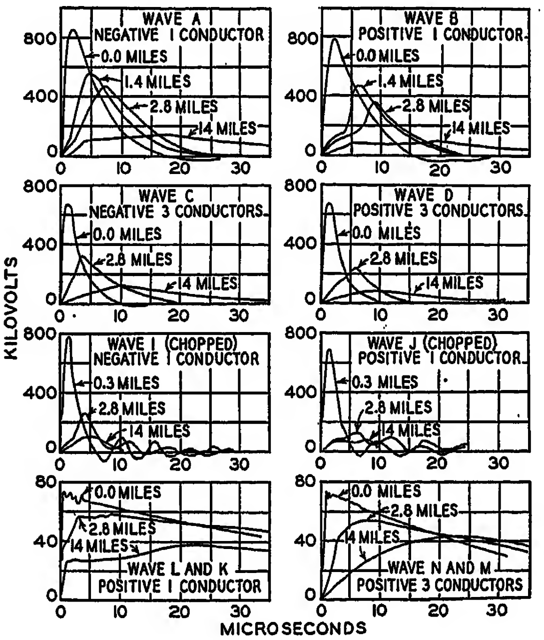

Fig. 2—Replot of Oscillograms to Show Change of Wave Shape

It will also be noticed that the front of the low-voltage waves becomes less and less steep as the surge proceeds along the line. A constant series resistance in the line leaves the perpendicular wave front unchanged; but a constant resistance is not physically possible with a perpendicular front on account of skin-effect which would make the resistance infinite for such a front. This effect makes the resistance higher, the steeper the slope of the wave, and so has the effect of decreasing the slope of a very steep wave. The effect on wave shape as calculated theoretically[3] is shown in Fig. 3b. These effects are all observable in the oscillograms of waves L and N (Fig. 1).

A striking fact is that for a surge on three conductors the slope of front decreases more rapidly than for a surge on one conductor. A possible explanation of this is that the higher concentration of current in the earth in the case of three conductors increases the effective ground resistance. Another cause may be found in the mutual effect between conductors. This effect will be discussed more fully in Section II of this paper.

Distortion. Returning to the case of the high-voltage surges, it will be seen that in these also the maximum voltage is decreased while the whole wave is increased in length. Frequent attempts have been made to calculate the effect of corona by assuming an equivalent leakance, suggested by the fact that the power loss for steady alternating voltages is approximately proportional to the square of the voltage. It is possible however, to get a physical picture of the effect of leakance in the same way as was done for series resistance, and it will be found that attenuation in this case is accompanied by a distortion which decreases the length of the wave. (In the so-called distortionless line of telephone theory, resistance and leakance balance

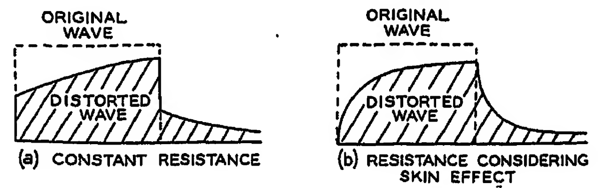

Fig. 3—Attenuation and Distortion Produced by Resistance (Theoretical)

each other in such a way that the distortion produced by one is exactly equal and opposite to the distortion produced by the other.) This is not the effect of corona observed experimentally. Closer consideration will show that the conception of corona loss as a leakance is erroneous, since the corona does not, except at very high voltages near sparkover, establish a conducting path by means of which charges may leak off the line. Usually the area surrounding a conductor becomes conducting only within an envelope of limited radius. As the voltage rises above corona voltage, charges enter this envelope thereby decreasing the voltage on the

front of the wave. The charge which enters this envelope will remain there as long as the voltage of the conductor is high; when it falls the charge will return to the conductor, thus holding up the voltage on the tail of the wave. In this exchange of charge between the conductor and the corona envelope *energy* is lost, but no *charge*.[11]

The process has some similarity with the discharge of one condenser into another through a resistance, the difference being that on the transmission line the quantities are distributed instead of lumped and the magnitude of these quantities depends upon the voltage.

Attenuation. In Fig. 4 are shown curves of maximum voltage plotted against distance of travel of selected surges. These curves may be called attenuation curves, but in fact they involve both attenuation and distortion unless we define attenuation in a very

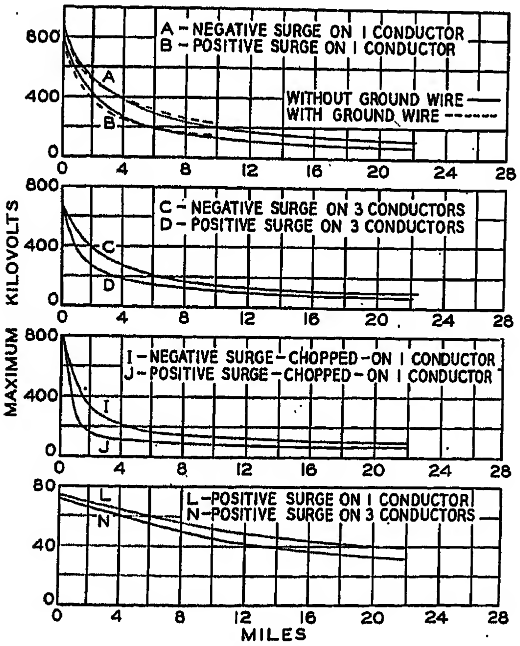

FIG. 4—CHANGE OF MAXIMUM VOLTAGE FOR SELECTED CASES

special way. The difficulty arises from the fact that the wave shape of the surge is continually changing, and as we have seen, the rate of decrease of maximum voltage depends on the wave shape.

It is however, possible to get a great deal of information from the curves in Fig. 4. They bring out very clearly the fact that positive surges are attenuated (and distorted) more rapidly than negative surges by corona. They show also that short waves are attenuated more rapidly than long waves (compare *e. g.*, waves *I* and *J* with waves *A* and *B*).

Surge on Three Conductors. A comparison of the rate of attenuation of a surge on three conductors with that of a surge on one conductor is not immediately possible because the wave shape of surge *C*, for example, is not the same as that of surge *A*. It is much shorter and would therefore be expected to attenuate more rapidly than *A*, other things being equal. A better comparison is obtained by considering surges *C* and *I* which are of approximately the same length at a voltage of about 600 kv. Considering the "attenuation" curves of Fig. 4 for these waves, it is seen that surge *C* attenuates more slowly than surge *I*; the same is true of surges *D* and J. Thus we are led to the conclusion that if the voltage and wave shapes are the same, a surge on three conductors will attenuate more slowly than the corresponding surge on one conductor. This is in accord with the fact that, if the same voltage is produced by a charge on a single conductor as by charge distributed on three conductors, the voltage gradient will be greater around the single conductor than around each of the three conductors; therefore corona would appear more readily in the former case than in the latter.

Ground Wires. The dotted curves in Fig. 4 (waves *A* and *B*) show the attenuation of these waves when one of the remaining conductors was grounded at frequent intervals along the line. The differences between this case and that without ground wire are slight, but the indications are that the ground wire increases the rate of attenuation at very high voltages but decreases it at the lower voltages. At very high voltages corona would appear on both the line and the ground wire, while at lower voltages corona will not appear on the ground wire, and the only effect of the ground wire would be to decrease the effect of ground resistance.

This is in agreement with the results obtained in 1929 where, because of the low voltages used, only the latter effect was noticed.

II. SURGE VOLTAGES INDUCED IN PARALLEL CONDUCTORS

The theory of induced surges in parallel conductors was very early developed by K. W. Wagner[4] for the case of the non-dissipative line. Wagner's formulation has been used by many authors, principally in the more recent theory of ground wires.[5] Briefly, this theory leads to a set of simultaneous linear equations expressing the surge voltage on any conductor in terms of the surge currents on all the conductors. (The usual convention is observed of regarding all voltages as being composed of two traveling waves proceeding in opposite directions.) The coefficients in these equations are known as the self and mutual surge impedances of the conductors.

Because of the linear relationships in these equations an induced surge will have exactly the same wave shape as the inducing surge. An isolated conductor will have a potential due to its position in the field but no current along the conductor.

During the summer of 1929 sphere-gap measurements of induced voltages on an isolated conductor were made and it was found that the ratio of induced to inducing voltage increased as the surge proceeded along the line. Since there was considerable attenuation, it was natural

to expect that the theory of the non-dissipative line would not hold accurately.

Further light is thrown on this phenomenon by the oscillograms now available. A selection from these is shown in Fig. 5. In these oscillograms it will be seen that the wave shape of the induced surge is appreciably

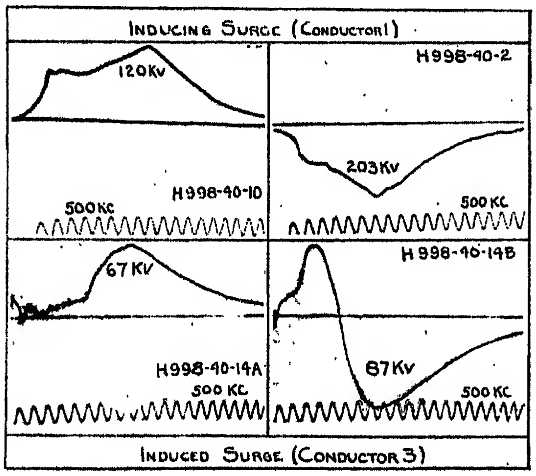

FIG. 5—OSCILLOGRAMS OF INDUCED VOLTAGE AT TOWER 93

different from that of the inducing surge. For both the positive and the negative wave the voltage of the induced surge is lower at the beginning and higher later on than that due to the field of the inducing surge; in the case of the negative surge the induced voltage is actually of opposite polarity at the beginning. It seems probable that the difference between positive and negative induced surges is due to the fact that the electric field is more affected by the corona envelope for the positive than for the negative surge.

That this peculiarity in the induced wave is not produced only when corona is present is shown by the fact that it also exists on low-voltage surges; Fig. 6 shows such a case. It resembles more nearly the negative high-voltage case than the positive, which is in accord with the fact that for the same voltage the negative corona envelope is smaller than the positive.

In Fig. 7 is shown the induced voltage at a point one

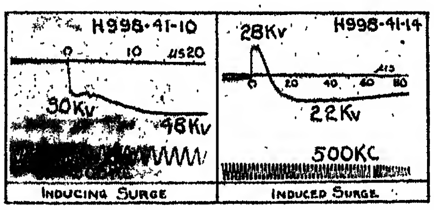

FIG. 6—INDUCTION ON FREE CONDUCTOR AT LOW VOLTAGE

mile from the impulse generator end of the line. Only the beginnings of a divergence from the wave shape of the inducing surge are here seen. The cumulative effect of this divergence is brought out very clearly in Fig. 8. It appears as though there is a continuous separation of charge taking place within the induced surge (for in no other way than by a separation can charge of a definite polarity appear on an isolated conductor). Charge of the opposite polarity to that of the main surge collects in the front of the induced surge, thus decreasing the resultant voltage there, while charge of the same polarity increases the voltage in the latter portion of the induced surge.

It is not easy to see the mechanism by which this separation takes place, but it seems that it must be in

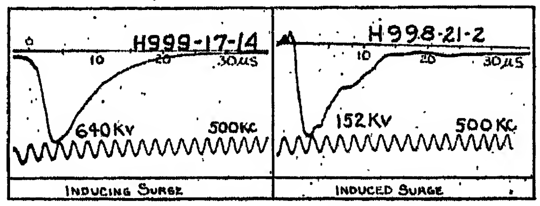

FIG. 7—OSCILLOGRAMS OF INDUCTION AT TOWER 10

some way connected with the process of attenuation and distortion of the main wave. Not only is the self surge impedance no longer a constant for wave propagation on a dissipative line, but also the mutual surge impedances have become more complicated operators.

It is now easy to see how an induced voltage of this nature will affect the propagation of a surge on three

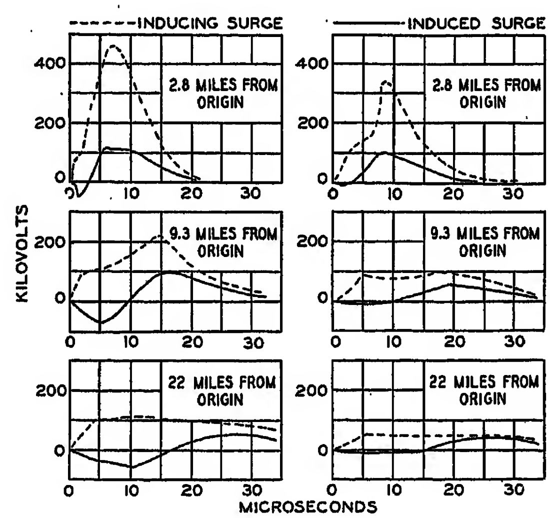

FIG. 8—REPLOT OF SELECTED OSCILLOGRAMS SHOWING INDUCTION ON FREE CONDUCTOR

conductors so as to produce that flattening of the front which was noticed in the foregoing section of this paper (Fig. 2, waves C, D and N).

III. COUNTERPOISE

General. The use of counterpoise has been suggested to improve grounding conditions in high resistance soil, especially in connection with ground wires.

The tests to be described in this section were conducted to determine the effectiveness of counterpoise and the relative merits of different arrangements.

An effort was also made to obtain a clearer insight into the manner of action of the counterpoise.

Considering for a moment the action of the counterpoise from the standpoint of theory it is evident that there will be two factors tending to reduce voltage at the tower footing; namely, the capacitance of the counterpoise to ground (in which true ground potential will occur at an appreciable depth below ground level) and a conductance effect due to the leakage of charge from the conductor into the soil. On surge voltages however, time will be necessary for the whole length of the counterpoise to come into action; there will in fact be a traveling wave effect on the counterpoise which will behave like a line with very high leakance; this includes the possibility of reflections from the remote (open) end making themselves felt at the tower footing end.

Method of Tests. For the purpose of these tests a location of very high soil resistivity was chosen. At a tower whose footing resistance measured (50 cycles) approximately 400 ohms, various arrangements of counterpoise were connected and a surge was discharged directly from the line into this grounding system. At a distance of 40 ft. from the tower leg a probe was driven to provide a reference ground and the voltage of the point at which the line was grounded was measured relative to this reference ground. In some tests the line was grounded through a known resistance; the voltage across the resistance was used to measure the current flowing into the ground.

A. Variation of Length of Counterpoise

Test Results. In Fig. 9 is shown replotted to a uniform scale, a number of oscillograms of surge voltages taken at the point of grounding with various lengths of counterpoise.

It will be seen that extending the counterpoise in two directions produces practically double the effect of extending it in one (the voltage of curve 1 is not half that of curve 2, but it must be remembered that the voltage at this junction involves also the surge impedance of the line).

Curve 3 coincides with curve 2 for practically six microseconds, after which curve 3 lies slightly higher. It seems reasonable to attribute this rise to a reflection arriving back from the end of the 1,050 ft. of counterpoise. A similar rise is evident in curve 4 but beginning much earlier; namely, about three microseconds from the start. This is in accord with the supposition that these rises are caused by reflections, since the times are approximately in the ratio of the respective lengths. (It is also interesting to note that this gives a velocity of propagation of approximately 375 ft. per microsecond in the counterpoise.) In curve 5 is noticed for the first time an increase in maximum voltage, because now the reflection has arrived back before crest voltage. There is also evidence of two reflections in this case.

In deciding how much advantage is to be gained by increasing the length of the counterpoise, it is clear that for this particular case no great advantage is gained by extending the counterpoise further than 1,000 ft; but this already involves two spans so that if we are dealing with a *region* of high resistance it involves going the whole distance from one tower to the next. How much shorter than 1,000 ft. the counterpoise can be made without losing a great deal of its effect will depend on the length of surge to be expected, particularly on the length of wave front; for example, a length of 500 ft. is quite satisfactory for the waves used on these tests, but would not be so for a wave having a six microsecond front. That a great deal can be gained by increasing the number of directions in which counterpoise is laid is evident, but there are obvious limitations to the distance possible in any direction other than parallel to the line because of questions of right-of-way.

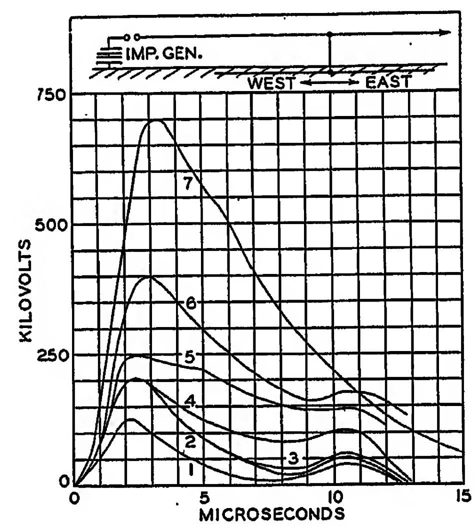

Fig. 9—Effect of Varying Length of Counterpoise

1. Counterpoise 1,527 ft. west and 921 ft. east
2. Counterpoise 1,527 ft. west
3. Counterpoise 1,056 ft. west
4. Counterpoise 529 ft. west
5. Counterpoise 225 ft. west
6. Tower footing alone
7. Line not grounded

B. Current and Voltage at the End of Counterpoise

In Fig. 10 are shown plots of current entering the ground (counterpoise) at the point of grounding, together with the voltage at this point. It is immediately apparent that current and voltage are not proportional; in fact, the ratio of current to voltage increases with time. We have here another case of non-constant surge impedance, chiefly on account of the high leakance of the counterpoise.

If a constant voltage be impressed at the end of a line with leakance, it is clear that the current at this end will increase with time although the voltage remains constant. This is due to the effect of the leakance at distant points being reflected back to the source at successive points of time; if the length of the line were infinite the current at the end of the line would ulti-

mately become infinite (provided the resistance of the line is zero). On the other hand, if the voltage at the end is not held constant but is supplied through a resistance (as in the case of grounding a rectangular wave of surge voltage on a line by a counterpoise, where the surge impedance of the line enters as a series resistance) the voltage will decrease as the current increases towards a constant value. Consequently a counterpoise becomes more and more effective as time goes on, provided the counterpoise is long enough. This is of importance in considering the effectiveness of counterpoise on different wave fronts. It is clear that for a counterpoise of unlimited length, the smallest effect on maximum voltage would occur on surges of shortest wave front. For a surge of rectangular front, leakance would have no effect initially, the entire effect of the counterpoise then being equal to that of a resistance $= \sqrt{\frac{L}{C}}$. Limitations of time allowed of no further tests being made to check this, but this conclusion from the voltage-current relationship is plain.

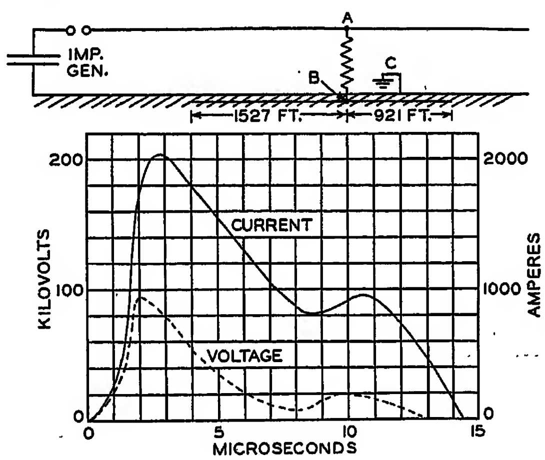

FIG. 10—CURRENT AND VOLTAGE AT POINT OF GROUNDING OF COUNTERPOISE

This last fact is important in view of the increasing weight of evidence that direct strokes are a major factor in producing outages. In this case flashover would be produced by the tower rising to a sufficiently high potential above that of the line and the object of the counterpoise would be to keep the tower footing impedance low enough to prevent this from happening. It is reasonable to believe that a direct stroke on a tower would produce a voltage wave of very steep front on the tower approximating the condition of rectangular front discussed above.

SUMMARY

Attenuation and Distortion

1. Lightning surges undergo considerable changes both in crest voltage and in wave shape in traveling along transmission lines.

2. Attenuation is more rapid above corona voltage than below.

3. Positive surges are attenuated more rapidly by corona than negative surges. This fact will influence the interpretation of lightning measurements made by surge recorders and other instruments inasmuch as surges of equal magnitude but opposite polarity at the point of origin will not be equal at the point of measurement.

4. On both positive and negative waves the attenuation is accompanied by a distortion which lengthens the wave.

5. The rate of voltage rise (or fall) decreases as the surge proceeds along the line.

6. Short waves attenuate more rapidly than long waves. This and (5) means that a lightning surge chopped by insulator flashover becomes relatively harmless after traveling one or two miles, since it will be of low crest voltage and will also have lost all abrupt changes in magnitude.

7. There is an appreciable attenuation of low-voltage (below corona) surges, probably mostly due to ground resistance.

8. A surge on three conductors in parallel is attenuated less rapidly by corona than a surge of the same voltage and wave shape on one conductor.

9. The front of a surge is slowed down more rapidly when the surge is on three conductors in parallel than when it is on a single conductor.

Surge Voltages Induced on Parallel Conductors

10. A surge on one conductor induces on a parallel isolated conductor a voltage which is initially of approximately the same wave shape as the inducing surge and of the order of magnitude to be expected from its position in the field of the inducing surge.

11. The ratio of the maximum voltage of induced surge to the maximum voltage of inducing surge increases as the surge proceeds along the line.

12. As the surge progresses there is a cumulative divergence between the wave shapes of the induced and the inducing surges in which the front of the induced surge tends to acquire a charge opposite in polarity to that of the inducing surge, while the latter portion gains a charge of the same polarity; it is this gain in the latter portion of the surge which accounts for the increased ratio noted in 11, while the tendency for the front to assume opposite polarity produces the flattening of front noted in (9) above.

Counterpoise

13. The impedance offered to surge voltages by tower footings is materially reduced by the addition of a counterpoise.

14. A counterpoise behaves like a line with very high leakance.

15. Depending on the length of the counterpoise, reflections from the remote end will be noticeable in the voltage at the end connected to the tower.

16. The first and all successive reflections will be attenuated to a negligible value if the counterpoise extends beyond a certain distance.

17. The ratio of current to voltage at the tower end of the counterpoise increases as the effect of the leakance of distant portions of the counterpoise is reflected back to this end.

18. An increase in the number of directions in which counterpoise is laid, increases the "surge admittance" of the counterpoise practically in proportion (this will be true as long as the mutual effect between the different conductor lengths is negligible).

19. The effectiveness of the counterpoise depends, among other things, on the length of front, as well as on the total length of the wave. The effectiveness decreases with increasing slope of front and for the extreme case of a perpendicular front is a minimum being equal to the surge impedance of the counterpoise with no leakance.

Acknowledgments

The authors wish to express their thanks to all those who contributed in collecting the results embodied in this paper. In particular they wish to mention Messrs. K. B. McEachron and J. G. Hemstreet, who initiated this series of tests, Messrs. E. J. Wade, W. J. Rudge and E. J. Shimek (of the General Electric Company) and Messrs. C. C. Tanner, F. D. Blackwell, and D. H. Geiger (of the Consumers Power Company) who took active part in all the tests, and all the operators and men of the Consumers Power Company whose cooperation was essential to the success of the tests.

They also wish to thank Mr. T. Brownlee (of the General Electric Company) for assistance in the preparation of this paper.

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Discussion

For discussion of this paper see page 1146.

Lightning Investigation on Transmission Lines—II

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Synopsis.—The results of a five years' investigation of the effects of lightning on transmission systems are reviewed. It is concluded that direct strokes play a prominent part in causing flashover. Induced strokes are also important.

Conventional overhead ground wires with low tower footing resistance take care of direct strokes, as well as induced strokes, and prevent flashover. The mechanism of the lightning stroke is not yet understood, but the indications are that polarity effects are highly important and must be given full consideration.

* * * * *

IN previous papers[1,2,3] the authors have summarized the results obtained for four years (1926, 1927, 1928 and 1929) in a surge voltage investigation conducted by the General Electric Company in cooperation with various power companies.

In 1929 and 1930 progress in this investigation was greatly accelerated. This was partly due to concentration of the work on a few systems and largely due to the liberal use of several new instruments especially developed by the General Electric Company for this investigation.

Instruments Used in the Investigation

The *surge-voltage recorder* of the two electrode type.[4] Approximately 250 of these instruments were in use.

The *cathode-ray oscillograph.*[5] Six of these instruments were in use.

The *field intensity recorder* and *rate of change of field intensity recorder.*[3] One instrument of each kind was in use in 1929, and three field intensity recorders in 1930.

The *severity meter*[6] introduced in 1930. Eleven instruments were in use.

The *lightning stroke recorder.*[3] About 800 of these instruments were in use in 1930.

The *surge indicator*[7] introduced in 1930. About 1,500 of these instruments were in service.

Systems on Which Investigation was Conducted

In 1929 and 1930 the work was concentrated mostly on the Pennsylvania Power & Light Company's Wallenpaupack-Siegfried line, the Ohio Power Company's Philo-Canton line, and the Atlantic City Electric Company's Deepwater-Pleasantville line (the last mentioned in 1930 only). In addition, an investigation was conducted with artificial lightning on a 45-mile, 70-kv. line of the Consumers Power Company in Michigan.

See companion papers for details of the investigation on these systems.[8,9,10]

The main change on the Wallenpaupack-Siegfried line in the 1930 set-up as compared with the 1929 set-up was the removal of the lightning laboratory from tower 1-3 near Wallenpaupack to tower 23-4 of the Siegfried-Roseland section (at Cherry Valley).

In this laboratory two cathode-ray oscillographs were installed, one operating with a 50-microsecond time axis and the other with a 2,000-microsecond time axis. Both oscillographs were connected to phase X of the transmission line.

In 1929, a "counterpoise" was installed parallel to the line conductors and trenched in the ground, connecting the bases of the towers from tower 11-4 to tower 14-1 of the Wallenpaupack tap section of the Wallenpaupack-Siegfried line. In 1930, four tower footing grounding cables, each 50 ft. long, extending out in four directions from the bases of the towers and trenched in the ground, were installed at each tower from tower 1-4 to tower 11-3 and from tower 14-2 to tower 20-2 of the Wallenpaupack tap section of the Wallenpaupack-Siegfried line.

Two "lightning diverter wires,"[11] 180,000 cir. mils A. C. S. R. extend over the line conductors from tower 22-1 to tower 25-4 (18 consecutive towers) of the Wallenpaupack tap section of the Wallenpaupack-Siegfried line. These were installed in June 1930.

Each tower of the Wallenpaupack-Siegfried line was equipped with a lightning stroke recorder and each insulator string with a surge indicator.

Results of Investigation

1. Crest Value of Surges due to Lightning and Switching.

Lightning surges up to 3,000 kv. in magnitude have been measured on the Wallenpaupack-Siegfried line, or 16.7 times normal crest value line-to-neutral voltage. This is approximately 58 kv. per ft. of height of conductor. On the Ohio system lightning surges up to 1,640 kv. or 15.2 times normal have been measured. It appears that the limit for these values is set by the line insulation, and that this limit increases as the line insulation is increased.

Fig. 1 gives a curve of lightning voltage measured by surge-voltage recorders (times normal line-to-neutral voltage) against percentage of time occurring. The data are based on a study of 678 surges due to lightning, occurring on 14 systems during the years 1926 to 1930 inclusive. Fig. 1 also shows a curve of switching surges measured by a surge-voltage recorder, based on 724 surges occurring during the years 1927 to 1930 inclusive on

*General Electric Co., Schenectady, N. Y.

1. For references see end of paper.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

practically the same systems from which the lightning data were taken.

2. Flashovers and Trip-outs Caused by Lightning.

The development and installation of the surge indicator and the use of the high-speed magnetic oscillograph at the line terminals, greatly assisted in the study of insulator flashover and trip-outs on the Pennsylvania system. By means of the data from these instruments supplemented by data from the slow-sweep (2,000 microseconds time axis) cathode-ray oscillograph and the lightning-stroke recorders, it was possible in 1930, for the first time, to locate the point of breakdown with reasonable certainty for about two-thirds of the lightning trip-outs.

On the 220-kv. lines in Pennsylvania for the five-year period, 1926 to 1930 inclusive, the Wallenpaupack-Siegfried line had 324 flashovers of insulator assemblies. In addition, there were 19 relief gap operations at Wallenpaupack and Siegfried. For the three-year period, 1928 to 1930 inclusive, the Plymouth-Siegfried line had six flashovers, the Conowingo-Plymouth line

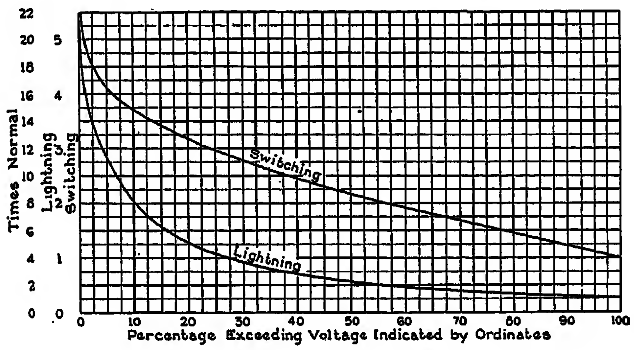

FIG. 1—CURVES OF VOLTAGE AGAINST PERCENTAGE OF TIME OCCURRING, FOR LIGHTNING AND SWITCHING

Based on data from 14 systems for years 1926 to 1930 inclusive

No. 1 three, and the Conowingo-Plymouth line No. 3, four flashovers.

Each outside conductor on the Wallenpaupack-Siegfried line suffered two to three times as many flashovers as the middle conductor. The majority of flashovers involved only one phase and one or two assemblies, but in one case as many as three phases and four assemblies were involved.

During the five-year period, 1926 to 1930 inclusive, the Wallenpaupack-Siegfried line tripped out 98 times. During the three-year period, 1928 to 1930 inclusive, the Plymouth-Siegfried line tripped out five times, the Conowingo-Plymouth line No. 1, four times, and the Conowingo-Plymouth line No. 3, five times.

On the Wallenpaupack-Siegfried line in about 80 per cent of the cases only one phase was involved in a trip-out.

3. Direct-Stroke Records.

In 1930, after June 1, 18 records were obtained on the Wallenpaupack-Siegfried line with lightning stroke recorders. These have been interpreted as indicating direct strokes.

The list is divided into two groups, the first group of 10 or 11 are associated with trip-outs and the second group of 8 or 7 are not associated with trip-outs. The strokes associated with trip-outs took place on the section of line not equipped with overhead ground wires, while 7 strokes not associated with trip-outs took place on the section of line equipped with overhead ground wires or diverter wires.

Of the 24 trip-outs experienced on the Wallenpaupack-Siegfried line in 1930, 8 took place before the lightning recorders were in operation. Of the remaining 16 trip-outs, 10 or possibly 11 were interpreted to be associated with direct strokes. In four cases, no lightning stroke records were obtained and the remaining one or two cases were indefinite.

On the Ohio Power Company's 132-kv. Philo-Canton line, 130 lightning stroke recorders were installed in the vicinity of Newcomerstown. Many interesting records were obtained, involving in some cases as many as five adjacent towers. On the Atlantic City Electric Company's system, 200 recorders were installed. Some records involved as many as six towers.

On the Philadelphia Electric Company's 220-kv. and 66-kv. systems about 65 recorders were installed and some interesting data obtained.

In a high percentage of the cases the direct strokes recorded on the Ohio Power, Atlantic City, and Philadelphia Electric Companies' systems were not accompanied by trip-outs. All of these lines were equipped with overhead ground wires.

4. Cathode-Ray Oscillograms.

During 1928, one cathode-ray oscillogram was obtained on the Wallenpaupack-Siegfried line; during 1929, 95 oscillograms and during 1930, 23 oscillograms. These are of a wide variety of shapes and durations. The origin of some of the surges was near and some far from the oscillograph.

In 1930, there seemed to be a tendency for the waves to have a certain general shape, if the phase on which the oscillograph was installed was involved in the fault, while another distinct shape of wave was recorded if the phase on which the oscillograph was connected was not involved in the fault.

One oscillogram was obtained of a direct stroke, which apparently hit the conductor to which the oscillograph was connected, on the span adjacent to the laboratory. About 3,000 kv. negative was recorded on this conductor, both by cathode-ray oscillograph and surge-voltage recorder. On the other two conductors not over 1,620 kv. negative and 720 kv. negative, respectively, were recorded by a surge-voltage recorder, and on the 50-ft. antenna adjacent to the transmission line no voltage was recorded.

5. Storm Severity

The average storm severity on an arbitrary scale as

measured by the storm severity meter at four stations in Pennsylvania and one station at Schenectady for 20 weeks, May to September, varied from one to two. At two stations in Colorado the average storm severity for eight weeks in July, August, and September, was 9.3 at one station and over 10 at the other.

6. Overhead Ground Wires

On the section of the Wallenpaupeck-Siegfried line equipped with overhead ground wires and counterpoise, tower WT-11-4 to tower WT-14-1 (14 towers in all), the flashed insulator assemblies for the years 1926 to 1930 were as follows (the counterpoise was in service in the last two years only):

TABLE I

Year	Flashed insulator assemblies
1926 (No overhead ground wires)..	6
1927 (Ground wires installed)....	9
1928...........................	14
1929 (Counterpoise installed).....	0
1930...........................	0

The Overhead Systems Committee of the Great Lakes Division, N. E. L. A., has gathered data for several years on the performance of transmission lines in its district, and has made a special study of the effect of overhead ground wires.

The miles of line under observation equipped with overhead ground wires have increased from 1,400 miles in 1927 to 2,660 miles in 1929. The miles without overhead ground wires have decreased in the same period from 759 to 673 miles. The interruptions per 100 miles of line due to lightning on the lines with overhead ground wires have increased from 5.8 in 1927 to 7.0 in 1929. On the lines not equipped with overhead ground wires interruptions per 100 miles have increased from 12.2 in 1927 to 20.9 in 1929. The indications from this study are that from two to three times as many interruptions occur on the lines without overhead ground wires as on the lines with overhead ground wires.

The American Gas & Electric Co. operates a very extensive 132-kv. network in Ohio, Indiana, West Virginia and Virginia (approximately 1,800 miles).[13] This network is divided into 24 lines, all but two of which are equipped with overhead ground wires. On the lines equipped with overhead ground wires, the outages (1928) averaged 11.7 per 100 miles. On the lines not equipped with overhead ground wires the outages for the same year averaged 38 per 100 miles.

The Public Service Elec. & Gas Co. of New Jersey operates an extensive system of 13.2, 26.4, 66, 132 and 220-kv. lines. The lines 66 kv. and 132 kv. are mainly on double-circuit steel towers with conductors vertically arranged and two ground wires over the outermost (middle) conductors. The ground wires are about 10 ft. above top conductor. The lines pass through farm, meadow, and swamp lands, and the towers have a naturally low-footing resistance. Whereas the total interruptions on this system in 1930 were 231, only two of these took place on the 66-kv. system and four on the 132-kv. system. The remainder took place on the 13.2- and 26.4-kv. systems. This territory is subjected to a normal amount of lightning.

The Philadelphia Electric Co. has a number of 66-kv. double-circuit steel tower lines, with two overhead ground wires placed vertically above the outermost (middle) conductors. These lines in general are subjected to a limited number of interruptions, but one line in particular, that from Philadelphia to Chester, a distance of 14 miles, has operated 14 years without an interruption due to lightning. The distinguishing feature of this line is that throughout the entire length the line follows a double-track railroad, with the tower structures located a short distance from the nearest track rail. In addition the line parallels a river throughout practically its entire length, the distance between the line and river varying greatly. The tower-footing resistances on the line are unusually low and uniform (of the order of 4 ohms). This line is in territory which is subjected to about the same number of storms per year as the Pennsylvania Power & Light Co. system and the storms are apparently of comparable severity as measured by the severity meters.

7. Direct and Induced Strokes.

It is quite important to know how relatively frequent are direct and induced strokes, the magnitudes of potentials placed on the power conductors by such strokes and the effect of overhead ground wires and other protective measures against such strokes. Unfortunately, reliable data on this subject are still largely missing. Some of the meager data that are available are examined.

In a recent paper[12] appear the statements given in the following three paragraphs:

Observations by skilled engineers show conclusively that lightning strokes within ½ to 2 miles from the line resulted in surges having values not over the line voltage.

In May, 1930, a photograph was taken of a lightning stroke which hit the earth 1,600 ft. (estimated by sound) from a cathode-ray oscillograph station at Roseland, New Jersey. This stroke had hit the earth within 400 feet of the line to which the measuring instruments were connected. Neither the klydonographs connected to this line nor the cathode-ray oscillograph indicated any substantial voltage, although these instruments were so calibrated that they would indicate any surge above the line voltage, which in this case was 200 kv.

In another case, a 33-kv. line in Arkansas was struck by lightning and the poles split from top to bottom. At right angles to this line was another line to which recording apparatus was connected. The instruments gave no record of any surge, although they were located only 700 ft. from the pole which was struck.

In 1929, at the Wallenpaupack laboratory an attempt was made to correlate the potential gradient, as measured with field intensity recorder, with distance

of the stroke from the laboratory. The distance was determined roughly by an observer in the cupola of the laboratory, with the assistance of a stop watch and a device for sighting on the stroke. Of 253 strokes over 15 kv. per meter recorded, it was possible to correlate in this manner only 81 strokes or 32 per cent of those occurring. The other strokes were too uncertain in correlation or else no satisfactory distance measurements were obtained. Fig. 19 of the authors' previous paper[3] shows the results of this attempt at correlation. It will be noticed that the correlation is very rough, but that there is a tendency for a curve showing greater field intensity with shorter distance from the laboratory. Field intensities as high as 280 kv. per meter (85 kv. per ft.) were recorded in this investigation.

From the standpoint of gradient alone, therefore, it appears reasonable to assume that induced voltages may be of such amplitude as to cause insulator flashover. It is appreciated, however, that induced voltage on a line conductor is a function of rate of change of field gradient as well as absolute field gradient. Information concerning rate of change of field gradient is not sufficiently complete to permit a conclusion regarding the amplitude of induced voltage.

In 1928, on the Philo-Canton line of the Ohio Power Co. a tree designated as tree No. 1, located 360 ft. from the transmission line at tower 176 was struck by lightning. A surge-voltage recorder located on the bottom conductor at tower 176 recorded a voltage of 2,100 kv. positive. The surge was accompanied by a line trip-out. The correlation between the transmission line record and the direct stroke at the tree is not certain.

In 1930 two trees in this same vicinity were struck, apparently during a storm which occurred on Sept. 23. Tree No. 2 was 500 ft. from the transmission line and tree No. 3, 1,300 ft. from the line. There was no surge-voltage recorder at tower 176 this year;the nearest recorder being at tower 172 (approximately one mile away). There were no trip-outs on the line or surges which correlated with these strokes recorded on instruments connected to the line.

On the Louisiana Power & Light Company's 110-kv. system, in 1929, lightning strokes occurred to two trees adjacent to the right-of-way of one of the lines. Tree No. 1 was approximately 550 ft. from the line and opposite one of the highly insulated wood pole *H* frame structures which had no guys, pole grounds, or bonding. The next structure south was similar. The second structure south was a guyed structure with a bonding wire along the cross arm, a grounding wire down each pole, except for a 13½-ft. gap in each grounding wire, and 18 ft. wood guy insulators in each guy. This structure was approximately 1,000 ft. from the tree that was struck. No evidence was left of flashover on the unguyed structures, but the guyed structure flashed over the 13½-ft. wood pole gap at the time of the lightning stroke.

At another time, tree No. 2 which was approximately 480 ft. from the line and 670 ft. from the same guyed structure was struck, and two of the 18-ft. wood guy insulators were flashed over with no indications of flashover on any of the unguyed structures.

On the Consumers Power Company's 140-kv. 30-cycle system in Michigan, a direct stroke hit the top of tower 4334. The line was single-circuit on steel towers with one overhead ground wire, conductors vertically arranged. Power arcs occurred from all three conductors to the tower. Potentials of 2,800 kv. positive were recorded on two phases at tower 4334. On the adjacent tower north (tower 4333) a positive potential of 1,375 kv. was recorded. On other towers (4323 and 4304) located one and three miles respectively, north, negative potentials of 1,480 and 900 kv. were recorded. On towers (4366 and 4385) located one and three miles respectively, south, negative potentials of 1,840 and 1,300 kv. were recorded.

The following explanation has been advanced for this occurrence: A direct negative stroke to the tower, resulted in a flow of current down the tower and out in both directions on the overhead ground wire. The flow of current down the tower encountered the tower footing resistance (55 ohms) and raised the tower to a potential above the conductors (conductors positive with respect to the tower). The conductors flashed to the tower, and the conductors became negatively charged, which accounts for the negative potentials at the towers one and three miles north and south of tower 4334. The positive potential on the adjacent tower 4333 may have been caused by the flow of current out on the ground wire and down the tower, raising the tower to a potential above ground and conductor.

On the right-of-way of the Wallenpaupack-Siegfried line of the Pennsylvania Power & Light Co. there were erected at five points highly insulated antenna wires about 300 ft. long and at different heights above the ground, varying from 12½ ft. to 100 ft. The antenna wires were approximately 75 to 100 ft. from the nearest line conductor.

A voltage (presumably induced) of 2,700 to 3,150 kv. positive was measured on the 100-ft. antenna at Wallenpaupeck, with a corresponding voltage of 350 kv. positive on the transmission line. In another case, a voltage of 2,700 kv. negative was measured on the 50-ft. antenna at High Knob with no registration on the transmission line. In no cases were high voltages measured on the transmission line which correlated with antenna registrations.

On June 26, 1930, the diverter wire was apparently subjected to a direct stroke at tower WT-22-1. At the same time a voltage of 2,400 kv. positive was measured on the conductor on *Y* phase.

In the past, numerous high-voltage potentials of positive polarity have been recorded on transmission lines, although high-voltage surges of negative characteristics were apparently in the majority. There is a tendency to attribute the high-voltage negative surges

to direct strokes. The high-voltage positive surges may be due to induced strokes, or they may be due to direct strokes to towers or ground wires, which cause sufficient current to flow down the tower to give an appreciable difference of potential between the tower and the conductors. In this case the conductor would have a positive polarity with respect to the tower.

Apparently, there is evidence to the effect that direct strokes play an important part in transmission flash-overs and outage. There is a great deal of evidence that induced strokes are also highly important. There is insufficient evidence at the present time to evaluate more exactly these two types of strokes. It may eventually be found that one type of stroke is more important than the other on lines of one voltage class and insulation, and that in another voltage class the other type of stroke becomes more prominent.

Discussion of Results

1. *Mechanism of Lightning Stroke.* There is, of course, as yet no adequate understanding of the lightning stroke even from a qualitative standpoint. Furthermore it is improbable that from existing data an indisputable explanation of the mechanism of the stroke can be built. However it appears to the authors that there are suggestions in the results already obtained which should be given careful consideration.

Measurements of cloud field potentials[3,14,15] have established beyond question that fields of both polarities are present. It has been shown that the field polarity at a point on the ground may change from positive to negative or from negative to positive during the same storm. Many storms have been shown to have a predominant polarity characteristic but in almost all cases both positive and negative fields are present to some degree.

Studies of polarity effects in gas discharges suggest that no satisfactory understanding of the mechanism of the lightning stroke will be reached until due consideration is given polarity effects.[16,17] Laboratory experimental work on the breakdown of gases between dissimilar electrodes[18] reveals that the shape of the positive electrode is of greatest importance in determining voltage breakdown. Lichtenberg figure studies[19] have supplemented this information and in addition have contributed valuable experimental proof that in most cases the propagation of the steamer tip away from the positive electrode is the essential aspect of the breakdown mechanism.

Laboratory studies[20,21] of the "protective zone" effect of the vertical lightning rod have shown that polarity is the determining factor. With the lightning rod polarity positive and the cloud negative a very high degree of protection is afforded. With the lightning rod negative relative to the cloud some protection is still obtained but the effectiveness of the rod is greatly reduced.

As previously stated all lightning-stroke recorder records of direct strokes to towers indicate that the top of the tower is negative with respect to the bottom and from this it appears reasonable to infer that these are cases of negative clouds and positive ground charges. With the tower structure positive with respect to the cloud, breakdown proceeds from the structure to the negative cloud. Numerous strokes terminate on the grounded structure under this polarity condition. Under conditions of tower structure negative with respect to the cloud the stroke would proceed from the cloud and the probability of the tower structure being struck would not be great. While this conception appears to reverse our picture of the cloud-to-ground stroke nevertheless it is consistent in that it is in accord with laboratory and field experience.

The direction of current in the lightning stroke has also been given considerable study.[22] As the breakdown streamer propagates rapidly away from the positive electrode electrons flow toward the positive electrode. The lightning stroke currents therefore classified in accordance with our practical definition always flow from the positive to the negative polarity.

2. *Ground Wires.* The theory regarding the particular mechanism of operation of the ground wire in case of induced charges is generally agreed upon. Ground wires are presumed to protect in this case because they reduce the induced charge on the line conductor and increase the capacitance of the conductor to ground.

However, in the case of direct strokes it appears that a somewhat different viewpoint is necessary. The fact of the elevation of the line structure above the ground plane causes many strokes to terminate on the line structure.

When the cloud polarity is negative and the ground positive all strokes within a distance of the order of ten times the height of the line on either side will strike the line. This, of course, is the condition obtained when there are no other objects projecting above the ground plane of the line within the distance mentioned.

When the cloud polarity is positive and the ground negative the projecting transmission line still functions to attract many strokes. However, in this case the number terminating on the structure will be smaller, the line attracting all discharges within a distance of 2 to 3 times the height of the line on either side instead of 10 times. This explains the great number of negative strokes indicated during the lightning investigations.

The overhead grounded structure (towers and ground wires) serves as the ground terminal for the cloud-to-ground stroke, forming a shield for the line conductors. The customary ground wire attached to the tower structure serves to extend the shielding effect provided by the towers to the space between towers. In addition the ground wire also lowers the impedance between tower top and ground. Current resulting from a direct stroke at the tower top may pass out over the ground wires as well as down the tower structure. The conventional ground wire therefore functions in two ways to afford protection to line conductors; first to provide

over the line conductors a grounded structure which can serve as a terminal for direct strokes, and second to reduce impedance to ground in case of direct stroke to the grounded structures.

The so-called diverting wires, one form of which is being experimented with in an actual installation, are ground wires which may or may not be insulated from the tower structure and in general are located at a greater distance above the line conductors than the conventional ground wire.

Diverting wires supported on the tower structures by metallic tower extensions can have no advantage over the conventional ground wire other than a possible increased effectiveness in providing a terminal for direct strokes due to greater height and more advantageous position over the line conductors.

Diverting wires located above the line conductors but insulated from the tower structure and provided with independent grounds of low resistance have the advantage that lightning currents are not conducted through the tower structure to ground. As will be briefly discussed later this advantage is a significant one.

Data obtained in the field investigations and extensive operational experience testify to the effectiveness of the conventional ground wires. The number of direct strokes recorded compared to the number of trip-outs on a line with overhead ground wires indicates that only in rare cases will a line conductor be struck when steel towers and conventional ground wires are used. Knowledge of the commanding influence of the polarity effects described above permits at least a partial understanding of these results.

3. Insulator Assembly Flashover. Conditions under which the line insulation may flash over as a result of direct strokes appear to be three in number. First, of course, a direct stroke to a line conductor will provide sufficient voltage for insulation breakdown. Second, a direct stroke to the tower will cause insulator flashover by raising the potential of the tower structure with reference to the conductor. The voltage drop across tower and footing impedance is responsible for this flashover voltage. Third, a direct stroke to the ground wire between towers may result in such potentials on the ground wire that flashover to the line conductor takes place.

Direct strokes to line conductors would appear to be of rare occurrence where steel towers and overhead ground wires are used. Insulator flashover with this construction would therefore appear to result in most cases through tower potentials due to high tower footing resistance. Under this condition the tower structure would almost invariably be raised to a negative potential with respect to the conductor.

Information gathered by the surge indicators on insulator assembly flashover shows that in the greatest number of cases a single assembly flashes over. Next to this condition in frequency of occurrence is the case where two adjacent assemblies on the same phase flash over. Data on flashover of insulator units is still too meager to permit more than speculation on the influence of the ground wire, or a conductor connected to the tower by insulator flashover, in extending or limiting the total number of assemblies flashed for one stroke. However, it appears that where flashover occurs from the tower to a conductor the spread of flashover to other insulator assemblies on the same tower may be prevented by the resulting reduced impedance between tower and ground. Also insulator assemblies on adjacent towers on the same phase may flash over as the surge proceeds along the conductor from the tower struck and where the first flashover occurred.

In natural lightning measurements and also in artificial lightning measurements in field and laboratory, it has been shown that insulator flashover is not necessarily followed by line trip-outs.[10] One key to an explanation of this is found in the fact that flashover must occur at a time when the power voltage to ground and the current flowing through the conductor both have sufficient amplitude and proper polarity relations to establish a permanent arc for the power current to follow. Experience has shown that in many cases a permanent arc is not established because these conditions are not satisfied.

4. Wave Shape. Oscillographic work on transmission lines under natural lightning conditions has shown that wave shapes to be expected at a specific point on the line range in time to crest from about one microsecond to several hundred microseconds and that the times on the wave tail extend over a wider range than times to crest. The lower voltage surges are of longer time duration.

Wave shapes measured at or near the origin of the surge have fronts which bring the conductor potential up to insulator flashover in the order of a microsecond. These high amplitude waves of steep front usually cause flashover and are therefore reduced very quickly to low potential values. It would appear that a voltage wave rising to a high value but still somewhat under insulator assembly flashover would be of longer time to crest and have a wave tail value of some 10 to 100 microseconds.

Much has been learned concerning attenuation or diminution of voltage amplitude of the surge as it travels along the conductor. For general purposes a formula which has been derived from natural lightning measurements[3] is of sufficient accuracy for practical calculation. This formula is as follows:

$$e = \frac{e_0}{K s e_0 + 1} \text{ and } A = -k e^2$$

where

e_0 = initial surge voltage at the point where the surge originated

K = a proportionality factor found empirically

S = distance in miles from the origin of the surge
e = voltage at distance s
A = attenuation in kilovolts per mile at any point where the line voltage is e

The factor k has been shown to vary between 0.0001 and 0.0007 depending upon the particular line under consideration. This formula agrees with results of attenuation studies where artificial lightning was used on an actual transmission.

Detailed studies have proved that attenuation is dependent upon polarity and wave shape. Positive surges decrease more rapidly in amplitude than negative surges and chopped waves of short time element attenuate more rapidly than longer and unchopped waves.

Change in wave shape above corona voltage has been shown to consist of a flattening of the wave front and a lengthening of the wave tail.[10] It has been shown that in effect this may be thought of as resulting from a lower velocity of propagation for the higher voltage portions of the traveling charge. Such a viewpoint is undoubtedly helpful but it must not be accepted as an explanation of the mechanism of surge propagation without a closer examination than has as yet been given. The influence of corona and polarity effects which have been shown to be great in connection with surge propagation are probably determining factors. Conditions of relative energy loss on the front, crest, and tail of the wave are little understood. The shift of energy relations within the wave may account for the wave changes which have been demonstrated.

In connection with attenuation studies of surge propagation by means of surge-voltage recorders distributed along a line conductor it has been noticed that very frequently positive surges of high amplitude are recorded at one station with very little or no voltage measurements at adjacent stations. These voltages have been explained in two ways.

The positive conductor voltage has been attributed to bound charge on the line from a negative cloud which charge is released when the stroke occurs. In other words, it is an induced voltage.

Another explanation which has been offered is that these voltage registrations are the result of ground surges accompanying the negative stroke to the tower or some object near the line. Under this condition the instrument would record a positive voltage between conductor and ground. In view of the preponderance of negative direct strokes obtained by field measurements and the agreement of these data with laboratory studies of discharge mechanism this explanation appears to justify close consideration.

Conclusions

The following conclusions are based on the data presented or referred to in this paper.

1. Many trip-outs of high-voltage lines are caused by direct lightning strokes to the line structure. This conclusion is based on approximately 16 records of direct strokes correlating with line trip-outs.

2. Many trip-outs on sections of transmission line not equipped with overhead ground wires are due to direct strokes terminating on the line conductor.

3. Direct strokes terminating on the overhead ground structure may cause flashover of the line insulation. This appears to be due to the voltage drop across the tower footing resistance.

4. Direct strokes to the transmission line structure may occur without a line trip-out. Records of approximately 37 lightning strokes which did not result in trip-outs bear out this conclusion.

5. Overhead ground wires and steel towers serve as a terminal for lightning strokes, thereby protecting the line conductor and in many cases preventing insulator flashover.

6. Insulator flashover may occur without sufficient power current flowing through the arc to result in a line trip-out.

7. Conventional overhead ground wires have proved effective in intercepting direct strokes and preventing line trip-out.

8. Low tower-footing resistance functions to improve protection when a direct stroke terminates on the overhead ground structure.

9. Where overhead ground wires are used, currents resulting from direct-lightning strokes flow to earth in many cases through several adjacent towers. Records have been obtained involving as many as six tower structures.

10. Lightning strokes of negative polarity terminating on the transmission line, far outnumber strokes of positive polarity. In fact, no strokes of positive polarity have been recorded.

11. Strokes terminating on objects projecting from the ground near the transmission line, produce high voltage on the conductors and in some cases are known to have caused flashover of the line insulation.

12. In a great majority of cases of flashover of line insulation, only one or two insulator assemblies are involved. When two are involved, they are usually on the same phase and adjacent towers.

13. At the origin of the disturbance the rate of voltage rise across the line insulation is of such order as to reach flashover value in a very short time, perhaps of the order of one microsecond.

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Discussion

LIGHTNING
(PEEK)

LIGHTNING DISCHARGES AND LINE PROTECTIVE MEASURES
(FORTESCUE AND CONWELL)

LIGHTNING INVESTIGATION
(BELL AND PRICE)

1930 LIGHTNING INVESTIGATIONS
(SPORN AND LLOYD, JR.)

LIGHTNING INVESTIGATION
(GROSS AND COX)

EXPERIMENTAL STUDIES IN THE PROPAGATION OF LIGHTNING SURGES ON TRANSMISSION LINES
(BRUNE AND EATON)

LIGHTNING INVESTIGATION ON TRANSMISSION LINES—II
(LEWIS AND FOUST)

J. H. Cox: Messrs. Lewis and Foust have discussed the question as to the proportion of surges caused by direct and induced strokes. This is a very important question since it has a direct bearing on the methods used in protection. Eighteen records of direct stroke measurements are listed. Those recorded on the unprotected part of the line were accompanied by trip-outs and those recorded on the ground wire protected part of the line were, in at least the majority of cases, not accompanied by trip-out. These data are a gratifying indication of the benefits derived from the use of a ground wire.

The authors further state, "First, of course, a direct stroke to a line conductor will provide sufficient voltage for insulation breakdown." I do not believe that present data justify this statement and feel it entirely possible for a direct stroke to be insufficiently severe to cause a flashover on a high-voltage line. Furthermore, the fact that all direct-stroke records obtained on the unprotected part of the line were accompanied by flashover does not indicate that no direct strokes occurred which were not recorded. With the methods of measurement used a direct stroke to a line conductor without an insulator flashover would not yield any direct-stroke record. It may be likely that a direct stroke would be to the tower but not at all certain.

Another factor brought out is that all the direct-stroke records indicated negative strokes and in conclusion 10 the statement is made that this indicates that negative direct strokes far outnumber the positive. It is entirely possible that positive strokes, even when direct to towers, do not cause currents sufficiently heavy to yield a record with the instrument settings used. So far as the data show, it still seems likely that the range of severity of direct strokes may be sufficiently great so that some do not cause flashover, even on unprotected lines, and also that some cause flashover on lines with the best protection now available.

In connection with the records obtained by Messrs. Lewis and Foust on antenna, I wish to point out that antenna measurements, unless the antenna is connected to ground through a resistance equal to the antenna surge impedance, which would be of the order of 500 ohms, indicate the cloud field gradient but do not yield a measure of the potential which would be induced on the line. This fact is demonstrated by the experience of the authors as stated by them: "In no cases were high voltages measured on the transmission line which correlated with antenna registrations." The antenna was close to the line. Another fact which indicates that induced voltages may be low, for at least certain kinds of strokes nearby, is the case mentioned of a direct stroke to the transmission line with no recordable induced voltage on the antenna, and in this case the antenna being connected to ground only through a high resistance the conditions were most favorable for a high induced voltage.

Regarding the strokes to trees adjacent to transmission lines as mentioned in the paper, these data may be misleading. For instance, a stroke to a nearby tree without a line surge, a direct stroke or a trip-out record indicates definitely that the induced voltage was below the required value. However, a stroke to a nearby tree with a correlated record of one kind or another does not definitely indicate that the record was caused by the particular stroke which hit the tree, unless the correlation was complete and an actual observation was made which tied the two points of evidence together, since it is possible that the record on the line and the injury to the tree were caused by two different strokes of the same storm. The greatest care should be taken to take into account possibilities of this nature which completely modify the interpretation.

The use of a buried cable, called a counterpoise by the authors, was suggested some time ago as a method of reducing tower footing resistance. The results of at least one year's experience on one line, as contained in Table I of the paper, which shows a perfect record, are very gratifying.

The authors state that the lightning stroke always starts from the electrode of positive polarity. Simpson suggested this in 1926 but more recent data indicate this to be incorrect. Torok's work at Trafford on arrested discharges indicate that a discharge streamer always started from the electrode of high potential, that is the electrode of smallest effective dimensions, whether such electrode is positive or negative.

I am unable to understand the discussion under "Wave Shape" of the short distance of surge propagation. If the surge is induced and consists of the release of a bound charge, this bound charge is actual and must be dissipated in some manner. Therefore, it should perform as a simple traveling

wave and should not dissipate in two or three towers. The second explanation is somewhat obscure but it seems rather likely that the extent of the ground surge due to the lightning current is not great. I feel that the correct explanation is contained in the Fortescue and Conwell paper. As pointed out there, in the case of a stroke to a ground wire without a flashover the potential on the line conductor is an induced part of the traveling wave and involves no actual charge. Thus, when the surge on the ground wire is dissipated, which it will be in a very few towers, the potential on the line conductor will disappear simultaneously.

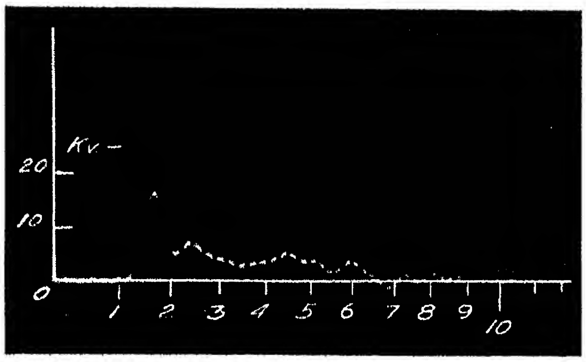

Fig. 1—Potential at Tower Struck

J. J. Torok: The calculations made in the R. N. Conwell and C. L. Fortescue paper were based upon the accepted theories of traveling waves. In setting up the equations it was assumed that the tower length was such that it could be neglected in these calculations. The authors of the paper have realized that in a general treatise it would be desirable to include the effects of the tower in these reflections, consequently, at their request the calculations were revised so as to include the effects of the tower.

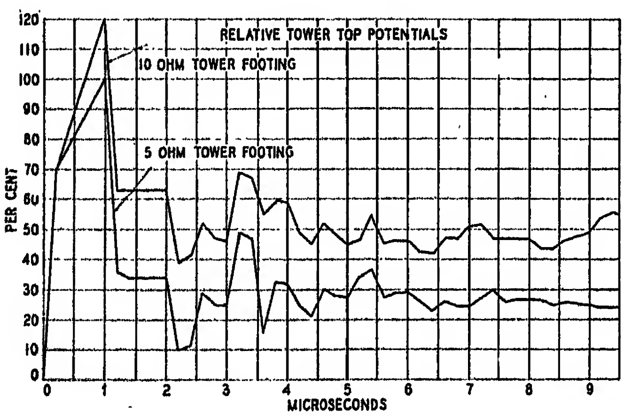

Fig. 2—Effect of Tower Footing Resistance on Potential at Tower Top

The surge impedance of the tower was determined to range from 75 to 150 ohms, the average being approximately 100 ohms. The assumptions in these calculations were that a tower to which two ground wires are attached was struck. Under this condition a train of traveling waves would be set up on the ground wires and on the tower. The waves on the ground wires will be reflected between adjacent towers while the waves on the tower will travel up and down the tower. In these calculations it was assumed that the nature of the wave traveling over the lightning channel was as follows: it rose to crest value in one microsecond and then dropped to half value in 50 microseconds.

The results of these calculations are shown in the following figures. Fig. 1 shows the potentials of the tower at both the top and bottom. A is the potential at the top and B the potential at the bottom. High over-shooting of the potential at the top of the tower during the first microsecond which is shown by curve A is due chiefly to the impedance of the tower itself. The tower having physical length will have, therefore, during the period of time that the current in the lightning stroke is increasing, a considerable potential across it. However, as soon

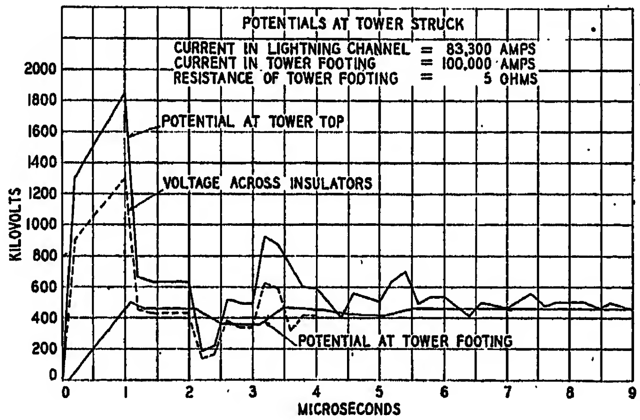

Fig. 3—Tower Potentials

as the wave ceases to increase in magnitude, the potential across the tower will drop to nearly the same potential as that of the tower footing.

The effect of the change in tower footing resistance is shown in Fig. 2. The tower top potentials have been shown with two conditions: (1) with the tower having a 5-ohm footing resistance, and (2) with a 10-ohm footing resistance. From this we see that by raising the resistance from 5 to 10 ohms the tower top potential has been increased only 20 per cent. These curves have been replotted on a voltage scale and are shown in Fig. 3. Three curves have been plotted: (1) the potential and the tower

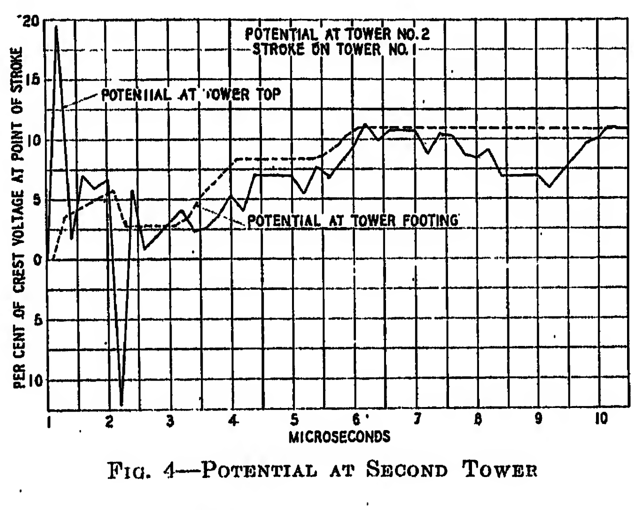

Fig. 4—Potential at Second Tower

top, (2) the potential at the footing of the tower, and (3) the potential across the insulator string. To obtain the potential across the insulator string a 30 per cent coupling was assumed between the ground wire and the conductors, thus the potential across the insulator string is approximately 70 per cent of the potential at the tower top. The maximum current in the lightning channel is assumed to be 83,000 amperes, the maximum current in the tower footing is 100,000 amperes, and the resistance of the tower footing is 5 ohms. The potential at the

adjacent tower has been found to be very much lower, having a volt-time characteristic as shown in Fig. 4. The highest voltage appearing at this tower is some 19 per cent of the potential at the first tower.

The relative currents in the first three towers are shown in Fig. 5. The highest current, of course, appears on the tower on which the stroke terminated.

In order to check these calculations the set up was made in the field in which tests were made on a non-energized line. A portable surge generator and cathode-ray oscillograph were

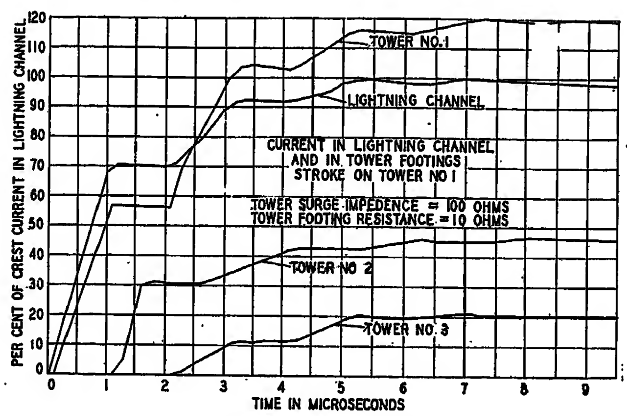

Fig. 5—Relative Currents

utilized in this test. A surge was impressed upon one conductor and allowed to travel over the line, finally reaching a point at which the conductor is grounded through a tower. The wave appearing at this tower had a three microsecond front and dropped to half value in 10 microseconds. This wave is not nearly as severe as that assumed in the calculations and due to experimental limitations a more severe wave could not be obtained. The wave traveling beyond this point was measured by cathode-ray oscillograms of the nature shown in Fig. 6. Inasmuch as the wave has a duration of only 10 microseconds it is to be expected that it could not maintain the potential of

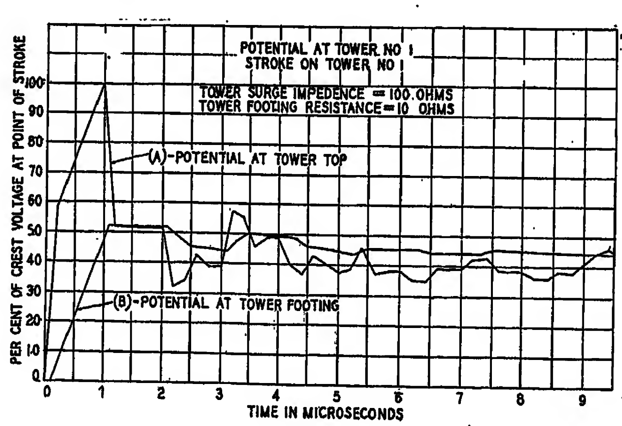

Fig. 6

the tower top to a value shown in the calculated waves, however, the first part of the wave is very similar to that obtained through calculations. The comparison of the two waves is shown in Fig. 7.

The analysis and accompanying curves as presented in this paper are of considerable importance, as they give for the first time a clear insight into the happenings on a transmission line at the time it is struck. From them the types of waves that are to be considered under different conditions can be ascertained. For example, a tower having low tower footing impedance (of the order of 5 ohms) when struck will stress the insulators with a wave having a high potential for one or two microseconds and then abruptly falling to a low value where it remains for a relatively long period of time. However, should the impedance of the tower footing be high (of the order of 50 ohms or more) the potential will rise to a high value and remain there for a considerable time. In the former case only the short time characteristics of insulators need be considered. In the latter, however, because of the flat top characteristics, the long as well as the short time characteristics must be considered. It is information of this type which will greatly facilitate the establishment of standard methods of adjudging insulation.

K. B. McEachron: In discussing the mechanism of lightning strokes Messrs. Fortescue and Conwell discuss the part played by water vapor in the production of sufficiently high charges to cause electrical discharge as in a thunder storm. I believe that it is generally understood that clouds consist of condensed water vapor in the form of actual drops and that these drops of water carry charges which are increased as the result of the action of the wind until finally an electrical discharge occurs.

The authors advance a partial theory of the mechanism of lightning strokes, indicating that the speed of formation is probably not greater than one-twentieth that of light and thus considerable time is involved in the production of the lightning

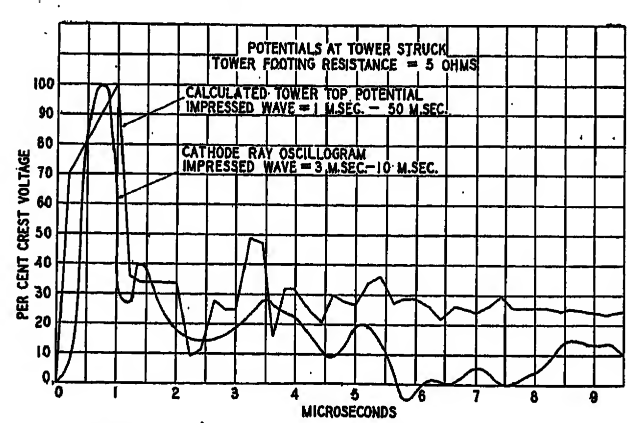

Fig. 7—Comparison of Calculated Potential and Cathode-Ray Oscillogram

discharge. They suggest that the energy in the streamer itself contains a very small amount of current until after contacting with some object on the surface of the earth. After such contact is formed the potential energy is changed to kinetic energy and a current wave moving at the velocity of light flows into the earth or the structure.

It seems to me it would be very helpful if the authors could give a more detailed explanation of just what they believe happens during the interval just before and after the lightning strikes. I do not believe that physicists are in agreement at the present time concerning the mechanism of the lightning discharge. Two different theories have been worked out in much greater detail than that advanced by the authors, one being developed by Simpson and the other by Dorsey. Simpson believes that the discharge channel is formed from the positive end, while Dorsey believes it is formed from the negative end. Dorsey in his "dart" theory suggests a mass of electrons and ions which move with high velocity at the front of the stroke following an irregular path which is determined more or less from each point of travel of the discharge.

It is appreciated that the calculations made by the authors in the first part of their paper are based on several assumptions for which little exact experimental data exist. For instance, it is assumed that the surge impedance of the lightning discharge path is 200 ohms and yet it is assumed that the continuation of this same discharge path on to ground after con-

tacting with the transmission line may be between 10 and 200 ohms. If the discharge path of the lightning can be considered as a surge impedance it undoubtedly is a variable quantity becoming smaller as we approach the surface of the earth. It seems to me that in making this calculation the authors assigned values to unknown quantities about which less is known than the unknown which is calculated, particularly as a cathode-ray oscillogram was taken giving some idea at least of the potential which might occur at the time of the direct stroke.

Based upon a considerable experience gained from the study of the operation of several transmission lines under lightning conditions the separations given between the line wires and the ground wires shown in Figs. 3 and 4 of the paper seem unnecessarily large, as many lines similar to that shown in Fig. 4 with much smaller clearances between line and ground wires with low tower footing resistance have had an excellent operating record. It is interesting to note that if these large spacings are necessary on extra high-voltage lines, equally large ground wire spacing is necessary on lower voltage lines if the spans are of equal length.

In connection with the study on the Siegfried-Roseland line the authors apparently conclude that high voltages cannot be obtained on a highly insulated transmission line except from direct strokes. This conclusion is based on the observation that direct strokes occurred close to the transmission line and yet voltages less than 100 kv. were observed on the transmission line. However, this does not mean that other direct strokes nearby might not cause high voltages on such a line. As an illustration of this situation reference should be made to the paper by Messrs. Sporn and Lloyd in which apparently two different lightning strokes did not give rise to induced voltage, while the third direct stroke apparently caused a recorded voltage at 2,100 kv. on the transmission line. These direct strokes occurred between 300 and 1,300 ft. from the transmission line. It may be that because of the variability of lightning discharges some do cause much higher induced voltages than others so that the conclusion to be drawn is that most direct strokes or at least many strokes are not responsible for high induced voltages.

Messrs. Lewis and Foust in discussing the mechanism of the lightning stroke point out the probability of discharges proceeding from the tower to the cloud when the cloud is negative. While this agrees with experiments made between a point and a plane in the laboratory, assuming the point to be positive, yet not all of the observations in the field confirm this point of view. If the clouds are usually negative for severe direct strokes one would expect it would be a common occurrence to witness long streamers from towers just before the flashover occurs. It is true that such streamers have been observed in a few cases, but apparently they are not a frequent occurrence. In the laboratory test it has been noted that the streamers extend away from the positive electrode and this led Simpson (*Proceedings of Royal Society A*, 1926 Vol. III, p. 56) to comment on the fact that only three out of more than two hundred photographs of direct strokes of lightning showed streamers upward, indicating that the earth was positive.

If long streamers do proceed upward from transmission line conductors and towers frequently some flashovers of transmission line insulation might be accounted for in this manner. This idea was originally suggested to me by Mr. H. P. Seelye as he had seen certain evidence of flashovers which led him to suggest the possibility of intense ionization on towers during lightning storms, which might cause flashovers even though no lightning discharge took place, which either struck the line or gave rise to induced effects.

Further study should be given to the determination of the manner of propagation of discharges in order that definite evidence may be obtained from which an adequate theory may be built up.

L. V. Bewley: There are seven main factors which characterize the magnitude and shape of induced voltage waves due to a lightning discharge.

They are:

Law (and time) of cloud discharge.

Distribution of bound charge.

Current in the stroke.

Charge on the cloud (amount and distribution).

Height of the cloud.

Field gradient.

Average height of the line.

Heretofore investigators have assigned values to some particular group of these variables, without respect to the effect of such arbitrary specifications on the remaining variables, and have arrived at unduly divergent conclusions as to the magnitude of induced voltages. However, as Mr. Peek's paper has so clearly shown, the above variables are not entirely independent of each other, but are definitely related, so that values cannot be arbitarily assigned to any one of them which violate the known confines of any of the others. He has made an outstanding contribution to our knowledge of the magnitude of induced strokes by tying all of these factors down with a single set of curves, Fig. 6 of his paper. By means of these curves, the influences of the time of cloud discharge, of the height of the cloud, and of the current in the stroke, can be readily estimated. Since the curves are based on a maximum field gradient of 100 kv. per ft. (an established value apparently agreed upon by all of the authorities on the subject), they are in conformity with that assumption.

It may be of interest to demonstrate the application of the equations in the Appendix of Mr. Peek's paper, to a specific case.

Cloud: A point charge (or uniformly charged sphere) Q at a height H.

Law of Cloud Discharge: Linear, in time T.

Ground Wires: None, therefore $\omega = 1.0$.

This particular example has previously been worked out by Mr. Fortescue.[1] It is included here merely to illustrate the general equations of which it is a special case. The vertical component of the field gradient near the surface of the earth due to a point charge (Q) at a height (H), and its image ($-Q$) at a depth ($-H$) below the surface, is

$$g(x) = \frac{2\,Q\,H}{(H^2 + x^2)^{3/2}}$$

and therefore the potential on the line at height h for an instantaneous cloud discharge is

$$f(x) = h \cdot g(x) = \frac{2\,Q\,h\,H}{(H^2 + x^2)^{3/2}}$$

If the cloud discharge is linear, then

$$F(t) = \begin{cases} t/T & \text{for } t \leq T \\ 0 & \text{for } t > T \end{cases}$$

and the current in the stroke therefore is

$$I = Q\,\frac{\partial F(t)}{\partial t} = \begin{cases} Q/T & \text{for} \leq t\ T \\ 0 & \text{for } t > T \end{cases}$$

Then by equation (6) in the Appendix of Mr. Peek's paper

$$e = \frac{h\,I}{H\,v}\left\{\frac{x + v\,t}{\sqrt{H^2 + (x + v\,t)^2}} - \frac{x - v\,t}{\sqrt{H^2 + (x - v\,t)^2}}\right\} \text{ for } t \leq T$$

where v is the velocity of propagation of waves on overhead lines, and is approximately 1,000 ft. per microsecond. This potential distribution is seen to consist of a pair of waves traveling in opposite directions. They superimpose for a maximum, at $x = 0$ and $t = T$, of

1. Discussion, C. L. Fortescue, A. I. E. E. TRANS. Vol. 49, p. 1503, October, 1930.

$$V = \frac{2}{v} \frac{h}{H} \cdot \frac{I}{\sqrt{1 + (H/v\,T)^2}}$$

and this increases with the time of discharge T, as the current is assumed to be constant. For the time of cloud discharge in microseconds large (say 3 times) compared with the height of the cloud in thousands of feet, and converting to practical units,

$$V \cong 60 \frac{h}{H} I \quad volts$$

where I is in amperes.

By equation (3) the reduction factor is

$$\alpha = \frac{V}{g\,h} = \frac{60\ I}{g\,H} \frac{1}{\sqrt{1 + (H/v\,t)^2}} \cong \frac{60\ I}{g\,H}$$

The equivalent rectangular bound charge, by equation (7), is

$$L = 2\,H = 2\ (cloud\ height) = (front\ of\ traveling\ wave)$$

The fact that the front of the free traveling wave is practically equal to the length of the equivalent rectangular bound charge was established by calculating the wave shapes for a variety of different distributions of bound charge and noticing that the relationship holds when the time of cloud discharge in microseconds is long compared with the length of the bound charge in thousands of feet—a condition that is true in most cases.

There are only a few simple cases which can be calculated directly by the analytic formulas, due to the difficulty of performing the prescribed operations. However, equation (1) can be expressed as the limit of a summation, and therefrom solutions obtained by tabular step-by-step and graphical methods.[2]

Theoretically, from an oscillogram of an induced lightning surge which has not suffered attenuation or distortion, the following information is obtainable:

1. (2) (*Height of cloud*) = (*length of bound charge*) = (*front of wave*)
2. (*Time of discharge*) = [(*total length of wave*) − (*front of wave*)]
3. *Law of cloud discharge* from the solution of the integral equation

$$e'(t) = \tfrac{1}{2} \int_0^t f\,[x + v\,(t - \tau)\,]\,\frac{\partial F(\tau)}{\partial \tau}\,d\,\tau$$

The function $f(x)$ may be taken as that due to a point charge, or even as a rectangle, without appreciable error; because the shape of $e'(t)$ is not sensitive to changes in $f(x)$. If an analytic solution cannot be obtained, then trial functions $F(t)$ may be tried until one is found which approximately satisfies the relationship. Since the form of $e'(t)$ for many representative functions $F(t)$ have been worked out and tabulated, a fair idea of $F(t)$ for a given $e'(t)$ is not hard to arrive at, within the permissible limits of error.

Since $F(t)$ must be such a function that it reaches a value of unity at $t = T$ (the completion of cloud discharge) and since the crest of $e(t)$ is given, the maximum value of $f(x)$ is also determined by this integral equation. Then

4. The *maximum gradient* is $g(x) = \dfrac{2\,f(x)}{h}$

5. The *reduction factors* are

$$\alpha = \frac{e_{max}}{h\,.\,g\,(o)} = \frac{e_{max}}{h\,.\,g\,(o)}$$

$$\alpha' = \frac{e'_{max}}{h\,.\,g\,(o)} = \frac{e'_{max}}{2\,f\,(o)}$$

6. The *charge on the cloud* (assuming a spherical cloud uniformly charged) is

$$Q = \frac{H^2\,g\,(o)}{2}$$

7. The *current in the lightning stroke* is

$$I = Q\,\frac{\partial F(t)}{\partial t} = \frac{H^2\,g}{2} \cdot \frac{\partial F(t)}{\partial t}$$

8. The cloud potential (assuming a spherical cloud of radius R) is

$$E = \frac{Q}{R} = \frac{H^2\,g}{2\,R}$$

It may be challenged that a sphere is not a true representation of a charged cloud. That is true, but by fortunate circumstances the shape of the cloud (or of the bound charge) does not seem to have much effect on the shape of the traveling waves, and consequently the assumption of a spherical cloud is as good as any for numerical accuracy, and has the distinct advantage of being relatively easy to handle mathematically.

As a result of their studies Messrs. Brune and Eaton have concluded that the counterpoise is in effect a very leaky short transmission line connected to the tower and open-circuited at its other end. This conclusion is evidenced by the highly attenuated and badly distorted reflections shown on the oscillograms. It is surmised, however, that the major benefit to be derived from the

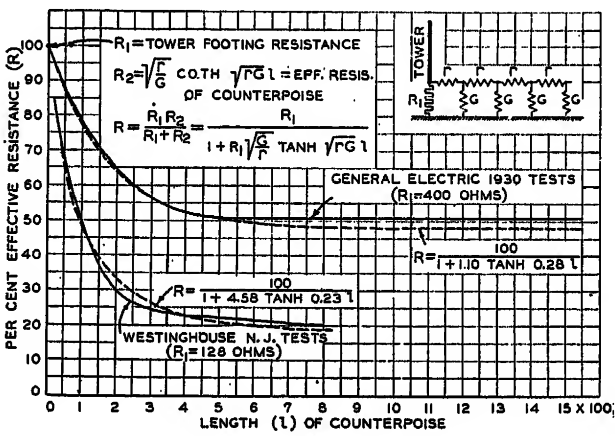

FIG. 8—EFFECT OF COUNTERPOISE

use of a counterpoise, is merely that of the reduction in true grounding resistance which it provides; and that the surge impedance and reflections are quite subsidiary effects. In support of this notion there have been plotted in Fig. 8 the field tests of two different investigations on the effects of varying the lengths of counterpoises, and the corresponding calculated curves based on assuming the counterpoise to be a simple distributed series resistance r and conductance to ground g. The constants used in the calculated curves were chosen to agree with the experimental results, so that the agreement between the test and calculated curves is of importance only in showing that they presumably follow the same law—in other words a distributed r and g is sufficient to account for the observed data within the limits of the experimental error in this type of field study. The equations are:

$$e = E\,\frac{\cosh \sqrt{r\,g}\,x}{\cosh \sqrt{r\,g}\,l} = \text{voltage at } x$$

$$i = E\,\sqrt{\frac{g}{r}}\,\frac{\sinh \sqrt{r\,g}\,x}{\cosh \sqrt{r\,g}\,l} = \text{current at } x$$

$$R_x = \frac{e}{i} = \sqrt{\frac{r}{g}}\,\coth \sqrt{r\,g}\,x = \text{effective resistance at } x$$

2. *Traveling Waves Due to Lightning*, L. V. Bewley, A. I. E. E., TRANS. Vol. 48, p. 1050, July 1929.

l = total length
x = distance from open end
r = resistance per unit length
g = conductance per unit length

The net resistance at the tower is the tower footing resistance R_T in parallel with the effective resistance of the counterpoise R_l, or

$$R = \frac{R_T R_l}{R_T + R_l} = \frac{R_T}{l + R_T / R_l} = \frac{R_T}{l + R_T \sqrt{g/r} \tanh \sqrt{r g}\, l}$$

which is the equation plotted in Fig. 8. In this equation it must be remembered that R_T and (l/g) under transient conditions are only about 75 per cent of the measured d-c. resistance; and that r is much greater than the measured d-c. resistance due to skin effect. Also the conductance g is difficult to measure, and probably varies over a wide range with the condition of the soil (moisture content), so that the practical utility of such an equation is very doubtful. Its chief merit lies in showing a reasonable correspondence with the observed experimental data, and therefore establishing to that extent the reason for supposing that a counterpoise is principally merely a low resistance to ground. But if that is so, then a given length of wire is most efficiently employed as a counterpoise by using it as a number of short lengths in parallel radiating out from the tower, rather than as a single length. And on this basis also, there is no particular virtue to the direction of the counterpoise—it might just as well be run perpendicular to the transmission line as parallel to it.

F. E. Andrews: The experience on the 132-kv. lines of the Public Service Company of Northern Illinois is of interest relative to the question of protection afforded by overhead ground wires. These lines are in general of double-circuit steel tower construction, using a single ground wire only over the one circuit installed on the single-circuit portions of these lines, and using two ground wires, one over each circuit, on the double-circuit sections. The single-circuit lines with one ground wire are all built on double-circuit towers, one side being left vacant for the future circuit.

The following tabulation summarizes this experience:

TWO GROUND WIRES

Year	No. interruptions	No. circuit miles	Trip-outs/100 miles due to lightning
1926	0	47.57	0.00
1929	1	158.06	0.63
1930	*3	158.06	1.90
Interruptions per 100 circuit miles per year over entire period			1.10
Interruptions per 100 tower-line miles per year over entire period			2.20

*Two of these simultaneous flashovers, third flashover from 132-kv. line to 33-kv. line on same right-of-way.

ONE GROUND WIRE

Year	No. interruptions	No. circuit miles	Trip-outs/100 miles due to lightning
1925	0	9.10	0.00
1926	0	9.10	0.00
1927	3	84.93	3.54
1928	3	112.25	2.67
1929	2	102.27	1.96
1930	9	132.59	6.80
Interruptions per 100 miles per year over entire period			3.78

ALL LINES SINCE GROUND WIRES WERE INSTALLED

Year	No. interruptions	No. circuit miles	Trip-outs per 100 miles due to lightning
1925	0	9.10	0.00
1926	0	9.10	0.00
1927	3	84.93	3.54
1928	3	159.82	1.88
1929	3	260.33	1.15
1930	12	290.65	4.13
Interruptions per 100 miles per year over entire period			2.58

It will be noted that with the lines protected by two ground wires, interruptions per 100 circuit miles per year average 1.1, whereas with the lines protected by one ground wire only the average is 3.78 interruptions per 100 miles per year. If it is considered on the double-circuit lines that the exposure to lightning is proportional to tower line miles rather than circuit miles, the interruptions per 100 tower-line miles per year with two ground wires would be 2.2. Either of the two figures for the lines equipped with two ground wires indicates a material improvement in performance in comparison with the lines equipped with one ground wire only.

Another interesting instance showing the protection of the ground wires can be cited on our Chicago Heights—Joliet line which had a four-mile section of ground wire removed during 1930 in the vicinity of the Frankfort lightning oscillograph station. During 1927, 1928, and 1929 there had been no trip-outs due to lightning in this section protected by the ground wire although during the two years previous to this, *i. e.*, in 1925 and 1926 during which there was no ground wire installed, there had been a number of lightning interruptions through this section. During 1930, with the ground wire removed, there were three lightning trip-outs in this section.

C. L. Fortescue: The first part of Mr. Peek's paper which seems to me to be a plea for the induced stroke theory is written in a convincing manner. If it were not for the fact that I have what I believe to be good physical grounds for disagreeing with him I might even be convinced myself. The basis on which the conclusion was arrived at by the group of engineers working on the lightning investigation, with which I have been associated, was that induced surges were probably not a factor in producing outages due to lightning on high-voltage lines. Last summer I made a mathematical analysis of the magnitude of induced voltages that would appear on a line due to a stroke to earth nearby and the value I obtained for a cloud discharging at the rate of 200,000 amperes was of the order of 120 kv. Mr. Peek says in his paper that the low value which I obtained was due to my assumption that the cloud was 5,000 feet high and he states, though I do not clearly follow his reasoning, that if a lower height had been chosen the induced voltage obtained would be much higher. If I understand his reasoning, it is based on the fact that with 20 coulombs the field due to the cloud at the lower height would be very much more intense. However, the 20 coulombs and the height chosen will give a field of 50 kv. per foot at the earth's surface which is a fairly intense field and with lower heights the same field gradient will require correspondingly lower charges.

It would seem reasonable that the formation of the lightning streamer would be associated with some minimum value of field intensity and therefore the clouds at the lower height would be apt to discharge at a lower energy content and therefore produce lower induced surges than clouds of higher altitude. This seems to be in accord with field experience, in that in the parts of the country where the ceiling is low the thunderstorms are more frequent but apparently not so violent as in those parts where the ceiling is high.

As to the physical conditions which I feel absolutely prevent the cloud from discharging at the rates required to produce a high induced voltage, the thundercloud itself is bipolar, that is to say it has a positively and a negatively charged portion. This has been determined by actual observation and is an experimental fact. The very nature of the thundercloud requires that its atmosphere shall be a good insulator. In fact, it is probable that the cloud atmosphere is a better insulator than atmospheric air at the earth's surface on account of the absence of free electrons. This atmosphere of the thundercloud consists of discrete particles of water vapor charged with electricity. There are several theories accounting for the presence of these charged particles and the bipolar nature of the thundercloud. That of Dr. Simpson of the British Meteorological Department, is perhaps the best known and the one that is generally accepted. According to this theory the large drops of water are broken into spray by the wind and in doing so the spray is negatively charged and carried upward by the wind and in descending forms the negative portion of the cloud, the first portion of the cloud consisting of large water drops becomes positively charged. The two portions of the cloud atmosphere are kept separated by the wind due to a difference in their mobility. However, if free electrons were present in large quantities these would not be affected by the wind but only by the electric field in the cloud, as a consequence of which they would discharge the two portions of the cloud as fast as they become charged. It is a fact that the particles of water vapor have the faculty of capturing the stray electrons that make possible the formation of the thundercloud.

The particles of water vapor have very little mobility and consequently when the lightning channel is formed between cloud and earth they cannot by their motion discharge the cloud. The only way that the charge of the cloud can be carried to earth is by ionization of the cloud atmosphere itself and this is a slow process requiring a high gradient, and the velocity of discharge instead of being of the order of the velocity of light will probably be of the order of one-twentieth this velocity.

I cannot follow Mr. Peek's reasoning regarding the relation between the time from zero to crest of the wave and the height of the thundercloud. I can see no relation whatever myself. For example, in the case which I considered last summer with an infinite line and the cloud discharging at a uniform rate, the analysis shows that the induced voltage will continue to increase until the discharge ceases and then it will decrease logarithmically. Here, where I took a uniform rate of discharge of the cloud of 200,000 amperes, the total quantity discharged being 20 coulombs, the time from zero to crest would be 100 microseconds. It also seems to me that the assumption that the undistorted positive surge, where no outage had taken place on the line, is induced, is not tenable, for in probably one-half the cases where a direct stroke takes place to the line no outage occurs even though the insulator flashes over.

Regarding the time of discharge of surges due to direct strokes, as I have pointed out many times, a cloud discharge may or may not reach earth depending upon whether the energy given up by the cloud is sufficient to support the streamer in its path towards earth; consequently, the lightning strokes to a line may be weak and of short duration or intense and of long duration. I think it may be stated quite definitely that those of short duration have very little potential while those of long duration have a high potential. To support Mr. Peek's theory of induced voltage this condition would have to be reversed, that is to say, intense strokes would have to be of short duration and the weak strokes would have to be of long duration. These, in brief, are the physical reasons why I do not believe that it is possible to get a very high voltage induced in a transmission wire due to a stroke of lightning to the earth that does not hit the wire.

Referring to the paper by Lewis and Foust, I would like to point out that one is not justified in assuming that a stroke which reaches the earth near a transmission line will not also make contact with the line itself. In fact it is highly probable that even though the main part of the stroke hits the earth a streamer may strike the line or that a second stroke following in general the path of the first may make electrical contact between cloud and line in either of which cases, of course, the surge resulting cannot be considered as an induced surge. Furthermore on transmission lines well protected with ground wires when the ground wire is struck the conditions are ideal for producing a high-induced surge were such a surge possible, since there is a bound charge on the line wires which is released when the lightning stroke is discharged to earth through the ground wires and towers. However, we have no record of any such high-induced surges despite the fact that our cathode-ray oscillographs have been connected to lines which were known to be struck by lightning.

In the paper by Mr. Lloyd the means by which he arrived at the 670,000 amperes of lightning current, shown in this table, is not permissible. I understand that the currents in his six towers were added directly. Since these maxima occur at different times and a considerable time after the maximum crest of the lightning stroke they cannot be added directly. As far as I can see, if the measurements are correct the current that can be assigned to the lightning stroke is not over 300,000 amperes.

F. W. Peek, Jr.: Contrary to Mr. Fortescue's impressions, the first part of my paper is not a plea for induced voltages. I have shown how induced voltages are affected by such factors as height of cloud, time of discharge, current, height of line, gradient, etc. Then, by inserting *measured values* of current, time, etc., I have shown the possible range of induced voltage values on lines. It is quite evident that much higher induced voltages result from certain types of lightning discharges than from others. Some discharges near a line would be expected to cause high voltages while others would not. The relative occurrence of these types and therefore the frequency of high-induced voltages is being determined statistically by cathode-ray measurements. High-induced voltages can occur. How often are the conditions such that they do occur?

A similar analysis has been made in my paper of direct strokes and methods of determining their effect developed.

In general it can be said that as the operating voltage, or rather the line insulation is increased, the direct stroke becomes an increasing cause of outage. For the low-voltage line the induced voltage is the principal cause of trouble. It cannot be said that there is any definite dividing voltage at which one or the other is the more important. There are too many varying factors involved for different locations. *For high-voltage lines the design must be made with full consideration of direct strokes.* Fortunately, direct-stroke remedies are also good induced voltage remedies. I have given a number of examples in my paper.

Mr. Bewley in his discussion has brought out some very important facts. One of these that I also wish to emphasize is that the different variables such as time, voltage, current, are definitely related and that erroneous conclusions will be reached if arbitrary values are assigned to one group without reference to the other. This has been largely done in the past by some investigators.

Perhaps the most interesting fact brought out in my paper is that complete information regarding a lightning stroke including height of cloud, voltage, etc., is given by the oscillogram of the traveling wave on a transmission line. Such a wave is in fact an illustrated autobiography of the stroke.

Mr. Fortescue has asked how a cloud lower than 5,000 ft., which he assumed in his example cited in my paper, could cause a higher induced voltage. Mr. Fortescue will find that the equation which he used in calculating his example can be reduced to the following form:

$$V \simeq 60 \frac{h}{H} I \text{ volts}$$

Where h = height of line in feet
H = height of cloud in feet
I = current in amperes

With the equation in this form it can be readily seen that if the cloud height H is reduced with a given constant discharge current the voltage is increased. For example, with a cloud 5,000 ft. high the induced voltage at 200,000 amperes discharge is 120,000 volts on a 50-ft. line wire; with a cloud 1,000-ft. high the induced voltage at 200,000 amperes discharge is 600,000 volts on a 50-ft. line wire. Examples of this are given in Table II of my paper.

Mr. Fortescue speculates on the time that it takes a cloud to discharge. I have avoided this speculation and have considered values of time measured by the cathode-ray oscillograph. The time of cloud discharge should be no longer than the total time recorded by the oscillograph. (See Fig. 2 of my paper.) Mr. Fortescue states that his 5,000-ft. cloud discharging at a velocity of 1/20 of that of light would require 100 microseconds to complete its discharge. A great majority of the cathode-ray oscillograms of natural lightning are much less than 100 microseconds and some of the more severe ones are of the order of 10 microseconds.

In support of his belief that no relationship holds between the front of the wave and the height of the cloud Mr. Fortescue states that the final crest is not reached until after 100 microseconds. It is quite true that the final crest is not reached until 100 microseconds have elapsed in his example. However, if the calculated curve is plotted it will be seen that the voltage reaches 90 per cent of its final value at the end of 10 microseconds and it takes 90 microseconds to reach the remaining 10 per cent. *Practically speaking, therefore, the crest* of the wave is reached in 10 microsec. Thus,

$$2\,H = 10 \text{ microsec.}$$

or

$$H = 5 \text{ microsec.} = 5{,}000 \text{ ft.}$$

In other words, the height of the cloud is determined from the time from zero to approximate crest of the wave.

If the bound charge had been rectangular the front of the wave would have been exactly the length of the bound charge. For other distributions it is very nearly so.

The energy necessary to complete a lightning discharge is less for short streamers than for long ones. It follows, therefore, that although the energy of a low cloud may be less than for a high cloud a discharge may nevertheless take place.

Mr. Fortescue has questioned Mr. Lloyd's method of arriving at the total current in the lightning stroke by adding the currents measured in the adjacent towers. I have analyzed this method and illustrate the results in Fig. 9 of my paper. This analysis shows that with moderately low tower footing resistances approximately correct values of total current are obtained by adding the values measured in the several separate towers. Correction may be made from Fig. 9.

C. L. Fortescue: I pointed out in my discussion of Mr. Peek's paper that the nature of the thunder cloud does not permit of very quick discharge of its energy, and, therefore, the induced surges are not of high enough magnitude to affect high-voltage transmission lines. It is quite possible, however, that for the lower voltage lines, those below 66,000 volts, the surges produced by induction may be a factor in causing outages.

In answer to Mr. McEachron's questions I think that I have made it plain in my discussion why I believe it impossible for a thunder cloud to discharge at a high rate. However, I have mentioned the speed of propagation of the lightning channel between ground and earth as being of the order of one-twentieth the velocity of light. My basis for this estimated velocity is the speed of propagation of streamers between two spheres. In tests with suppressed charges this velocity was estimated to be of the order of one-twentieth the velocity of light. This, however, is merely an approximate figure. It is probable that the speed of propagation is not as high as this. If I remember correctly, Dr. Toepler estimates it even lower than this. I think his figure, based on analysis, gave it as of the order of one-thousandth the velocity of light.

With regard to the nature of the lightning discharge during the progress of the streamer, the space charge on the outside of the streamer is progressively fed by the actual current in the channel which is comparatively small. However, the air in the core of the channel is probably highly conducting through thermionic disassociation of the air molecules. On account of the fact that current in this core is small most of the energy in the channel before it reaches the earth is potential energy in the form of space charge. However, when the streamer reaches a transmission line, this potential energy is at once changed into kinetic energy which results in a surge propagated in the transmission line away from the channel and a receding surge propagated in the channel changing the potential energy into kinetic energy, or, in other words, producing current flow in the channel. When this receding wave reaches the cloud, it at once sets up a high gradient in the cloud causing the release of more of the cloud's energy. This gradient is, of course, propagated into the cloud producing ionization and even streamer formations.

I do not agree with Dorsey's dart theory nor do I believe with Simpson that the streamer is initiated at the positive end. I believe that the streamer is set up by conditions at the surface of the cloud which produces high enough gradient to cause ionization and that this condition will persist so long as the cloud is able to furnish sufficient energy to keep the streamer progressing. This means that at the end of the streamer space charge must be formed high enough to cause progressive ionization. The streamer, therefore, progresses as fast as it can with the energy available, the speed being controlled by the rate at which ionization can take place.

After the channel has been established between the cloud and earth, it then forms a conducting path and the propagation of the energy from the cloud to the earth along this channel is determined by the law of propagation of surges along conductors. However, as Mr. McEachron remarks, surge impedance along the channel is not uniform but will depend upon the height above earth, but the only value that we are concerned with in estimating the effect on transmission lines is the value at the point of the stroke which I have estimated to be 200 ohms. This figure may be too high or too low, but it is convenient to use as a basis for estimating the surge potentials that may rise on transmission lines due to lightning. Of course, just at the instant before the streamer strikes the transmission line, its actual potential will be double that assigned to the kinetic surge, which conforms to the law of surge propagation. I agree with Mr. McEachron that research should be carried out to determine the proper value to give to this surge impedance and also the mechanism of the streamer formation should be given considerable study, but we cannot wait for the results of these investigations in designing transmission lines.

Mr. McEachron also remarks that the values given for spacing between ground wires and lines in the middle of the span seem to him excessive. This may be true and there are lines with maximum spacings of the order of 15 to 17 ft. which are giving a good record in certain parts of the country. Some recent laboratory work indicates that a lower spacing will be satisfactory, perhaps a maximum of from 30 to 35 ft. This lower spacing results from the fact that the coupling between ground wire and line wires is much higher than that obtained from the physical dimensions of the wires, on account of the large diameter due to corona formation on the ground wires at the instant of stroke. In short, I am in accord with most of what Mr. McEachron says and agree with him that we have a lot more to

learn about lightning though we have made very marked progress iu the last few years. However, I feel that it is better to err on the safe side and use somewhat larger clearances than necessary than to use too small clearances and regret it later on.

In his discussion of the induced stroke versus direct stroke theory, Mr. McEachron assumes that when the earth or a tree is struck in the neighborhood of a transmission line that the high voltage appearing on a transmission line as a result of the stroke is necessarily induced. I wish to suggest that when a high resistant object is struck the potential of the streamer may have a value of 40,000,000 volts. Under these conditions it is quite possible for a side streamer to also hit the transmission line. The lightning stroke as we see it is made up of a succession of discharges, some of which might hit the tree and others of which might be diverted to the transmission line. The conclusions that outages on high-voltage transmission lines are caused by direct strokes and not by induced surges are not only based on observations of the past few years but also on analysis. In order to produce potentials of the order of several million volts, it will be necessary to have discharge current in the lightning channel of the order of several million amperes. Such measurements as have been taken in the past show no indication of discharge currents of such large order of magnitude.

W. W. Lewis and C. M. Foust: Referring to Mr. Cox's discussion, the antennas adjacent to the Pennsylvania line are connected to ground through a few megohms resistance. They are intended to give a measure of the cloud field potential at the height of the antennas and not to measure the potential which would be induced on the line.

In the next to the last paragraph, Mr. Cox states that Mr. Torok's work indicated that a discharge streamer always started from the electrode of high potential (gradient), that is, the electrode of smallest effective dimensions, whether such electrode is positive or negative. This is equivalent to stating that polarity has no importance in connection with breakdown between electrodes of dissimilar shape.

As pointed out in our paper, a considerable amount of published data, showing the importance of polarity effects on such breakdown is available. Attention is especially called to Reference 18. The authors have presented considerable evidence which demonstrates the vital importance of polarity in the mechanism of breakdown in an article, entitled, "Direct Strokes to Transmission Lines," *G. E. Review*, August 1931.

The final paragraph of Mr. Cox's discussion refers to a suggested explanation for the high-positive conductor potentials recorded at one station with very little or no voltage at adjacent stations. We agree with Mr. Cox that the apparent rapid attenuation indicates that these are not induced surges of the commonly accepted kind. The second explanation the authors feel to be more reasonable. There are considerable data showing that the tower structure and adjacent ground are elevated to high potentials when the direct-stroke currents pass through the tower structure. The conductor tends to remain at ground potential. The measuring instrument connected between the conductor and the bottom portion of the tower measures a potential between the highly-charged tower structure and the conductor. The instrument, of course, indicates a conductor polarity opposite to that of the tower itself. The line conductor record of positive polarity on this basis indicates a direct stroke to the tower structure of negative polarity.

Mr. McEachron refers to Simpson's classification of streamers branching away from and toward the cloud. Further consideration of this classification might indicate somewhat different conclusions. The large number of unbranched discharges would appear to the authors to be logically classified as strokes between negatively charged clouds and positively charged earth. The upward streamer branching is very likely entirely concealed from the observer, as it takes place in the cloud itself. The preponderance of branched discharges found among collections of photographs is, undoubtedly, influenced by a desire to retain the more spectacular pictures.

The field distribution between cloud and transmission line tower is such that any extension of breakdown streamers from the tower toward the cloud would result in an increasing gradient at the streamer tip and, thus, a rapidly increasing streamer length. Long streamers proceeding from the tower top and terminating in space would not be common on this basis.

An Electric Analog of Friction

For Solution of Mechanical Systems Such as the Torsional-Vibration Damper

BY H. H. SKILLING[1]

Associate, A. I. E. E.

Synopsis.—When the mathematical solution of a problem is so difficult as to be impracticable it sometimes is possible to obtain the desired answer from the behavior of an analogous electric circuit. This article presents an analog of "coulomb friction," and demonstrates the application of this analog to the previously unsolved problem of the torsional-vibration damper.

SOLUTION of the equation of motion for the torsional-vibration damper represents one of the mathematical problems which is extremely difficult to solve. Almost endless complications are involved on account of the peculiar combination of vibratory motion and friction which the problem contains. An approximate solution can be arrived at by introducing several assumptions, but this is not entirely satisfactory; in the more intricate cases, approximate solutions are of no value at all. In order to predict the effectiveness of new designs, mechanical models sometimes are built and tested. Difficulty often is experienced, however, in the construction of these models, and in addition, the results obtained have been found to be somewhat uncertain.

In this article is proposed a method of solution wherein electric analogs are set up for the various types of mechanical motion involved. An electric circuit can then be set up with constants analogous to the mechanical characteristics of the apparatus being studied. The principal advantage in treating the problem in this manner is in dealing with a system easily put together and altered, and with quantities easily metered. Although the analogous method in general is not new, difficulty has been experienced previously in representing mechanical friction electrically. In the present investigation this obstacle has been overcome by the use of gaseous conduction tubes.

The torsional-vibration damper is in effect a flywheel that fits loosely on a shaft. A common application is on the crank-shaft of a Diesel engine, where there is danger of torsional stress in the shaft becoming extremely large because the natural frequency of oscillation of the shaft may be the same as the firing frequency of the cylinders. The damper flywheel is placed on the shaft at the point where torsional vibration is greatest; it is not keyed fast but turns with the shaft because of the friction between the two. When rapid torsional vibration of the shaft exists, the friction force is not great enough to accelerate the flywheel as rapidly as the shaft; hence the shaft slips within the flywheel and in so doing dissipates sufficient energy to prevent resonant oscillation of the shaft. In actual design, the damper although simple is not so elementary as this account has indicated. A detailed description may be found in a paper by Den Hartog and Ormondroyd (*A. S. M. E. Trans.*, Vol. APM-52-13, p. 133) which also treats its action by an approximate mathematical solution.

With ordinary dry friction—so-called "coulomb friction"—which is involved in the torsional-vibration damper, the frictional force depends upon the nature of the rubbing surfaces; also upon the force holding them together, but not upon velocity. An electric analog for this application must therefore produce a constant voltage drop, variable at will but independent of the amount of current flowing.

TABLE I—MECHANICAL-ELECTRICAL ANALOGIES

Mechanical quantity	Electrical quantity
Torque	Voltage
Velocity (angular)	Current
Displacement (angular)	Charge
Moment of inertia	Inductance
Elastance (torsional)	Elastance
Mechanical hysteresis	Resistance

A mechanical system which is the practical equivalent of a Diesel engine may be seen in Fig. 1. The main flywheel is shown at A; moment of inertia of the pistons, cranks, and other moving parts is represented by another flywheel B; the damper flywheel which fits loosely on the shaft is shown at C. Input torque from the cylinders, which may be considered as a steady torque with a superposed alternating torque, enters the system at the pistons. Any load on the engine will be applied to the left of the main flywheel.

The electrical analogy to this mechanical arrangement is shown also in Fig. 1 where, with the exception of friction between the shaft and the damper flywheel, the electrical quantities used are as shown in Table I. Analogous to the shaft is condenser C; analogous to the piston flywheel B is inductance L. These cooperate to form the oscillating system, since the main flywheel is so large that for the present it can be considered infinite and therefore impassable by any vibrations. The damper flywheel C is represented by the inductance L, but because of its looseness on the shaft a shunt must be

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connected around the inductance to make the analogy complete. In this shunt is the apparatus which, to be analogous to friction of the flywheel on the shaft, must prevent current from flowing until a certain voltage is reached, and then permit current to pass without any further increase of voltage.

The peculiar property of constant voltage drop is possessed by a mercury arc; it is also characteristic of glow discharge under special conditions. For this

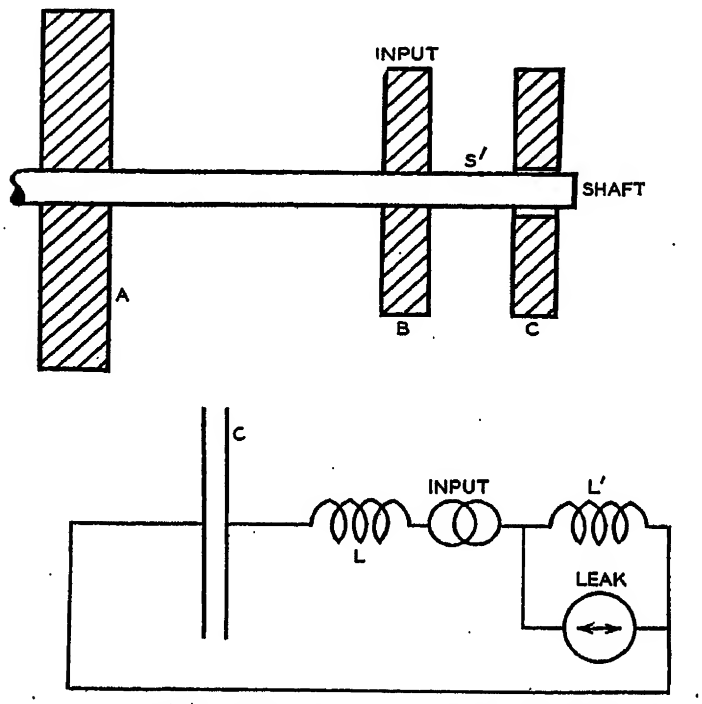

Fig. 1—(Above) Mechanical System Schematically Representing the Diesel Engine; (Below) the Electric Analog of this Mechanical System

investigation raytheon gaseous conduction tubes were employed. Two tubes were used, one to pass current in each direction; the voltage drop in the tubes as they were connected in this case was 85 volts independent of current. In series with each tube was a source of potential which either added to or subtracted from the 85-volt tube drop depending upon other conditions. For the input to the circuit an alternator was used the speed and voltage of which could be varied over an extremely wide range; its resistance, together with that of the rest of the system, was small. For mechanical hysteresis losses in the elastic parts, and for any viscous friction that may exist, resistance is sufficiently analogous to permit its use.

The validity of the complete analogy may be seen from the differential equations of the two systems which are adapted from Den Hartog. For the mechanical system,

$$I \frac{d^2 \alpha}{d t^2} + I' \frac{d^2 \alpha}{d t^2} + k \alpha = M \sin \omega t$$

when the damper flywheel is not slipping, and

$$I \frac{d^2 \alpha}{d t^2} + k \alpha \pm T = M \sin \omega t$$

when this flywheel is slipping.

I = moment of inertia of system being damped
I' = moment of inertia of damper flywheel
k = elastance of shaft
α = flywheel displacement from neutral
$M \sin \omega t$ = input torque as a function of time

The analogous electrical equations, when the shunt is not working and when it is, are respectively,

$$L \frac{d^2 q}{d t^2} + L' \frac{d^2 q}{d t^2} + \frac{1}{C} q = e \sin \omega t$$

$$L \frac{d^2 q}{d t^2} + \frac{1}{C} q \pm v = e \sin \omega t$$

All terms are well known except v, the voltage drop in the leak, which is added or subtracted depending upon the direction of the current. The analogy of corresponding terms, displacement, and charge, for example, may be seen to agree with Table I.

The purpose of making a detailed study of damper performance, whether by mathematics, model, or analogy, is to determine whether a particular damper

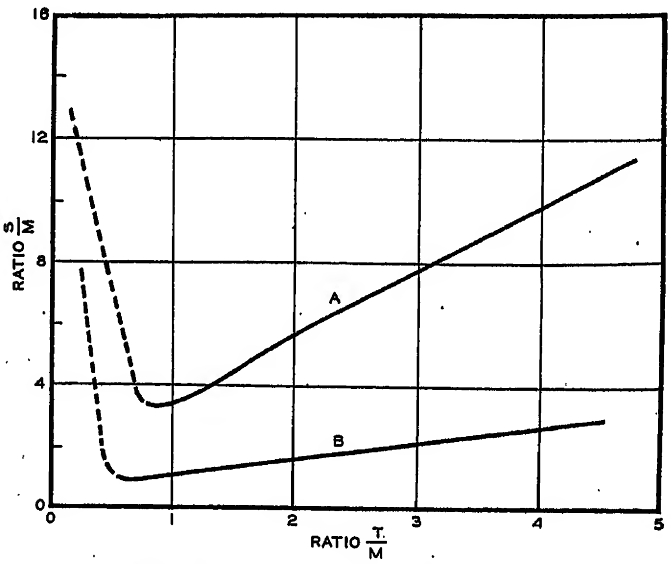

Fig. 2—Performance Curves for the Torsional-Vibration Damper as Determined by Electrical Analogy. (A) With Rigid Damper Shaft; (B) With Elastic Damper Shaft

is so proportioned as to prevent dangerous stresses in the shaft. Momentary stresses in a shaft are exceedingly difficult to measure, but the analogous voltages across the condenser in the electric circuit may be read quite easily on a crest-reading voltmeter; thus for any given combination of quantities and dimensions an indication of maximum stress is obtained immediately by simply reading that meter.

The crest voltage of condenser C (Fig. 1) is analogous

to the maximum torque in the shaft and will be designated by S; analogous to the torque required to slip the shaft inside the damper flywheel, is the voltage drop in the leak which will be denoted by T. Results of a study by such analogy may be plotted showing S as a function of T, where curves similar to those of Fig. 2 will be obtained. If the value of S at the lowest point of the curve represents a safe stress, the damper is large enough to be useful; the value of T corresponding to this minimum value of S gives the optimum adjustment for tightness of the damper flywheel.

To avoid changing back and forth from mechanical to electrical units, it is well to avoid absolute values of quantities. Relative values need not be so transformed. Let M be the amplitude of the alternating input torque; instead of plotting S as a function of T, S/M may be plotted as a function of T/M. This scheme involves neither volts nor foot-pounds but only ratios, the ratio of voltage to voltage being of course numerically equal to the analogous ratio of torque to torque. The scale

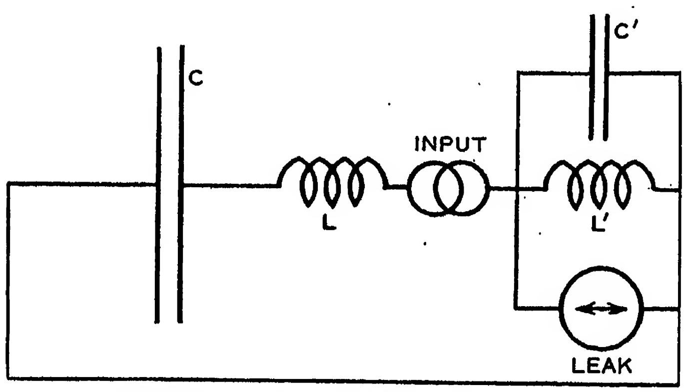

FIG. 3—ELECTRIC ANALOG OF THE TORSIONAL-VIBRATION DAMPER WITH ELASTIC DAMPER SHAFT. COMPARE WITH FIG. 1

of the electrical model is equally unrestricted; only the proper ratios between parts need be specified. The frequency of the input voltage is of course the frequency causing the most severe stress; in the simple mechanical system the crucial stress will occur when the frequency of firing is the same as the natural frequency of the vibrating system.

As a typical example, assume a damper with moment of inertia of the damper flywheel equal to the moment of inertia of the system being damped. In the analog any convenient capacitance and inductance may be used to represent C and L (Fig. 1), but they must be resonant at some frequency attainable by the alternator used for *input*. The inductance L' equal to L is then added, and the vacuum-tube leak is connected across L'. The input voltage is adjusted to any convenient value, and the voltage drop in the leak circuit made comparatively large (for instance, 15 volts input and 85 volts drop in the leak). Now, as the frequency of the input voltage is varied, the crest voltage across condenser C will rise to a maximum and then decline. The maximum value of this voltage (in this particular case about 208 volts) is analogous to the maximum torque in the engine shaft. Dividing this by the maximum value of the input voltage, (about 21.2 volts) and the result S/M is about 9.8; divide the 85-volt drop also by 21.2 and the quotient, which is 4, is the value of T/M. By varying the leak and input voltages, enough points can be determined to give curve A of Fig. 2. This curve gives the whole story of the operation for this particular damper. Moreover, it gives the entire story of operation for all dampers having the same moment of inertia as has the system being damped. In present practise, however, a damper has from one-fifth to one-tenth the amount of inertia of the system being damped, and S/M is proportionately larger.

So far, the damper has been considered as being applied to the simplest possible vibrating system. This simple system may be solved mathematically, although with some difficulty if approximations are avoided. Accordingly, this case was used as a general check on the method, and the results were compared with Den Hartog's computed values; agreement was so close as to be within the limits of experimental error.

With the more complicated systems, however, solution by analogy is of far greater value. When the elastance of that part of the shaft labelled S' (Fig. 1) is considered, a mathematical solution is out of the question; in the analog a condenser of the proper size is simply shunted around L', and the solution is as simple as before. The study of this circuit disclosed an interesting and important relationship that previously was not recognized. As it is a typical analog, it will be used as a further example of the application of the method.

The circuit used with C' representing the elastance of the damper-shaft is shown in Fig. 3. The elastance of condenser C' must bear the same proportion to condenser C as elastance of the damper-shaft does to the main shaft. Now as the input frequency is varied, the voltage measured across C no longer has one maximum, but three. The reason for this is apparent from the diagram, since there are now two resonant parts of the circuit, consisting of (1) C and L and (2) C' and L'. At some definite frequency the reactance through C and L in series is zero; at two additional frequencies the reactance of the complete circuit is zero. However, if the natural frequency of C' and L' is the same as that for C and L, when the reactance through C and L is zero, the reactance through C' and L' is infinite; thus the circuit current under these conditions is small and one maximum eliminated. The other two maxima, one falling at a higher and one at a lower frequency, must be prevented from becoming serious by the action of the leak.

Experiments with this second analog showed that the damper is much more effective with an elastic

damper-shaft than it is with a rigid shaft as in the first case. When the damper flywheel has one-fifth the moment of inertia of the system being damped, and when the damper-shaft is one-fifth as stiff as the main shaft, the analog method gives results as shown in curve *B*, Fig. 2. To make due allowance for the light weight of the damper, all ordinates have been multiplied by the ratio of moments of inertia (one-fifth); this makes the curves comparable to those of Den Hartog and Ormondroyd if a factor of $1/\pi$ is introduced into the vertical scale.

Since the primary purpose of this article is to point out the merits of the electrical means of solution, no particular attention is given to the mechanical merits of this new design of damper. Certain relationships were discovered by study of the analog, however, which seem worthy of mention. The "elastic damper-shaft" is not in the nature of a spring, for a shaft one-fifth to one-tenth as rigid as the main shaft of a Diesel engine is needed. The torque on the damper-shaft never can exceed T, the slip torque of the damper, and the shaft may be designed accordingly. The smaller the damper, the more effective will be a proper damper-shaft in improving its performance.

In conclusion, the principal advantages to be gained by the use of the electric analog are that (1) it shows the extent to which an approximate mathematical solution may be trusted, and (2) it offers a new method of approach to designers of dampers and other frictional devices. It is particularly useful in the study of new designs and at present there appears no limit to the possible applications of the method.

Vertical Networks in Metropolitan Office Buildings

BY A. H. KEHOE[1] and BASSETT JONES[2]
Fellow, A. I. E. E. Fellow, A. I. E. E.

Adoption in New York City

A TYPE of a-c. distribution known as vertical network service has been adopted for the supply of twenty-two large buildings in the Borough of Manhattan, New York City, from the distribution system of The United Electric Light and Power Company. This kind of installation was first offered by companies comprising the New York Edison System in December 1928, and at that time, the arrangement was accepted by the owners of the Irving Trust Company building at 1 Wall Street as suitable for the electric supply to that building. Similar arrangements were made almost simultaneously with several other buildings which at that time either were being planned or were under construction. Since then, this type of service has been adopted for practically all new buildings in New York City of more than forty stories, as well as for some lower structures. A few high buildings of the tower type have been built having comparatively small floor areas in which case there is slight advantage in this type of service.

On January 1, 1931, there were in Manhattan ten completed buildings having this type of service, seven were under construction, and five were in the planning stage of development. The accompanying Table contains detail information regarding these installations. The arrangement in general may be described as an adaptation of the multiple-feed secondary a-c. network to the supply of tall or large buildings. Fig. 1 shows typical examples of street and building network supply systems.

High-Tension Supply Above the Street Level

Alternating-current distribution for densely-loaded urban areas in recent years has been made multiple-feed network in many localities. The first installation of this type in New York City was completed in 1922. This arrangement gives an economical and reliable system which satisfies the electrical distribution requirements of most urban districts. In many cities networks are of the three-phase, four-wire type, supplied at 120/208 volts. The distribution from the distribution transformers over which the energy at such voltages can be economically transmitted is definitely limited if both a reasonable amount of copper for distribution and satisfactory regulation are to obtain.

In a number of cities, there has been a decided tendency in recent years to increase the height of buildings; this has resulted in an increase of total floor area many times the ground area, with a corresponding increase in the electrical density of distribution. While in these large buildings the electrical-load density per sq. ft. of the floor area has increased somewhat, the percentage increase, of course, is much less than the increase in load density based on ground area.

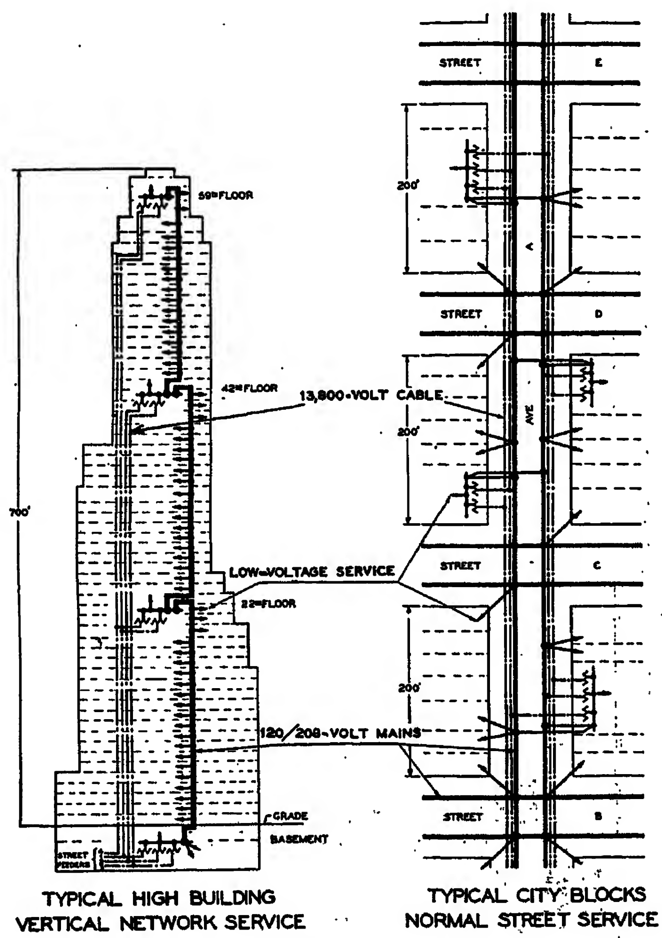

Fig. 1

While this load can be considered as a dense load at the street level, yet the area may be so located that it is cheaper in over-all costs to supply it from distribution transformer capacity in the upper floors of the building than to distribute the energy at 120/208 volts from the street distribution system on which the building faces. Thus it might be considered that the utility has been provided with the equivalent of a large addition to its territory to be supplied. This will be readily under-

1. The United Electric Light and Power Co., New York, N. Y.
2. Consulting Engineer, New York City.
Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

Date: 1-1-31

VERTICAL AND HORIZONTAL NETWORK SERVICES

Name and type of building	Stories	Gross floor area (sq. ft.)	Total feeders supplying service 13.8 kv.	Total feeders supplying service 3 kv.	Vault location (floor)	No. of banks	Size of banks (kva.)	Initial capacity in vault (kva.)	Character of inter-vault ties	Terminal points of inter-vault ties	Sizes of inter-vault ties	Load and estimated demand	Status of installation
CHRYSLER	77 & Tower	1,160,000	5	3	Base	4	300	1,200	Non-fused	St. N. W.-Base	7-4/0	Light =2,460 kw.	Complete
(Office)					30th	4	300	1,200	None	Base-30th	None	Power =3,378 hp.	Complete
					60th	4	150	600	None	30th-60th	None	Demand =2,500 kw.	Complete
					74th								Held pending flood-light development
BANK OF MANHATTAN (Bank & Office)	72 & Tower	800,000	5		Base	5	300	1,500	Non-fused	St. N. W.-Base	7-4/0	Light =3,241 kw	Complete
					15th	5	150	750	None	Base-15th	None	Power =3,765 hp	Complete
					65th	5	150	750	None	15th-65th	None	Demand =2,000 kw	Complete
DAILY NEWS	36	723,500	4		Base	4	150	600	Non-fused	St. N. W.-Base	2(7-4/0)	Light =1,115 kw	Complete
					25th	4	150	600	None	Base-25th	None	Power =1,292 hp Demand = 900 kw.	Complete
(Office and Printing Plant)	9	314,090	5		Base	5	300	1,500	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light = 639 kw Power =2,782 hp.	Complete
					2nd	4	300	1,200	Fused Non-tapped	Base-2nd	2(7-4/0)	Demand =1,500 kw	Complete
DOWNTOWN CLUB	42	520,154	4		Base	4	150	600	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light =1,043 kw Power =1,117 hp.	Complete
(Club House)					19th	4	150	600	None	Base-19th	None	Demand = 805 kw	Complete
IRVING TRUST CO.	50	500,000	4		Base	4	150	600	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light =1,379 kw Power =4,047 hp.	Complete
					21st	3	300	900	Fused Tapped	Base-21st	4(4-300 MCM V.C.)	Demand =1,500 kw	Complete
(Bank & Office)					37th	3	300	900	Fused Tapped	21st-37th	4(4-300 MCM V.C.)		Complete
BARBIZAN PLAZA	40	460,000	3		Base	3	150	450	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light = 742 kw Power =1,120 hp.	Complete
(Hotel)					37th	3	150	450	None	Base-37th	None	Demand = 645 kw	Complete
SALMON TOWERS	60	649,403	4		Base	3	300	900	Non-fused	St. N. W.-Base	7-4/0	Light = 974 kw Power =1,928 hp.	Complete
					22nd	2	150	300	Fused Tapped	Base-22nd	2(4-300 MCM V.C.) 1(4-400 MCM V.C.)	Demand = 940 kw	Complete
					42nd	2	150	300	Fused Tapped	22nd-42nd	3(4-250 MCM V.C.)		Complete
(Office)					59th	3	150	300	Fused Tapped	42nd-59th	3(4-3/0 V.C.)		Complete
	38	460,000	4		Base	3	150	450	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light =1,170 kw Power =1,208 hp.	Complete
					19th	2	150	300	Fused Tapped	Base-19th	3(4-500 MCM)	Demand = 750 kw	Complete
(Office)					38th	3	150	450	Fused Tapped	19th-38th	3(4-500 MCM)		Complete
LEFCOURT	43	703,000	4		Base	2	150	300	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light =1,214 kw Power = 770 hp.	Complete
					22nd	2	300	600	Fused Tapped	Base-22nd	2(11-4/0)	Demand = 522 kw	One bank cut in
(Apt. Hotel)					42nd	2	150	300	Fused Tapped	22nd-42nd	2(8-4/0)		Complete
	30	330,000	4		1st	1	300	300	Non-fused to St. N. W.	St. N. W.-1st	7-4/0	Light = 650 kw Power =1,090 hp.	Complete
(Office)					21st	3	300	900	Fused Non-tapped	1st-21st	4-600 MCM	Demand = 630 kw	Under construction
FARMER'S LOAN & TRUST CO.	56	863,821	4		Base	4	300	1,200	Non-fused to St. N. W.	St. N. W.-Base	7-350 MCM	Light =1,296 kw Power =4,201 hp.	Complete
					21st	4	300	1,200	Fused Non-tapped	Base-21st	4-500 MCM	Demand =1,780 kw	Under construction
(Bank & Office)					54th	3	300	900	Fused Non-tapped	31st-54th	4-500 MCM		Under construction
	37	800,000	3		Base	2	450	900	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Light =1,322 kw Power =1,430 hp.	One bank cut in
(Office & Loft)					36th	3	300	900	Fused Non-tapped	Base-36th	2(3-800 MCM) 2(1-400 MCM)	Demand =1,110 kw	Two banks cut in
CONTINENTAL	44	335,545	4		Base	1	450	450	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Light = 503 kw Power =1,138 hp.	Complete
(Office)					43rd	3	300	900	Fused Non-tapped	Base-43rd	4-500 MCM	Demand = 583 kw	Two banks cut in
NELSON TOWERS	48	477,720	3		Base	3	300	900	Non-fused to St. N. W.	St. N. W.-Base	1(7-4/0) 1(7-2/0)	Light = 715 kw Power =1,420 hp.	Complete
(Office & Loft)					48th	3	300	900	Fused Non-tapped	Base-48th	4-500 MCM	Demand = 715 kw	Complete

Date: 1-1-31

VERTICAL AND HORIZONTAL NETWORK SERVICES—*Continued*

Name and type of building	Stories	Gross floor area (sq. ft.)	Total feeders supplying service 13.8 kv.	Total feeders supplying service 3 kv.	Vault location (floor)	No. of banks	Size of banks (kva.)	Initial capacity in vault (kva.)	Character of inter-vault ties	Terminal points of inter-vault ties	Sizes of inter-vault ties	Load and estimated demand	Status of installation
R. C. A.	50	462,238	3		Base	3	150	450	Non-fused to St. N. W.	St. N. W.-Base	7-350 MCM	Light = 693 kw. Power =1,577 hp.	Two Banks cut in
					22nd	3	150	450	Fused Non-tapped	Base-22nd	4-500 MCM	Demand = 727 kw.	Under construction
(Bank & Office)					49th	3	150	450	Fused Non-tapped	22nd-49th	4-500 MCM		Under construction
Empire State	85 & Tower	2,754,180	5		Base	5	600	3,000	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Light =4,431 kw. Power =9,593 hp.	4 Banks cut in
					41st	4	600	2,400	Fused Non-tapped	Base-41st	4-500 MCM	Demand =5,031 kw.	Complete
(Office)					84th	4	600	2,400	Fused Non-tapped	41st-84th	4-500 MCM		Complete
Stone & Webster	Initial 24 Ultimate 54	Initial: 350,212 Ultimate: 501,012	Initial 3 Ultimate 4		Base	1	450	450	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Initial: Light = 525 kw.	Under construction
					24th	2	300	600	Non-fused Non-tapped	Base-24th	2(7-4/0)	Power = 914 hp. Demand = 407 kw.	Under construction
					52nd	1	300	300	Non-fused Non-tapped	24th-52nd	3(7-4/0)	Ultimate: Light = 752 kw. Power =1,270 hp. Demand = 637 kw.	Planning
(Office)													
River House	27	481,000	3		Base	1	450	450	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Light = 633 kw. Power = 523 hp.	Planning
(Club & Apartment)					15th	2	300	600	Non-fused Non-tapped	Base-15th	2(7-4/0)	Demand = 371 kw.	Planning
City Service	63	1,046,000	4		Base	3	450	1,350	Non-fused to St. N. W.	Cedar St. N. W.-Base Pine St. N. W.-Base	2(7-350 MCM) 2(7-350 MCM)	Light =1,345 kw. Power =4,725 hp. Demand =1,963 kw.	Planning
					18th	3	300	900	Fused Tapped	Base-18th	2 Tubular Busses		Planning
					31st	3	300	900	Fused Tapped	18th-31st	1 Tubular Bus		Planning
(Office)					61st	3	300	900	Fused Tapped	31st-61st	1 Tubular Bus		Planning
Port of N. Y. Authority Inland Stat. No. 1 (Offices & Stores) & Loft & (Warehouse)	15	Offices & Stores, 8th & 9th Aves. = 273,966 Loft Mfg. = 997,767 Warehouse = 997,767 Total = 2,269,500			Base (8th Av.)	0	0	Sec. Ser. Pt.	Non-fused to St. N. W.	St. N. W.-Sec. Ser. Pt.	4(3-500 MCM) (1-4/0)	Light =2,258 kw. Power =7,775 hp.	Planning
					Roof (8th Av.)				Non-fused Non-tapped	Sec. Ser. Pt. Roof (8th Av.)		Demand =4,147 kw.	Planning
					Base (center)				Non-fused to St. N. W.	St. N. W. Base (center)	4(3-500 MCM) (1-4/0)		Planning
					Roof (center)				Non-fused Non-tapped	Roof (c)-Base (c) Roof (c)-Roof (8th) Roof (c)-Roof (9th)			Planning
					Base (9th Av.)	0	0	Sec. Ser. Pt.	Non-fused to St. N. W.	St. N. W.-Sec. Ser. Pt.	4(3-500 MCM) (1-4/0)		Planning
					Roof (9th Av.)				Non-fused Non-tapped	Sec. Ser. Pt. Roof (9th Ave.)			Planning
Metropolitan Life	(Unit No. 1) 29	1,012,564			Base (4th Av.)				Non-fused to St. N. W.	St. N. W.-Base (4th)	4(3-500 MCM) (1-4/0)	Light =1,519 kw. Power =9,455 hp. Misc. = 35 kw.	Planning
					15th (Part 1)				Non-fused Non-tapped	Base-15th (Part 1)		Demand =3,475 kw.	Planning
	(Unit No. 2) 84	2,144,165			Base (Mad. Ave.)				Non-fused to St. N. W.	St. N. W.-Base (Mad.) Base (4th)-Base (Mad.)			Planning; Unit No. 2 Will be postponed approx.
					15th (Part 2)				Non-fused Non-tapped	Base (Mad.)-15th (P2) 15th (P1)-15th (P2)			4 years
					45th				Non-fused Non-tapped	15th (P2)-45th			" "
(Office)					80th				Non-fused Non-tapped	45th-80th			" "
N. Y. C. R. R. St. John's Park Freight Terminal	12	3,529,000			3rd (Clarkson St.)				Non-fused to St. N. W. Non-fused Non-tapped	St. N. W.-3rd (Clar.) 3rd (NC)-3rd (Clar.)	4(3-500MCM;1-4/0)	Light =1,484 kw. Power =5,666 hp. Demand =2,035 kw.	Planning
					3rd (N. Cent.)				Non-fused to St. N. W. Non-fused Non-tapped	St. N. W.-3rd (N. C.) 3rd (S. C.)-3rd (N. C.)	4(3-500 MCM; 1-4/0)		Planning
					3rd (So. Cent.)				Non-fused to St. N. W. Non-fused Non-tapped	St. N. W.-3rd (S. C.) 3rd (Sp.)-3rd (S. C.)	4(3-500 MCM; 1-4/0)		Planning
(Freight Terminal)					3rd (Spring St.)				Non-fused to St. N. W. Non-fused Non-tapped	St. N. W.-3rd (Sp.)	4(3-500 MCM-1-4/0)		Planning

stood when it is realized that building heights of from 500 to 1,000 ft. above the street are becoming commonplace and such buildings require large banks of elevators, the motors of which are located in the upper stories of the building. Some of these elevators have motors ranging in size from 100 to 150 hp. for the higher buildings, with short period demands several times these values.

With such dense loads at an appreciable distance from the street level, it is found to be definitely economical in over-all costs to distribute with multiple high-voltage supplies feeding a low-voltage network throughout the building. This system is identical in principle with the street network system.

Early Considerations, Economic Aspects

The introduction of this type of service in New York City with transformer vaults on the upper floors of buildings did not mark the first use of transformers above the street level in this territory. A few installations had previously been made on the radial supply system with transformers located as high as the sixth floor.

The tapped riser type of building wiring was first considered in 1920 when an economic study was made of a 50 story building for the New York Telephone Co. at 140 West Street. This study contemplated the installation of supply transformers on the floors of the building where the elevator machine rooms were to be located and a low tension tie between transformer rooms was to be tapped to supply load on each floor. A considerable saving was estimated for this system of wiring over the old scheme wherein all building feeders originate at the house switchboard located in the basement, but certain restrictions made it impossible to make use of this type of installation at that time.

In April 1928, comparative estimates of building wiring costs for the Irving Trust Company Building, 1 Wall Street showed a saving of $35,000, for the vertical network system as compared with the system in which the service transformers are all located in the basement. However, some modifications in plans were made at the request of the local inspection authority and all of the above estimated saving was not realized.

In the Empire State Building, the network system has been applied to the individual floors. Four-wire 120/208-volt feeders from one or more centers of distribution, where they are fused, feed into a four-wire main circling the floor. Unfused branch circuits are tapped off this main to feed one or more bays.

With the radial system of building wiring, enough copper must be run from the house switchboard to each load group (say three floors, for lighting, an elevator bank, or other motor group) to carry the maximum load of that group. With the network system using a tie bus between transformer banks, the advantage of load diversity is realized in reducing the amount of copper required. The utilization factor can be worked out by the known probability factors for the case (see "Power Calculations for Elevators by the Method of Probabilities," *G. E. Review*, October-November 1930). This method applies not only to elevators but is applicable also to variable loads of any kind for which an "operating factor" can be determined. The fifteen-minute demand (r. m. s. value of the load over this period) on the usual elevator bank is about 35 per cent of the connected load. With the old radial system of wiring, a separate feeder or duplicate feeders must be run to each such bank having at least the load capacity of the fifteen-minute demand of each bank.

Ownership of Service Installations and Supply Feeders

Since a distribution installation in a building cannot be considered to have the permanency of a street system, it being subject to the whims of the building owners, and since it cannot be utilized for any service other than for that particular building, there is naturally a distinct limitation to the amount of investment that the utility company can make for such an installation. On the other hand, the building owner realizes a definitite saving in building wiring if the vertical distribution network is used instead of distributing throughout the building from street services. It is thus reasonable to have a different division of expense for vertical distribution than that which would be used in the case of general distribution throughout a city.

The division used in New York is to have the utility company supply, own, and maintain the high-voltage cables and the distribution transformers with their associated network equipment. These are placed in position by, and at the expense of, the building owners in ducts and vaults supplied as part of the building installation. All low-voltage distribution conductors and ducts are owned and installed by the building owners except the 120/208-volt a-c. network ties from the street mains to the first distribution point within the building. While the building owners install the transformers and high-voltage cables, all connections are made by the utility company. This division makes it possible for the utility company to recover all its equipment in case the building is abandoned, and does not materially change its costs from those that would be incurred in supplying the building direct from the street system.

The cost of the wiring installation in the case of high buildings is considerably reduced. Certain expense, of course, is incurred in using what might otherwise be usable in building space for vaults for distribution transformers but in most cases space yielding little of value can be used for this purpose.

Low-Voltage Tie Connections Between Transformer Vaults

If all conditions of supply for vertical distribution were the same as those found in the common network system, the installation would be unusual only from

the standpoint of establishing the division of ownership and maintenance between the building owner and the utility. However, as the types of installations made to date will indicate, there are numerous conditions which cause these services to differ. The vertical networks used so far can be classified according to the arrangement of the secondary connections within the building; such as isolated networks without connecting ties, networks with untapped ties, and networks with

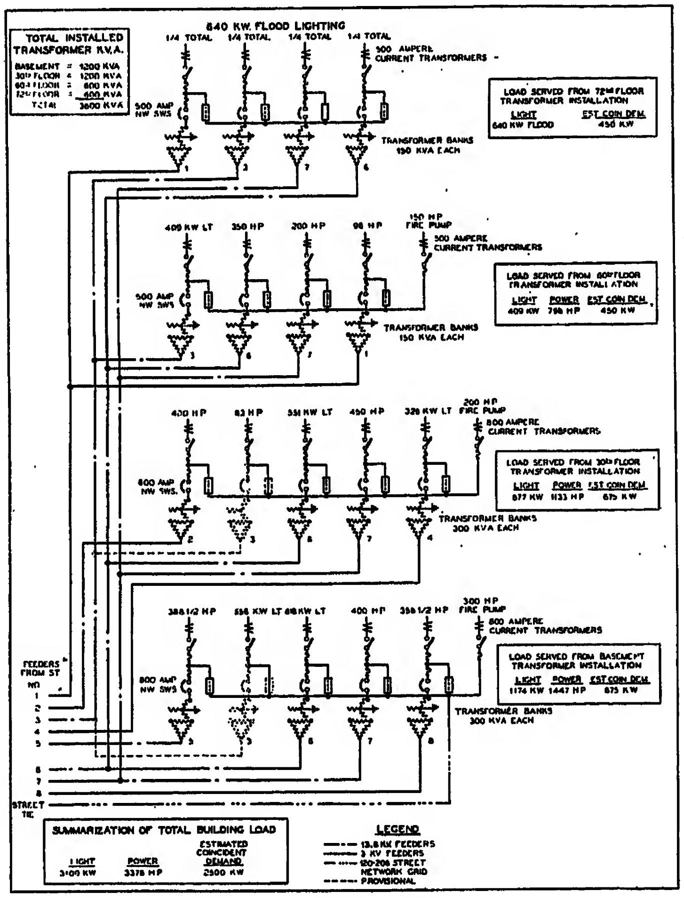

Fig. 2

tapped ties. (Figs. 2 and 3.) In the isolated or spot network, which also has been called a junior network, the energy used at any one point in the building is supplied solely from a plurality of step-down transformers at a single location as there are no secondary ties to any other network. The network with untapped low-voltage ties is similar, except in this case the ties do connect to other sources and there does not necessarily have to be a multiplicity of step-down transformers at the location as the ties form a network with other sources. However, in this case, service has to be supplied from the one location over special utilization feeders in the same manner as with a spot network. In the case of the network with tapped ties, the ties are used to distribute load to the several floors between transformer locations. Each of these types of installation has been used to meet conditions which existed at the time they were planned. None of the tapped ties has been installed without fuses and there is little incentive for the owner to install the secondary ties in conduit buried in concrete when he installs fuses which make it unnecessary to surround the conduit with concrete. Recent installations are being made with untapped and unfused ties installed in conduit surrounded by two or more inches of concrete; and in such cases, the building service is supplied locally from each step-down location.

Future Considerations

One of the advantages obtained with vertical network service is the ability to increase the electrical capacity in the building in the future without expensive reconstruction of the original wiring. The ability to double, or triple the supply capacity, if needs be, results from the fact that each of the several high-voltage cables supplying the building is required for mechanical

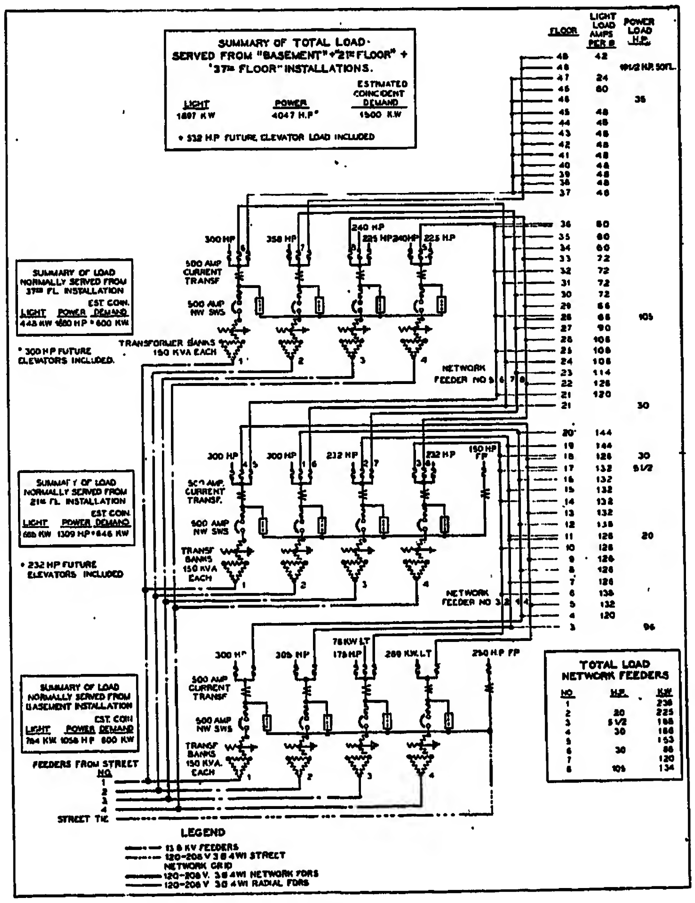

Fig. 3

reasons to have considerably more carrying capacity than is needed to serve any of the buildings contemplated up to the present time. Of course, the installation of additional distribution transformers would be required either at existing vaults or at new locations nearer to the utilization point than the present vaults; and, in addition, there would be required the necessary utilization wiring to deliver the new energy to the places in the building where it would be used. Such

a procedure is much simpler and less expensive than would be required if the increase were to be supplied from the street network.

All vertical high-voltage feeder installations planned to date have used a dry type of three-conductor shielded cable with an outside wrapping of steel wires with which the cable is held up at the top of its run. It is possible, and obviously desirable, that future developments should make it feasible to use standard types of high-voltage cables for this purpose.

Considering the operating experience in New York City covering a period of about two years, the vertical network installations being made, are regarded to be more than adequate for the service requirements now foreseen. Improvements in methods and in equipment which will simplify the wiring installation and reduce its cost may be expected in the future as inspection authorities, architects, and supply company engineers become better acquainted with the special requirements of this type of installation.

Effects of Electric Shock—II

BY W. B. KOUWENHOVEN* Member, A. I. E. E. and O. R. LANGWORTHY† Non-member

INTRODUCTION

IN a paper[1] published in January 1930 the authors discussed the results of a series of experiments on electric shock in which the electrodes were applied to the head and tail of animals. In those tests rats were used as the experimental animal and continuous and alternating voltage shocks at 110, 220, 500 and 1,000 volts were given. At each voltage the duration of the contact was increased until it was impossible to resuscitate the subject by means of artificial respiration.

The prior set of experiments brought to light certain facts which apply to the present series and, therefore, these will be recapitulated here, namely:

1. That a 1,000-volt continuous-current circuit was more dangerous to rats than an alternating circuit of the same effective voltage.
2. That a large rat withstood a greater shock than a small rat and still survived.
3. That there was no difference between the sexes in their susceptibility to electric shock.
4. That the a-c. experiments were characterized in many cases by paralysis of the hind legs, caused by hemorrhages in the spinal cord.

It is well recognized that a factor of importance in cases of electric injury is the points on the body where contact is made with the circuit. The location of the contacts determines largely the pathway of the current flow. Once inside the body the current spreads out in a more or less fusiform shape, but it is difficult to believe that the current density is high in organs distant from the main path. For example, in man, if the contact with the circuit is made with one arm and a leg, it is scarcely possible that considerable current will pass through the brain.

With the electrodes on the head and tail of an animal, the brain lies directly in the current path. In the previous study it was found that the brain appeared to be injured readily by the passage of an electric current. After the death of the rats Dr. Langworthy[2] found evidences of macroscopic and microscopic hemorrhages in the brain and also of considerable damage to nerve cells in many of the rats. In legal electrocution one electrode is placed upon the skull.

In industrial accidents, however, the current rarely passes through the head. In the 479 cases discussed by Maclachlan[3] the path of the current was commonly from the hands to the feet. The present series of experiments was undertaken to study the influence of the current path upon the effects produced by an electric shock. Are injuries to brain cells produced when the current does not pass through the head? Rats again were chosen as animals for study, both because of our knowledge of their behavior and because a rat seldom, if ever, dies of damage to heart after an electric injury, but rather from respiratory failure.

In the present series, 102 rats were shocked with a 1,000-volt circuit, half with alternating current and half with direct current. In all cases the duration of the contact was two seconds. The position of the electrodes was varied in these experiments. A few tests were made with the electrodes on the head and tail of the animals to check the results of the prior series. Other current paths were, right fore-leg to tail, left fore-leg to tail, right fore-leg to left hind-leg, left fore-leg to left hind-leg, right fore-leg to left fore-leg, and right hind-leg to left hind-leg.

TECHNIQUE

Preparatory to the shock the rats were given ether until they were quiet. The points of contact were moistened thoroughly with saline solution and the electrodes, voltmeter clips filed smooth, were attached. When connection was made to the head, a special electrode was employed which made contact only on the upper surface of the skull, and did not interfere with breathing.

After the electrodes were in place, the rat was allowed to recover from the effects of the anaesthetic before the closure of the switch in the high-tension circuit. The circuit was held closed for two seconds and the current flowing through the animal was recorded. Time was measured with a stop watch.

Immediately following the shock, the animal was removed from the circuit. If there was no evidence of breathing, artificial respiration was applied. This consisted of the application of digital pressure upon the chest, reproducing as far as possible the Schaefer prone-pressure method in man. The tongue was pulled out, the vagus nerves in the neck were stimulated by stroking with the fingers, and efforts were made to keep the respiratory tract free from mucus.

OBSERVATIONS

Following the a-c. injuries, the chest of the animal was fully expanded. This seemed to be an important aid to the resumption of spontaneous breathing or to adequate artificial respiration. On the other hand, after contact with the d-c. circuit the chest was collapsed and it was difficult to get air into the lungs. The a-c. circuit produced a very strong contraction of muscula-

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1. For references see Bibliography.

ture of the body and this in the cases of males was accompanied almost invariably by an emission. Clonic movements of the muscles of the leg were observed in many of the a-c. injuries. They were probably produced by irritation of the nerve cells due to asphyxiation. Bleeding of the conjunctiva occurred in many cases.

The results are arranged in groups in accordance with the location of the electrodes upon the surface of the body. The data for the principal groups are stated in tabular form. The tables give the animal number, sex, weight in grams, the type of circuit, the current, whether or not artificial respiration was required, and the condition of the animal following the injury.

Electrodes on Head and Tail

Six rats in the present series and ten in the prior series were subjected to a two-second contact with a 1,000-volt a-c. circuit with the electrodes on the head and tail. As may be seen from Table I, artificial respiration was applied in 88 per cent of the cases, and five, or 31 per cent, recovered. The balance died or were paralyzed by the shock. Three of the five that recovered were normal. After breathing was established it was rapid, shallow, and somewhat irregular for several hours following the injury. Later it became normal.

Only one of the six rats in this series was paralyzed as against five of the ten in the prior series. In fact in this present group paralysis was less common and fewer hemorrhages in the spinal cord were found than in the tests of a year ago. No explanation for this discrepancy can be given.

In the tests previously published no rat survived a contact of one second or over with a d-c. circuit of 1,000 volts. Therefore, in this series only two animals were given a two-second shock at this voltage. Neither of them survived, although their hearts beat strongly for several minutes following the contact. Autopsy showed only few abnormalities. The microscopic examination of the brains of the rats of the prior series revealed, however, that the current had caused nerve-cell injuries incompatible with life.

The observations in the present group check those of the prior experiments where the electrodes were applied to the head and tail of the animal. It is clearly evident that contact with a 1,000-volt d-c. circuit is much more deadly to a rat than contact with an a-c. circuit of the same effective voltage. In this series, artificial respiration was applied to all of the animals tested with continuous current, and 100 per cent of them died. In the a-c. tests, however, artificial respiration was needed in only 88 per cent of the cases; 31 per cent of the animals died; 38 per cent were paralyzed; and 31 per cent recovered from the injury.

Electrodes on Right Fore-Leg and Tail

Twelve rats were given a two-second shock with 1,000 volts alternating current the electrodes being on the right fore-leg and tail. Five of these rats breathed as soon as the circuit was opened, and artificial respiration was required in the other seven cases. Death occurred either immediately or within a few minutes in 42 per cent of the cases, while three rats were paralyzed. One of the paralyzed rats died four hours after the injury, another nine hours, but the third survived until killed preparatory to autopsy.

This group was of particular interest in emphasizing the fact that an animal was not necessarily on the road to recovery when breathing was first started. This phenomena is mentioned often in records of human electric shock. Breathing was started in three of the rats, Nos. 1, 2, and 8 of Table II, only to cease after a few minutes. A second application of artificial respiration saved only one. Several of the rats in this group were hyper-irritable to all sensory stimuli for some minutes following the injury. Bleeding from the eyes was a common occurrence. The paralyzed rat that

TABLE I—TWO-SECOND SHOCK—1,000 VOLTS ELECTRODES—HEAD AND TAIL

Rat No.	Circuit	Sex	Wt. Gms.	Current Milliamp.	Artificial Respiration Required	Result	Remarks
1	A. C.	M	240	700	Yes	Recovered	Breathed rapidly
2	A. C.	F	206	840	Yes	Recovered	Breathed rapidly
3	A. C.	M	155	800	Yes	Died	Never breathed
4	A. C.	M	140	720	Yes	Died	Never breathed
5	A. C.	M	130	750	Yes	Died	Few breaths
6	A. C.	M	238	740	Yes	Paralyzed	
7	A. C.				No	Paralyzed	Prior series[1]
8	A. C.	F	230		Yes	Paralyzed	Prior series[1]
9	A. C.			650	Yes	Paralyzed	Prior series[1]
10	A. C.	F	190	800	Yes	Recovered	Prior series[1]
11	A. C.	F	230	830	No	Paralyzed	Prior series[1]
12	A. C.	F	240	900	Yes	Recovered	Prior series[1]
13	A. C.	M	200	950	Yes	Died	Prior series[1]
14	A. C.	F	270	980	Yes	Paralyzed	Prior series[1]
15	A. C.	F	180	1,000	Yes	Died	Prior series[1]
16	A. C.	F	210	1,000	Yes	Recovered	Prior series[1]
17	D. C.	F	162	750	Yes	Died	Never breathed
18	D. C.	F	131	700	Yes	Died	Never breathed
19	D. C.	M		730	Yes	Died	Prior series[1]

TABLE II—TWO-SECOND SHOCK—1,000 VOLTS ELECTRODES—RIGHT FORE-LEG AND TAIL

Rat No.	Circuit	Sex	Wt. Gms.	Current Milliamp.	Artificial Respiration Required	Result	Remarks
1	A. C.	M	130	740	Yes	Died	Few breaths
2	A. C.	F	180	750	Yes	Recovered	
3	A. C.	F	170	830	Yes	Paralyzed	Died 9 hr. later
4	A. C.	F	130	750	Yes	Died	No breaths
5	A. C.		132	760	No	Recovered	Hyper-irritable
6	A. C.	M	237	840	No	Recovered	Breathed at once
7	A. C.	M	228	800	No	Paralyzed	Hyper-sensitive
8	A. C.	M	137	680	Yes	Died	Started breathing, died
9	A. C.	M	266	700	Yes	Died	Cyanosis
10	A. C.	M	139	600	No	Paralyzed	Died 4 hr. later
11	A. C.	F	208	720	Yes	Died	Died 5 min. after
12	A. C.	M	134	720	No	Recovered	Breathed at once
13	D. C.	F	80	600	No	Recovered	Breathed at once
14	D. C.	M	124	950	No	Recovered	Breathed at once
15	D. C.	M	58	500	No	Recovered	Breathed at once
16	D. C.	M	40	950	Yes	Died	Started breathing, stopped
17	D. C.			1,000	No	Paralyzed	Breathed at once
18	D. C.	F	156	850	No	Paralyzed	Breathed at once
19	D. C.	F	110	700	No	Paralyzed	Breathed at once
20	D. C.	F	125	900	Yes	Paralyzed	
21	D. C.	F	68	500	Yes	Died	Few gasps
22	D. C.	F	201	950	Yes	Died	Never breathed
23	D. C.	M	145	840	Yes	Recovered	Breathed almost at once
24	D. C.	F	135	880	Yes	Recovered	Breathed almost at once

survived had a hemorrhage in the spinal cord and there was no chance of its permanent recovery.

With the alternating current flowing through the animals from right fore-leg to tail, four of the rats, 33 per cent, recovered; 42 per cent died; and 25 per cent were paralyzed. The injury was slightly less severe than when the brain formed part of the current path as in the previous series.

An equal number of rats was tested on the continuous-current circuit with the electrodes on the right fore-leg and tail. In this test the brain did not lie directly in the current path and the results were vastly different from those of Table I where one contact was made with the top of the skull. Here six of the rats breathed spontaneously when the circuit was opened, whereas none were saved in the first series even after long applications of artificial respiration. There was only one rat in this group of twelve that did not attempt to breath after the injury.

Artificial respiration was applied in 50 per cent of the d-c. cases; five rats or 42 per cent recovered; 25 per cent died; and 33 per cent were paralyzed. These figures do not, however, tell the whole story. The burns on the d-c. circuit were much more severe than those observed with the alternating. Because of these burns, it was not possible to keep any of the animals that survived alive for a period longer than three days following the d-c. shock. On the other hand, any rat that survived an a-c. shock without paralysis appeared quite normal twenty-four hours later. They ate, drank, and were active. It is also interesting to note that in this d-c. group there were two cases, No. 16 and 21—Table II, where breathing was started, only to stop again. In both of these instances a second application of artificial respiration was unsuccessful. From these results it is clearly evident that in cases where the continuous current path does not include the brain there is at least a fifty-fifty chance of resuscitating the animal.

Electrodes on Left Fore-Leg and Tail

Twenty-four rats were tested with the electrodes on the left fore-leg and tail, half with alternating and half with continuous current. The results are given in Table III. Under alternating current artificial respiration was applied in 75 per cent of the experiments; 58 per cent recovered, 17 per cent were paralyzed, and 25 per cent died. With direct current, artificial respiration was used in only 58 per cent of the cases; 42 per cent recovered, 33 per cent were paralyzed, and 25 per cent died.

The results of this series also emphasized the fact that breathing will not necessarily continue after having been established either spontaneously or by artificial respiration. The reactions and behavior of the rats in this group were closely identical with those where the current path was from right fore-leg to tail. It was, however, not quite as dangerous to life. The d-c. results here also confirm the importance of eliminating the brain from the current path if any hope of the recovery of the animal is to be entertained.

Electrodes on Both Fore-Legs

Eight rats were tested with the electrodes on the fore-legs, thus involving the heart and lungs directly in the path of the current flow. Four were shocked with alternating, and four with continuous current. Their average weight was 144 grams, and the average current was 1.24 amperes.

Although artificial respiration was tried in every case, no animal survived. This is not strange when one considers the current that flowed for two seconds. The chest was warm to the touch when the circuit was

opened. In only one case, however, was there no heart beat following the shock. It is remarkable that there was any heart beat at all after this injury, and that two of the rats attempted to breathe following the a-c. contact and two following the d-c. contact. There is no possibility of survival with this current path.

Electrodes on Both Hind-Legs

Fourteen rats were tested at 1,000 volts with electrodes attached to the hind legs, seven on an a-c. circuit and seven on a d-c. circuit. Their average weight was 127 grams and the average current that flowed for two seconds was 1.17 amperes.

Only one rat died as a result of the two-second contact with the a-c. circuit and two from that of the d-c. circuit, when the circuit path involved principally the caudal region of the body. The rat that died after a-c. contact had started breathing spontaneously as soon as the circuit was opened. He was considered as recovered and had been placed back in the cage; ten minutes later he was found dead, probably from neglect.

Artificial respiration was not applied to any of the seven rats that were injured with alternating current. All breathed spontaneously and 86 per cent recovered completely. They were all very active and lively within a few minutes following the shock. One, or 14 per cent, died. The authors believe that his life could have been saved had artificial respiration been applied at the proper time. The a-c. shocks were accompanied by an emission in the case of the males.

Following the d-c. shock, five of the animals breathed at once; artificial respiration was applied in but two cases, 29 per cent. These two animals died after a few attempts at breathing. The other five, 71 per cent, recovered and in a few minutes became very active.

This series demonstrates clearly the fact that when none of the vital organs lie directly in the path of the current the electrical injury usually is small.

Electrodes on Fore-Leg and Hind-Leg

The effects produced by the flow of current from the right fore-leg to the left hind-leg were studied in eight rats; from the left fore-leg to the left hind-leg in eight other rats; and in a third group of eight the electrodes were on the left fore-leg and the right hind-leg. Half of these were subjected to the a-c. circuit and the remainder to the continuous-current circuit. The results are given in Table IV.

Not a single rat was permanently saved after contact with the d-c. circuit. One, No. 24, was resuscitated only to die two and a half hours later. In several cases no heart action could be detected and, at most, many of the animals gave only a few gasps. There was greater success in saving those that had been injured with the alternating circuit. In five cases breathing was established by a long application of artificial respiration, one of these rats, however, was paralyzed. They all, including the paralyzed one, became quite active soon after the shock.

The results may be summarized for the current path from fore-leg to hind-leg as follows:

Artificial respiration was applied in every instance, both for the a-c. and continuous-circuits. With the alternating current, 33 per cent of the animals recovered, eight per cent were paralyzed and 59 per cent died. With continuous current 100 per cent of the animals died.

Normally one would expect that there would be little, if any, difference between the effects produced by a current flowing from either fore-leg to tail or by one flowing from either fore-leg to either hind-leg.

TABLE III—TWO-SECOND SHOCK—1,000 VOLTS ELECTRODES—LEFT FORE-LEG AND TAIL

Rat No.	Circuit	Sex	Wt. Gms.	Current Milliamp.	Artificial Respiration Required	Result	Remarks
1	A. C.	M	149	750	Yes	Recovered	
2	A. C.	M	195	900	Yes	Recovered	
3	A. C.	M	155	700	Yes	Died	No breaths
4	A. C.	M	197	730	Yes	Died	No breaths
5	A. C.	M	114	680	Yes	Paralyzed	Fluid from mouth
6	A. C.	M	121	720	No	Recovered	Breathed at once
7	A. C.	M	214	770	No	Recovered	Breathed at once
8	A. C.	M	258	860	No	Died	Died 1 hr. later
9	A. C.	M	104	800	Yes	Recovered	
10	A. C.	M	131	720	Yes	Paralyzed	Died 4 hrs. later
11	A. C.	M	124	800	Yes	Recovered	
12	A. C.	M	115	620	Yes	Recovered	
13	D. C.	F	144	950	Yes	Died	Few gasps
14	D. C.	F	120	750	No	Paralyzed	Breathed at once
15	D. C.	F	175	1,050	No	Paralyzed	Irregular breathing
16	D. C.	F	115	750	No	Paralyzed	Irregular breathing
17	D. C.	F	154	800	No	Paralyzed	Irregular breathing
18	D. C.	F	120	900	Yes	Recovered	
19	D. C.	F	110	700	No	Recovered	Breathed at once
20	D. C.	M	131	850	Yes	Recovered	Died 35 hr. later
21	D. C.	F	135	760	Yes	Recovered	Rapid breathing
23	D. C.	M	160	840	Yes	Recovered	Shallow breathing
23	D. C.	F	95		Yes	Died	Died 20 min. after
24	D. C.	F	85	840	Yes	Died	Never breathed

TABLE IV—TWO-SECOND SHOCK—1,000 VOLTS ELECTRODES—FORE-LEG TO HIND-LEG

Rat No.	Circuit	Sex	Wt. Gms.	Current Milliamp.	Artificial Respiration Required	Result	Remarks
				RIGHT FORE-LEG—LEFT HIND-LEG			
1	A. C.	M	240	1,140	Yes	Died	Started breathing, stopped
2	A. C.	M	145	1,100	Yes	Died	No breaths
3	A. C.	M	143	1,060	Yes	Paralyzed	Active soon
4	A. C.	M	114	1,140	Yes	Recovered	Active soon
5	D. C.	F	122	1,450	Yes	Died	Few gasps
6	D. C.	F	69	1,700	Yes	Died	No heart beat
7	D. C.	F	264	1,700	Yes	Died	No heart beat
8	D. C.	M	127	1,200	Yes	Died	Heart beat
				LEFT FORE-LEG—LEFT HIND-LEG			
9	A. C.	M	54	920	Yes	Died	Fluid from nose
10	A. C.	M	155	1,060	Yes	Died	Fluid from nose
11	A. C.	F	130	680	Yes	Recovered	Clonic movements
12	A. C.	M	125	940	Yes	Died	Few gasps
13	D. C.	M	223	1,300	Yes	Died	Few breaths
14	D. C.	F	135	1,700	Yes	Died	Few breaths
15	D. C.	F	60	1,300	Yes	Died	No heart
16	D. C.	F	105	1,250	Yes	Died	Few gasps
				LEFT FORE-LEG—RIGHT HIND-LEG			
17	A. C.	F	130	1,060	Yes	Died	Never breathed
18	A. C.	M	150	1,110	Yes	Died	Never breathed
19	A. C.	F	102	820	Yes	Recovered	Clonic movements
20	A. C.	M	120	1,000	Yes	Recovered	Clonic movements
21	D. C.	M	95	840	Yes	Died	Never breathed
22	D. C.	F	95	820	Yes	Died	Few gasps
23	D. C.	M	100	1,500	Yes	Died	Fluid from mouth
24	D. C.	F	110	930	Yes	Died	Died 2½ hr. later

Yet in the first group with the current path from fore-leg to tail, 44 per cent of the rats recovered; in the second group, where one electrode was attached to a hind-leg instead of the tail, only 16 per cent were saved. Wherein, then, lies the deadliness of this current path from fore-leg to hind-leg? The authors believe that the high death rate found here is caused by the low resistance path that the animal offers to the current flow when its tail is eliminated from the circuit. The average resistance of the current paths from fore-legs to hind-legs for the 24 rats was less than 70 per cent of that found for the 48 rats (Tables II and III) where the current paths were from either fore-leg to tail. The higher resulting current is believed responsible for the deaths of the animals. A second argument supporting this conclusion is the almost complete absence of serious injury to the rats when the current passed from one hind-leg to the other, even though the current values were high.

Conclusion

These experiments confirm Urquhart's[4] conclusion that when an electric current passes through the brain, a temporary physiological block was produced in the respiratory center, and that spontaneous breathing ceased for a time. He stated that if no serious injury to the heart occurred, adequate artificial respiration might give time for the center in the medulla to recover and for normal breathing to begin. Maclachlan[3] reports in his field notes a case of recovery from shock following eight hours of continuous application of the Schaefer prone-pressure method, which also confirms Urquhart's findings. Urquhart,[5] in a second series of experiments, demonstrated that a temporary block could be produced in the spinal cord or in a nerve trunk by the passage of an electric current.

Langworthy[2] found, by his microscopic examination of sections of the brains of the rats injured by high-voltage electric currents, that in many cases the damage to the cells was such as to be incompatible with life. Microscopic study of the central nervous systems of all of the rats in this present series will be made.

Results clearly demonstrate the presence of a temporary respiratory block in many of the rats. The location of the block, whether in the brain stem or in the vagus nerves, depends upon the current path. Resuscitation is possible provided the damage done does not require a recovery time too long to permit life to be supported in the interval.

The effects produced by the various current paths through the animals are summed up in Table V. From this table and from the results presented, the following conclusions may be drawn:

1. A 1,000-volt continuous-current circuit is more dangerous to rats than an alternating circuit of the same effective voltage, irrespective of the current path.
2. The chances of recovery are best when the brain does not lie directly in the pathway of the current.
3. The injury produced by the current path from left fore-leg to tail is less severe than that caused by the same current flowing from right fore-leg to tail.
4. When the main pathway of the current did not

TABLE V—CONTACT WITH 1,000-VOLT CIRCUIT FOR TWO SECONDS

Circuit	Position of Electrodes		Number Shocked	Per Cent Requiring Artificial Respiration	Per Cent Immediate Recovery	Per Cent Paralyzed	Per Cent Immediate Death
Alternating	Head	Tail	16	88	31	38	31
Continuous			3	100	0	0	100
Alternating	Right fore-leg	Tail	12	58	33	25	42
Continuous			12	50	42	33	25
Alternating	Left fore-leg	Tail	12	75	58	17	25
Continuous			12	58	42	33	25
Alternating	Either fore-leg	Tail	24	67	46	21	33
Continuous			24	54	42	33	25
Alternating	Right fore-leg	Left fore-leg	4	100	0	0	100
Continuous			4	100	0	0	100
Alternating	Right hind-leg	Left hind-leg	7	0	86	0	14
Continuous			7	29	71	0	29
Alternating	Either fore-leg	Either hind-leg	12	100	33	8	59
Continuous			12	100	0	0	100

include the brain, spinal cord or nerves required for respiration, most of the animals breathed at once and were active a few minutes after the injury.

5. The presence of a respiratory block has been demonstrated, and the fact that when natural breathing is first established the animal is not necessarily on the road to complete recovery.

6. The respiratory block is less severe, if not absent, when the current traverses the posterior portion of the body.

7. Burning of the tissues with continuous current is much more severe than occurs for the same effective alternating current.

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